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ECE-205

Exam 2

Winter 2015

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 ________/13

Problem 2 ________/13

Problem 3 ________/20

Problem 4 ________/30

Problem 5 ________/24

Total ____________

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1) (13 points) Fill in the non-shaded part of the following table. You do not need to show any work.

	Linear? (Y/N)	Time Invariant? (Y/N)	BIBO Stable? (Y/N)
$y(t)=\int_{-\infty}^{t+1}\lambda^2 x(\lambda)d\lambda$	Y	N	
$y(t)=3+\int_{-\infty}^{t}e^{-(t-\lambda)}x(\lambda)d\lambda$	N	Y	
$\dot{y}(t)+\cos(t)y(t)=3x(t)$	Y	N	
$y(t)=\dot{x}(t)+2x(t)$	Y	Y	
$y(t)=x(\lvert t\rvert)$	Y	N	Y
$y(t)=\ln(\lvert x(t)\rvert+1)$			Y
$y(t)=\cos(2x(t+1))$			Y

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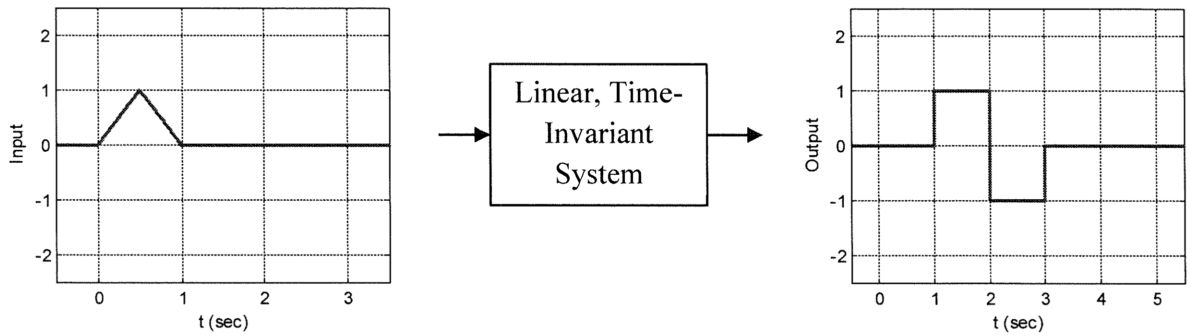
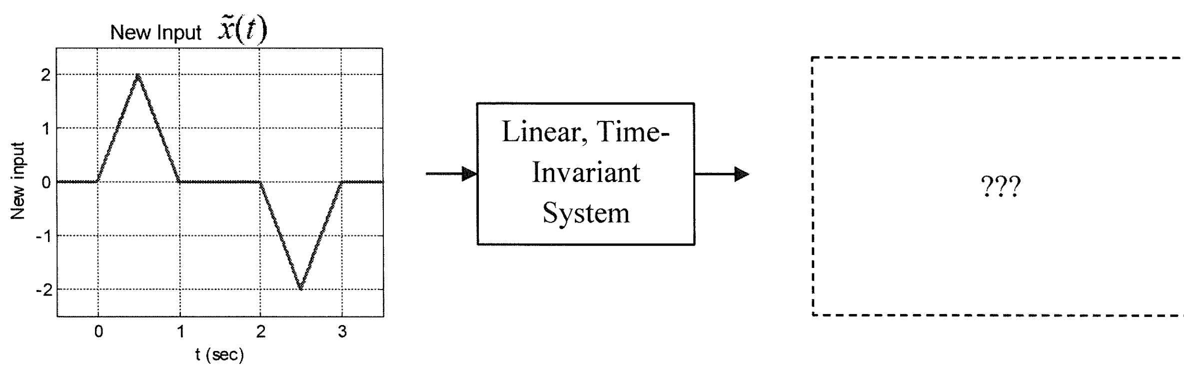
2) (13 points) A linear and time-invariant system with the following input $x(t)$ produces the output $y(t)$ below:

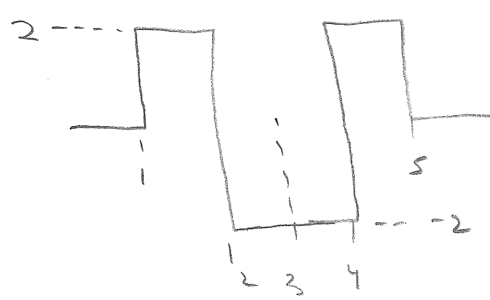


If the following new input $\tilde{x}(t)$ is fed in, sketch the corresponding system output.
(*Hint: Note that* $\tilde{x}(t)$ *is a linear combination of* $x(t)$)



$\tilde{x}(t) = 2x(t) - 2x(t-2)$

$\tilde{y}(t) = 2y(t) - 2y(t-2)$



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3) (20 Points) Simplify the following as much as possible. Be sure to include any necessary step functions.

a) $y(t) = \delta(t-1) * \delta(t+1)$ (Note: $*$ denotes the convolution)

$$y(t) = \int_{-\infty}^{\infty} \delta(t-\lambda-1)\,\delta(\lambda+1)\,d\lambda = \delta(t) \int_{-\infty}^{\infty} \delta(t-\lambda-1)\,d\lambda = \boxed{\delta(t)}$$

b) $y(t) = \int_{-\infty}^{t+1} \lambda \cdot \delta(\lambda - 3)\,d\lambda$ $= \int_{-\infty}^{t+1} 3\,\delta(\lambda-3)\,d\lambda$

$-\infty \quad 3 \quad t+1$

$t+1 > 3$

$t-2 > 0$

$= \boxed{3u(t-2)}$

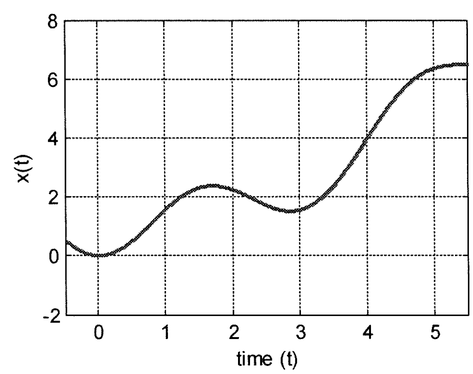
c) $y(t) = \int_{t-1}^{\infty} \delta(\lambda)\delta(t-\lambda)\,d\lambda$ $= \delta(t) \int_{t-1}^{\infty} \delta(\lambda)\,d\lambda$

$t-1 \quad 0 \quad \infty$

$t-1 < 0$

$= \delta(t)\,u(1-t) = \boxed{\delta(t)}$

For the remaining parts (d) and (e), suppose a signal $x(t)$ is given by the following:



d) Determine $x(t)\delta(t-4)$ $\boxed{4\,\delta(t-4)}$

e) For what value(s) of t_0 is $x(t)\delta(t-t_0) = 0$? $\boxed{t_0 = 0}$ since $x(0) = 0$

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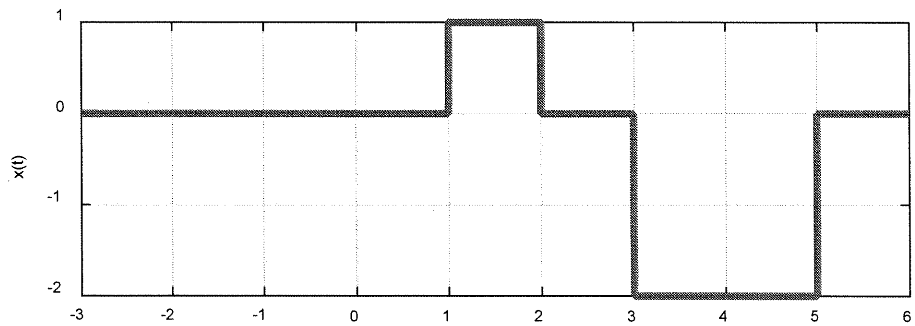
4) (30 points) Consider a linear time invariant system with impulse response given by

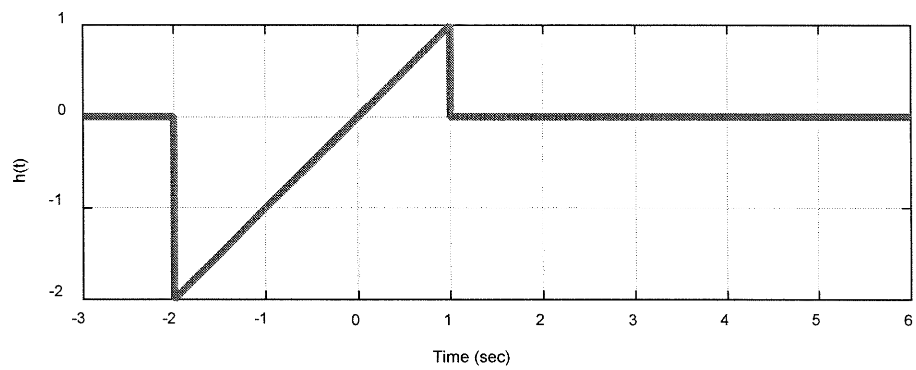
$$h(t) = t[u(\mathrm{t}+2) - u(t-1)]$$

The input to the system is

$$x(t) = [u(t-1) - u(t-2)] - 2[\mathrm{u(t-3) - u(t-5)}]$$

These two functions are shown below:





Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals **cannot contain any unit step functions**
- ***DO NOT EVALUATE THE INTEGRALS!!***

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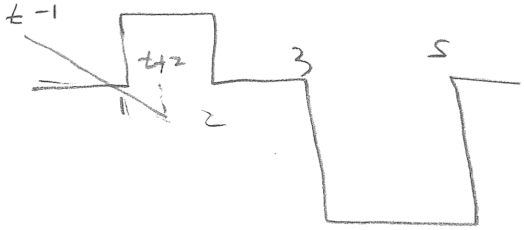
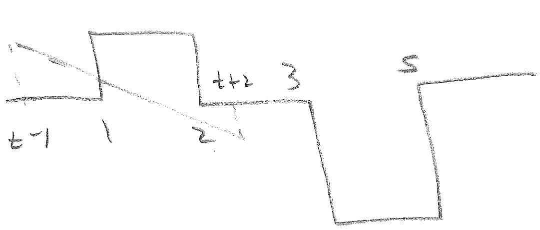


$y(t) = 0 \quad t+2 \le 1$ or $\boxed{t \le -1}$

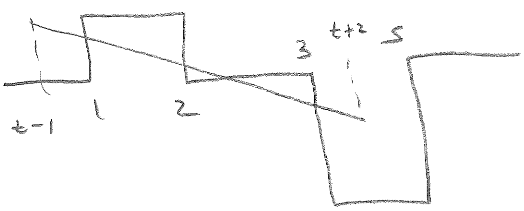
$y(t) = \int_1^{t+2} (t-\lambda)(1)\,d\lambda \qquad 1 \le t+2 \le 2$

$\boxed{-1 \le t \le 0}$



$y(t) = \int_1^{2} (t-\lambda)(1)\,d\lambda \qquad 2 \le t+2 \le 3 \qquad t-1 \le 1$

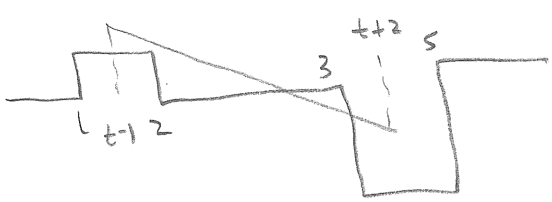
$\boxed{0 \le t \le 1} \qquad t \le 2$



$y(t) = \int_1^{2} (t-\lambda)(1)\,d\lambda + \int_3^{t+2} (t-\lambda)(-2)\,d\lambda \qquad t-1 \le 1 \quad 3 \le t+2 \le 5$

$t \le 2 \quad 1 \le t \le 3$

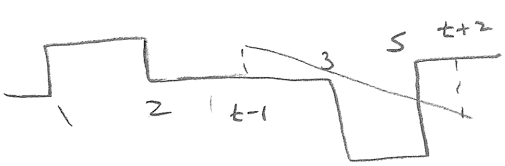
$\boxed{1 \le t \le 2}$



$y(t) = \int_{t-1}^{2} (t-\lambda)(1)\,d\lambda + \int_3^{t+2} (t-\lambda)(-2)\,d\lambda \qquad 1 \le t-1 \le 2 \quad 3 \le t+2 \le 5$

$2 \le t \le 3 \quad 1 \le t \le 3$

$\boxed{2 \le t \le 3}$

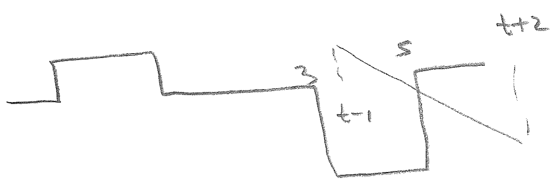


$y(t) = \int_3^{5} (t-\lambda)(-2)\,d\lambda \qquad 2 \le t-1 \le 3 \qquad t+2 \ge 5$

$\boxed{3 \le t \le 4} \qquad t \ge 3$

$y(t) = \int_{t-1}^{5} (t-\lambda)(-2)\,d\lambda \qquad 3 \le t-1 \le 5$

$\boxed{4 \le t \le 6}$



$y(t) = 0 \quad \boxed{t \ge 6}$

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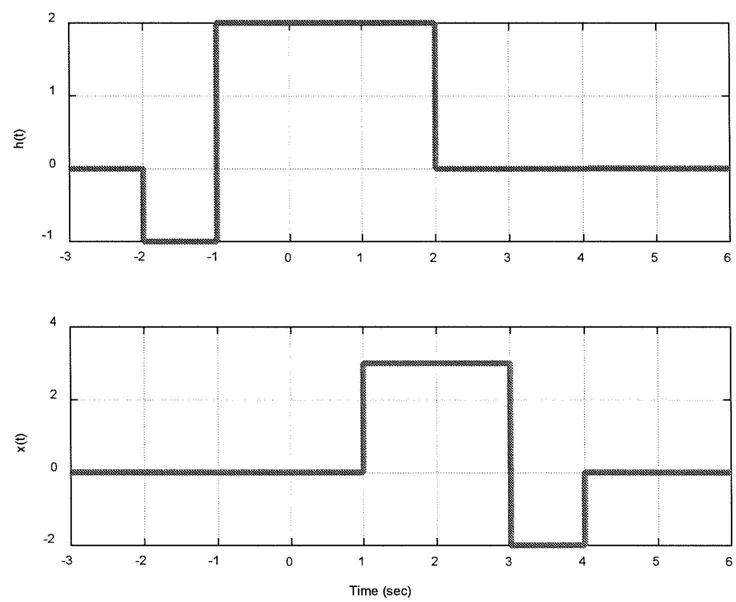
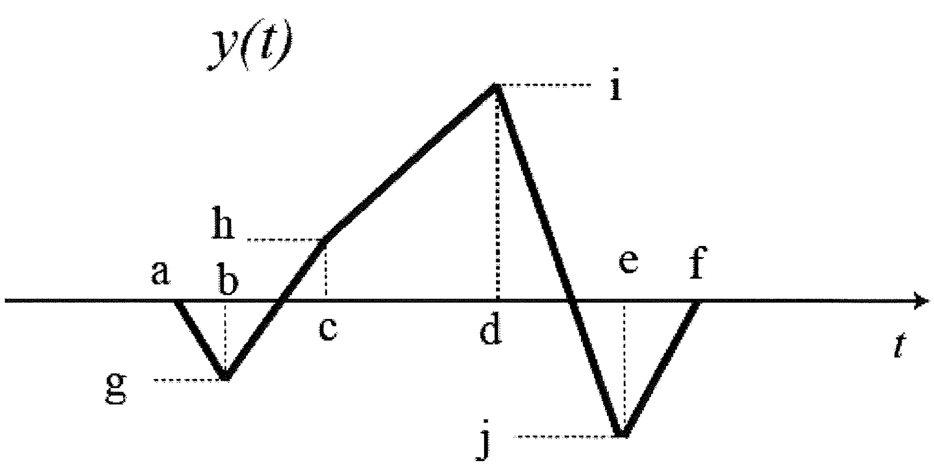
5) (24 Points) An LTI system has input, impulse response, and output as shown below. Determine numerical values for the parameters *a-j*. Note that parameters *a-f* correspond to *times (not equally spaced, these are located where the slopes change)*, and *g-i* correspond to *amplitudes.*

Hints:

- *Note that the output is not drawn to scale, it just represents the general shape of the output.*
- *A good way to check your answer is to flip and slide one of them, then flip and slide the other one.*
- *It is probably much easier to get the times correct than the amplitudes.*





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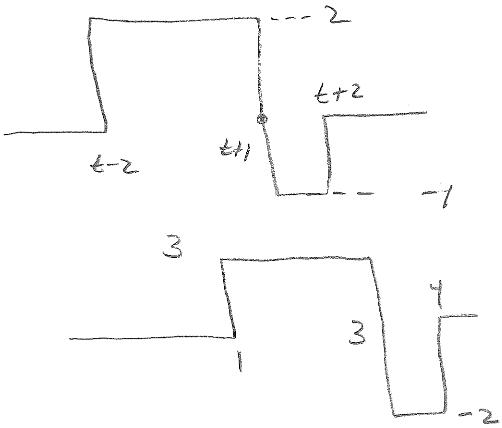

$t+2=1 \quad t=\boxed{-1=a}$

$t+1=1 \quad t=\boxed{0=b}$

$g=3\cdot(-1)\cdot 1=\boxed{-3=g}$

$t+2=3 \quad t=\boxed{1=c}$

$h=3(-1)\cdot(1)+3\cdot(2)(1)=\boxed{3=h}$

$t+2=4 \quad t=\boxed{2=d}$

$i=(-2)(-1)(1)+2\cdot 3\cdot 2=\boxed{14=i}$

$t-2=3 \quad t=\boxed{5=e}$

$j=(2)(-2)(1)=\boxed{-4=j}$

$t-2=4 \quad t=\boxed{6=f}$