Name Solutions Mailbox

ECE-205

Exam 1

Winter 2015

Calculators can only be used for simple calculations. Solving integrals, differential equations, systems of equations, etc. does not count as a simple calculation.

You must show your work to receive credit.

Problem 1	**_________/15**
Problem 2	**_________/30**
Problem 3	**_________/10**
Problem 4	**_________/10**
Problem 5	**_________/15**
Problem 6	**_________/20**

Total _____________

Name ______________________________ Mailbox ____________

1) (15 points) Assume we have a first order system with the governing differential equation

$$5\dot{y}(t) + 2y(t) = x(t)\,.$$

The system has the initial value of 0, so $y(0) = 0$. The input to this system is

$$x(t) = \begin{cases} 0 & t < 0 \\ 3 & 0 \le t < 2 \\ 1 & t \ge 2 \end{cases}$$

Determine the output of the system in each of the above time intervals. *Simplify your final answer as much as possible and box it.*

$y(t) = [y(t_0) - KA]e^{-(t-t_0)/\tau} + KA$

$5\dot{y}(t) + 2y(t) = x(t)$ $\quad$ $\frac{5}{2}\dot{y}(t) + y(t) = \frac{1}{2}x(t)$

$\tau = 5/2 \quad K = 1/2$

$0 \le t < 2$ $\quad$ $t_0 = 0 \quad y(t_0) = 0 \quad A = 3 \quad KA = 3/2$

$y(t) = [0 - 3/2]e^{-2t/5} + 3/2 = \boxed{\frac{3}{2}\left[1 - e^{-2t/5}\right] = y(t)}$

$2 < t$ $\quad$ $t_0 = 2 \quad y(t_0) = \frac{3}{2}\left[1 - e^{-4/5}\right] = 0.826 \quad KA = 1/2$

$y(t) = [0.826 - 0.5]e^{-\frac{2}{5}(t-2)} + 0.5$

$\boxed{y(t) = 0.326e^{-\frac{2}{5}(t-2)} + 0.5}$

Name ______________________________ Mailbox ______________

2) (30 Points) For the following differential equations the initial conditions are $y(0) = \dot{y}(0) = 0$

Determine the solution to each of the following differential equations and put your final answer in a box. Be sure to use the initial conditions to solve for all unknowns. You must show all your work to receive credit.

a) $\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = x(t), \quad x(t) = 6u(t)$

$2y_f(t) = 6 \qquad \boxed{y_f(t) = 3}$

$r^2 + 3r + 2 = 0 = (r+1)(r+2)$

$y(t) = c_1 e^{-t} + c_2 e^{-2t} + 3$

$\dot{y}(t) = -c_1 e^{-t} - 2c_2 e^{-2t}$

$y(0) = c_1 + c_2 + 3 = 0$

$\dot{y}(0) = -c_1 - 2c_2 = 0$

adding $-c_2 + 3 = 0 \quad c_2 = 3$

$c_1 = -2c_2 = -6$

$$\boxed{y(t) = -6e^{-t} + 3e^{-2t} + 3 \quad t \geq 0}$$

b) $\ddot{y}(t) + 6\dot{y}(t) + 13y(t) = 2x(t), \quad x(t) = 13u(t)$

$13y_f(t) = 26 \qquad \boxed{y_f(t) = 2}$

$r^2 + 6r + 13 = 0$

$(r+3)^2 + 2^2 = 0 \quad r = -3 \pm 2j$

$y(t) = ce^{-3t}\sin(2t + \theta) + 2$

$y(0) = c\sin(\theta) + 2 = 0$

$c = \dfrac{-2}{\sin(\theta)}$

$\dot{y}(t) = -3ce^{-3t}\sin(2t+\theta) + 2ce^{-3t}\cos(2t+\theta)$

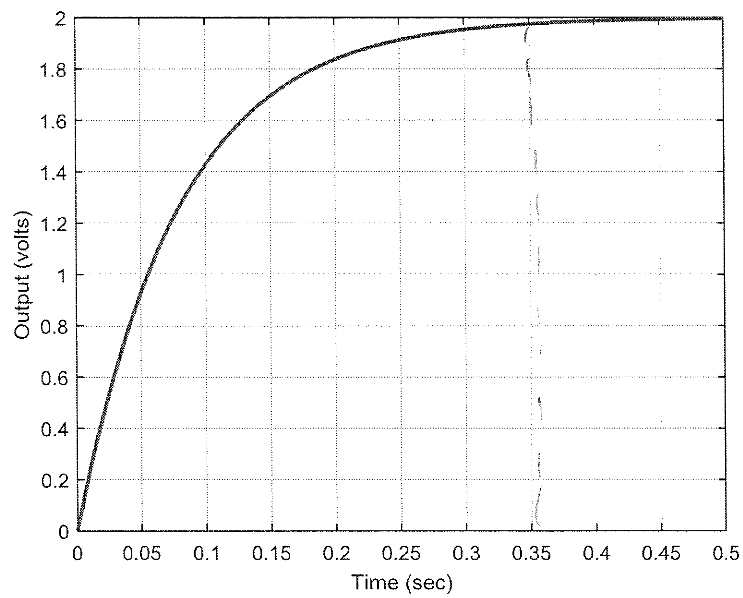
$\dot{y}(0) = -3\sin(\theta) + 2\cos(\theta) = 0 \quad \tan(\theta) = 2/3 \quad \theta = 33.69°$

$c = -3.61$

$$\boxed{y(t) = 2 - 3.61e^{-3t}\sin(2t + 33.69°)}$$

Name ______________________________ Mailbox ____________

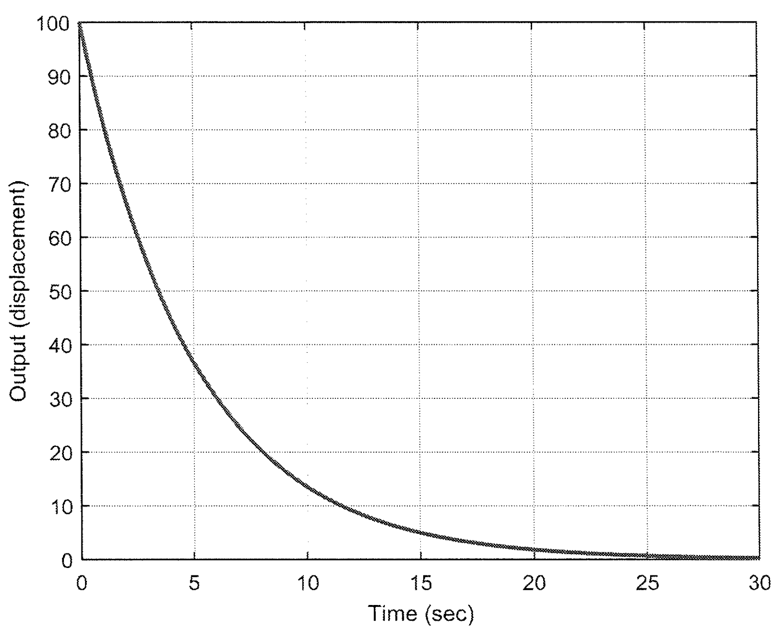
3) (10 Points) The following graphs showing the response of two different first order systems to a step input (top graph) and due only to initial conditions (bottom graph). Estimate the ***time constants*** of each system. (The time constants are different for each of the systems.)



$0.98 \times 2 = 1.96$

$4\tau = 0.35$

$\tau = \frac{0.35}{4} = \boxed{0.088 = \tau}$



$0.02 \times 100 = 2$

$4\tau = 20$

$\boxed{\tau = 5}$

Name ____ Mailbox __________________

4) (10 points) Using the integrating factor method, determine the expression of the response $y(t)$ for the following system:

$$\dot{y}(t) = 2t \cdot y(t) + e^{t^2} x(t).$$

The initial condition is $y(0) = 1$ with $t_0 = 0$. *Simplify your answer as much as possible.*

$$\frac{dy(t)}{dt} - 2t\,y(t) = e^{t^2} x(t)$$

$$\frac{d}{dt}\left[y(t)e^{-t^2}\right] = e^{-t^2} e^{t^2} x(t) = x(t)$$

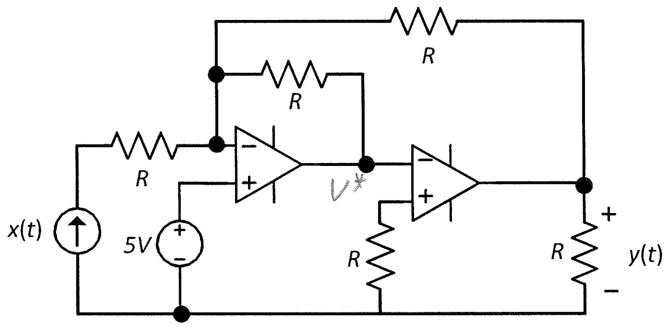
$$y(t)e^{-t^2} - y(0) = \int_0^t x(\lambda)\,d\lambda$$

$$y(t)e^{-t^2} = 1 + \int_0^t x(\lambda)\,d\lambda$$

$$\boxed{y(t) = e^{t^2} + e^{t^2}\int_0^t x(\lambda)\,d\lambda}$$

Name ______________________________ Mailbox ______________

5) **(15 points)** We can write $y(t) = Gx(t) + C$ for the following op-amp circuit. Determine expressions for G and C.



Careful: Be sure to account for the 5V voltage source at the positive terminal of the first op-amp.

$V^+ = V^- = 5V$ (1^{st} op amp) $V^* = 0$ (2^{nd} op amp)

at V^- of 1^{st} op amp

$$x(t) + \frac{V^*(t) - 5}{R} + \frac{y(t) - 5}{R} = x(t) - \frac{5}{R} + \frac{y(t) - 5}{R} = 0$$

$$Rx(t) - 10 + y(t) = 0$$

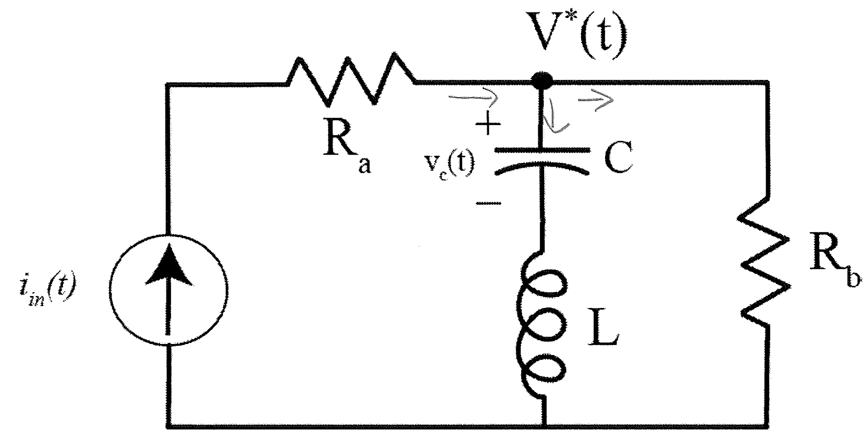
$$y(t) = -Rx(t) + 10$$

$\boxed{G = -R}$ $\boxed{C = +10}$

Name ____ Mailbox ___________________

6) (20 Points) Determine the governing 2nd order differential equation for the following circuit. The output should be the voltage across the capacitor, $v_C(t)$.

Hint: Determine two expressions for the voltage $V^*(t)$ and then eliminate this voltage.



$$i_{in}(t) = C\frac{dV_c(t)}{dt} + \frac{V^*(t)}{R_b}$$

$$R_b\, i_{in}(t) = R_b C\frac{dV_c(t)}{dt} + V^*(t)$$

$$V^*(t) = V_c(t) + L\frac{di_L(t)}{dt}$$

$$= V_c(t) + L\frac{d}{dt}\left[C\frac{dV_c(t)}{dt}\right]$$

$$= V_c(t) + LC\frac{d^2V_c(t)}{dt^2}$$

$$R_b\, i_{in}(t) = R_b C\frac{dV_c(t)}{dt} + V_c(t) + LC\frac{d^2V_c(t)}{dt^2}$$

$$\boxed{LC\frac{d^2V_c(t)}{dt^2} + R_b C\frac{dV_c(t)}{dt} + V_c(t) = R_b\, i_{in}(t)}$$