Name SOLUTIONS ______________________ Mailbox ______________

ECE-205

Exam 2

Winter 2013

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 ________/18

Problem 2 ________/20

Problem 3 ________/17

Problem 4 ________/25

Problem 5 ________/20

Total ____________

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1) **(18 points)** Fill in the non-shaded part of the following table. You do not need to show any work.

	Linear? (Y/N)	Time Invariant? (Y/N)	BIBO Stable? (Y/N)
$y(t)=3x(t)+\cos(t)$	N	N	
$\dot{y}(t)+e^{t}y(t)=\sin(t)x(t+1)$	Y	N	
$y(t)=tx(t-1)$	Y	N	
$y(t)=\int_0^t e^{\lambda}x(\lambda)d\lambda$			N
$y(t)=\int_{-\infty}^t \lambda x(\lambda)d\lambda$			N
$y(t)=t\cos\left(\dfrac{1}{x(t)}\right)$			N

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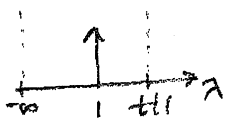
2) (20 points) Simplify the following as much as posible. Be sure to include any necessary unit step functions.

$$y(t)=[t^2+e^{(t-1)}]\delta(t-1) = [1^2+e^0]\,\delta(t-1) = 2\delta(t-1)$$

$$y(t)=\int_{-\infty}^{t+1}\delta(\lambda-1)d\lambda = \begin{cases} 0 & \text{if } t+1<1 \text{ or } t<0 \\ 1 & \text{if } t>0 \end{cases} = u(t)$$



$$y(t)=\int_{-t-3}^{3}\delta(\lambda-1)d\lambda = \begin{cases} 0 & \text{if } -t-3>1 \text{ or } t<-4 \\ 1 & \text{if } t>-4 \end{cases} = u(t+4)$$

$$y(t)=\int_{1}^{t} e^{-(t-\lambda)}e^{-2\lambda}d\lambda = \int_1^t e^{-t}e^{-\lambda}d\lambda = e^{-t}\int_1^t e^{-\lambda}d\lambda = -e^{-t}(e^{-t}-e^{-1})$$
$$= -(e^{-2t}-e^{-(t+1)})$$

$$y(t)=\int_{1}^{t-1} e^{-3(t-\lambda)}e^{-3\lambda}d\lambda = \int_1^{t-1} e^{-3t}e^{+3\lambda}e^{-3\lambda}d\lambda = e^{-3t}\int_1^{t-1}(1)d\lambda = e^{-3t}(t-2)$$

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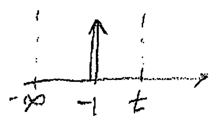
3) (17 Points) Determine the ***impulse response*** for the following systems. Don't forget any necessary unit step functions

a) $y(t) = x(t) + \int_{-\infty}^{t} x(\lambda+1)d\lambda$

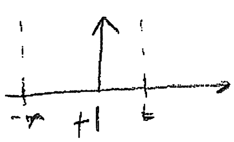
$$h(t) = \delta(t) + \int_{-\infty}^{t} \delta(\lambda+1)d\lambda = \delta(t) + u(t+1)$$



b) $y(t) = \int_{-\infty}^{t} e^{-(t-\lambda)} x(\lambda-1)d\lambda$

$$h(t) = \int_{-\infty}^{t} e^{-(t-\lambda)} \delta(\lambda-1)d\lambda = \int_{-\infty}^{t} e^{-(t-1)} \delta(\lambda-1)d\lambda = e^{-(t-1)} \int_{-\infty}^{t} \delta(\lambda-1)d\lambda$$

$$= e^{-(t-1)} u(t-1)$$



c) $\dot{y}(t) - 2y(t) = 4x(t+1)$

$$\dot{h}(t) - 2h(t) = 4\delta(t+1)$$

(Integrating factor)

$$\frac{d}{dt}\left(h(t)e^{\phi(t)}\right) = e^{\phi(t)}\left(\dot{h}(t) + \dot{\phi}(t)h(t)\right)$$

set $\dot{\phi}(t) = -2 \Rightarrow \phi(t) = -2t$

then

$$\frac{d}{dt}\left(h(t)e^{-2t}\right) = e^{-2t}\left(4\delta(t+1)\right) = 4e^{+2}\delta(t+1)$$

$$\Rightarrow \int_{-\infty}^{t} \frac{d}{d\lambda}\left(h(\lambda)e^{-2\lambda}\right)d\lambda = \int_{-\infty}^{t} 4e^{2}\delta(\lambda+1)d\lambda$$

$$\Rightarrow h(t)e^{-2t} = 4e^{2}\int_{-\infty}^{t} \delta(\lambda+1)d\lambda = 4e^{2}u(t+1)$$

$$\Rightarrow h(t) = 4e^{2(t+1)}u(t+1)$$

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4) (25 points) Use analytical evaluation to evaluate the following convolutions. Be sure to include any necessary unit step functions and simplify your answers.

a) $y(t) = \delta(t+1) * \delta(t-2)$

$$y(t) = \int_{-\infty}^{\infty} \delta(t-\lambda+1)\,\delta(\lambda-2)\,d\lambda = \boxed{\delta(t-1)}$$

b) $y(t) = u(t+1) * \delta(t-1)$

$$y(t) = \int_{-\infty}^{\infty} u(t-\lambda+1)\,\delta(\lambda-1)\,d\lambda = \boxed{u(t)}$$

c) $y(t) = \left[e^{-t}u(t)\right] * \delta(t-1)$

$$y(t) = \int_{-\infty}^{\infty} e^{-\lambda}u(\lambda)\,\delta(t-\lambda-1)\,d\lambda = \boxed{e^{-(t-1)}u(t-1)}$$

d) $y(t) = \left[e^{-(t-1)}u(t-1)\right] * u(t)$

$$y(t) = \int_{-\infty}^{\infty} e^{-(\lambda-1)}u(\lambda-1)\,u(t-\lambda)\,d\lambda$$

$$= \int_{1}^{t} e^{-\lambda}e^{1}\,d\lambda = e^{1}\left[-e^{-\lambda}\right]_{1}^{t} = e^{1}\left[-e^{-t} + e^{-1}\right] = 1 - e^{-(t-1)}$$

need $t > \lambda > 1$ $\quad t-1 > 0$ $\quad \boxed{y(t) = \left[1 - e^{-(t-1)}\right]u(t-1)}$

e) $y(t) = \left[e^{-t}u(t)\right] * \left[e^{-(t-2)}u(t-2)\right]$

$$y(t) = \int_{-\infty}^{\infty} e^{-(t-\lambda)}u(t-\lambda)\,e^{-(\lambda-2)}u(\lambda-2)\,d\lambda = \int_{2}^{t} e^{-t}e^{2}\,d\lambda$$

$$= e^{-(t-2)}\int_{2}^{t} d\lambda = e^{-(t-2)}(t-2)$$

need $t > \lambda > 2$ or $t-2 > 0$

$$\boxed{y(t) = (t-2)e^{-(t-2)}u(t-2)}$$

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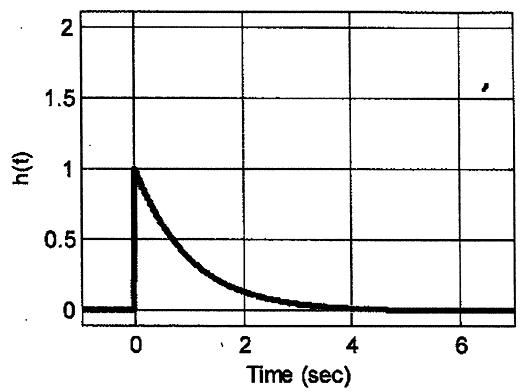
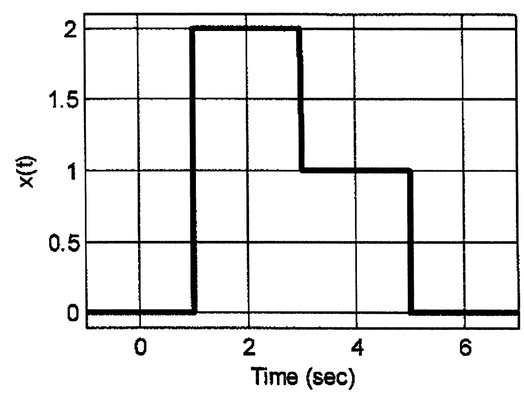
5) (20 points) Consider a linear time invariant system with impulse response given by

$$h(t) = e^{-t}u(t)$$

The input to the system is given by

$$x(t) = 2u(t-1) - u(t-3) - u(t-5).$$

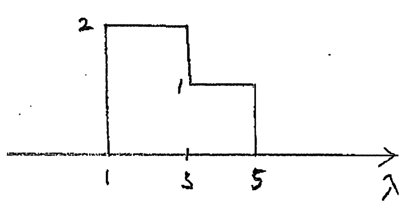




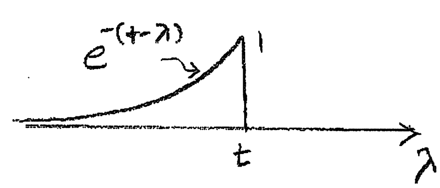
Using ***graphical evaluation***, determine the output $y(t)$. Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$.
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest.
- Determine the range of t for which each part of your solution is valid.
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals **cannot contain any unit step functions.**
- ***DO NOT EVALUATE THE INTEGRALS!!***

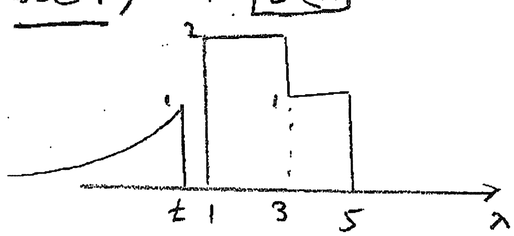
$x(\lambda)$



$h(t-\lambda)$



case 1) if $\boxed{t \le 1}$:



$\boxed{y(t) = 0 \quad \text{if } t \le 1}$

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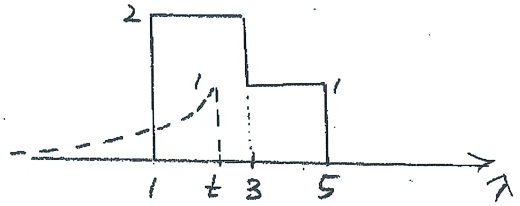
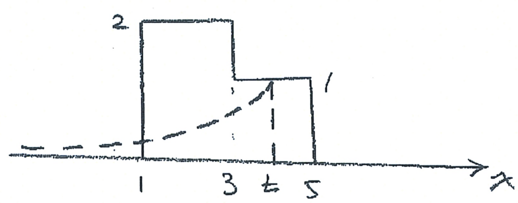
case 2) if $1<t\leq 3$



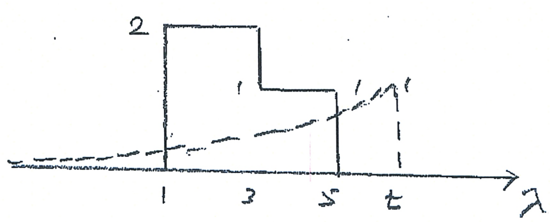
$$\boxed{y(t) = \int_1^t (2)\, e^{-(t-\lambda)}\, d\lambda \quad \text{if } 1<t\leq 3}$$

case 3) if $3<t\leq 5$



$$\boxed{\begin{aligned} y(t) &= \int_1^3 (2)\, e^{-(t-\lambda)}\, d\lambda \\ &\quad + \int_3^t (1)\, e^{-(t-\lambda)}\, d\lambda \quad \text{if } 3<t\leq 5 \end{aligned}}$$

case 4) if $t>5$



$$\boxed{\begin{aligned} y(t) &= \int_1^3 (2)\, e^{-(t-\lambda)}\, d\lambda \\ &\quad + \int_3^5 (1)\, e^{-(t-\lambda)}\, d\lambda \quad \text{if } t>5 \end{aligned}}$$

$$\therefore\ y(t) = \begin{cases} 0 & \text{for } t\leq 1 \\ \int_1^t 2e^{-(t-\lambda)}\, d\lambda & \text{for } 1<t\leq 3 \\ \int_1^3 2e^{-(t-\lambda)}\, d\lambda + \int_3^t e^{-(t-\lambda)}\, d\lambda & \text{for } 3<t\leq 5 \\ \int_1^3 2e^{-(t-\lambda)}\, d\lambda + \int_3^5 e^{-(t-\lambda)}\, d\lambda & \text{for } t>5 \end{cases}$$