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ECE-205

Exam 2

Winter 2012

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 ________/18

Problem 2 ________/15

Problem 3 ________/18

Problem 4 ________/16

Problem 5 ________/13

Problem 6 ________/20

Total ____________

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1) (18 points) Fill in the non-shaded part of the following table.

	Linear? (Y/N)	Time Invariant? (Y/N)	BIBO Stable? (Y/N)
$y(t)=\cos(t-1)x(t)$	Yes	No	
$\dot{y}(t)+x(t)y(t)=\sin(t+1)x(t)$	No	No	
$y(t)=x(2t)$	Yes	No	
$y(t)=\int_0^t e^{\lambda}x(\lambda)d\lambda$			No
$y(t)=\int_{-\infty}^t e^{-\lambda}x(\lambda)d\lambda$			No
$y(t)=\cos\left(\frac{1}{1-x(t)}\right)$			Yes

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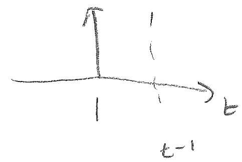
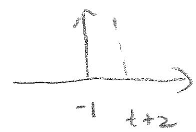
2) (15 Points) Determine the ***impulse response*** for the following systems. Don't forget any necessary unit step functions

a) $y(t) = x(t+1) + \int_{-\infty}^{t-1} x(\lambda - 1)\, d\lambda$



$t-1>1 \qquad t-2>0$

b) $y(t) = \int_{-\infty}^{t+2} e^{-(t-\lambda)} x(\lambda + 1)\, d\lambda$



$t+2>-1 \qquad t+3>0$

c) $2\dot{y}(t) + y(t) = 3x(t)$

(a) $\boxed{h(t) = \delta(t+1) + u(t-2)}$

(b) $\boxed{h(t) = e^{-(t+1)} u(t+3)}$

(c) $\dot{h}(t) + \frac{1}{2} h(t) = \frac{3}{2} \delta(t)$

$\frac{d}{dt}\left[h(t) e^{+t/2}\right] = \frac{3}{2} e^{t/2} \delta(t) = \frac{3}{2} \delta(t)$

$h(t) e^{t/2} = \frac{3}{2} u(t)$

$\boxed{h(t) = \frac{3}{2} e^{-t/2} u(t)}$

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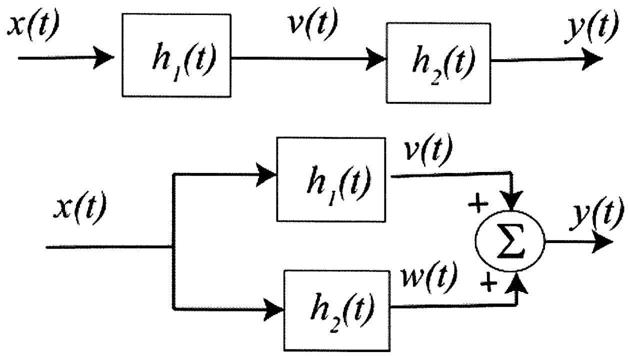
3) (18 points) For the following block diagram

For the following interconnected systems,

i) determine the overall impulse response (the impulse response between input $x(t)$ and output $y(t)$) and

ii) determine if the system is causal.



a) $h_1(t) = \delta(t-1),\ h_2(t) = \delta(t+2)$

b) $h_1(t) = e^{-t}u(t),\ h_2(t) = u(t-2) + \delta(t-2)$

Series Connections:

(a) $h(t) = \int_{-\infty}^{\infty} h_1(t-\lambda)\, h_2(\lambda)\, d\lambda = \int_{-\infty}^{\infty} \delta(t-\lambda-1)\,\delta(\lambda+2) = \boxed{\delta(\lambda+1) = h(t) \mid \text{not causal}}$

(b) $h(t) = \int_{-\infty}^{\infty} h_1(t-\lambda) h_2(\lambda)\, d\lambda = \int_{-\infty}^{\infty} e^{-(t-\lambda)} u(t-\lambda)[u(\lambda-2) + \delta(\lambda-2)]\, d\lambda$

$= \int_{-\infty}^{\infty} e^{-t} e^{\lambda} u(t-\lambda) u(\lambda-2)\, d\lambda + \int_{-\infty}^{\infty} e^{-t} e^{\lambda} u(t-\lambda)\,\delta(\lambda-2)\, d\lambda$

$= e^{-t} \int_2^t e^{\lambda}\, d\lambda + e^{-(t-2)} u(t-2) = e^{-t}[e^{t} - e^{2}]\, u(t-2) + e^{-(t-2)} u(t-2)$

$= [1 - e^{-(t-2)} + e^{-(t-2)}]\, u(t-2) = \boxed{u(t-2) = h(t) \mid \text{causal}}$

Parallel Connections:

(a) $h(t) = h_1(t) + h_2(t) = \boxed{\delta(t-1) + \delta(t-2) = h(t) \mid \text{not causal}}$

(b) $h(t) = h_1(t) + h_2(t) = \boxed{e^{-t} u(t) + u(t-2) + \delta(t-2) = h(t) \mid \text{causal}}$

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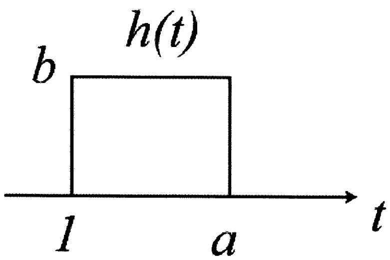
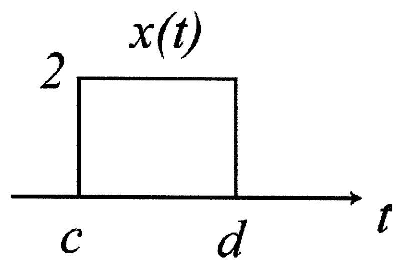
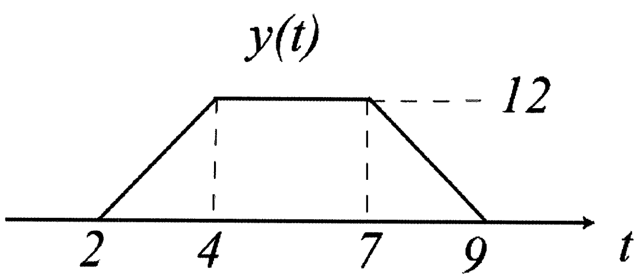
4) (16 Points) An LTI system has impulse response, input, and output as shown below. Determine numerical values for the parameters *a, b, c, and d.*

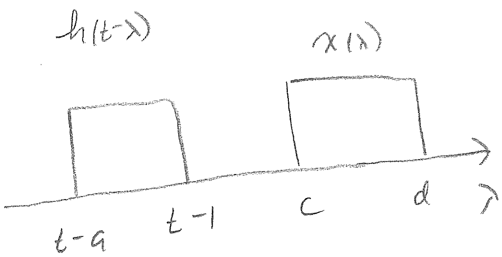
Hint: $a-1 < d-c$







h(t) narrower than x(t)



$t-1=c \quad t=c+1=2 \quad \boxed{c=1}$

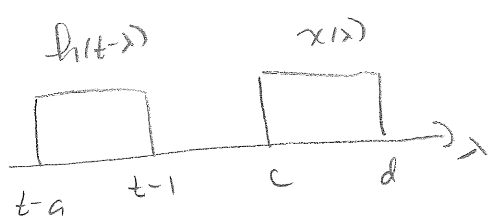
$t-a=c \quad t=c+a=4 \quad \boxed{a=3}$

$t-1=d \quad t=d+1=7 \quad \boxed{d=6}$

$12 = 2\cdot 2\cdot b \quad \boxed{b=3}$

width height of x(t)

x(t) narrower than h(t)



$t-1=c \quad t=c+1=2 \quad \boxed{c=1}$

$t-1=d \quad t=d+1=4 \quad \boxed{d=3}$

$t-a=c \quad t=a+c=7 \quad \boxed{a=6}$

$12=2\cdot 2\cdot b \quad \boxed{b=3}$

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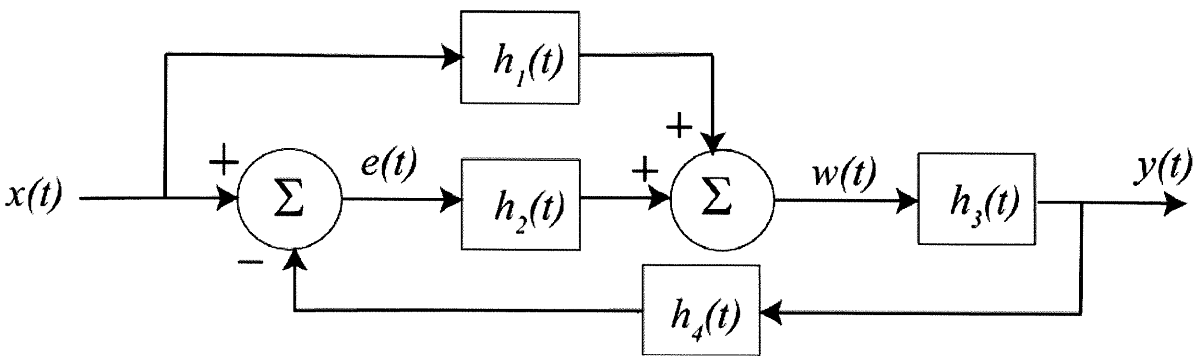
5) (13 points) The input-output relationship for the following system can be written as

$$y(t) * A(t) = x(t) * B(t)$$

Determine $A(t)$ and $B(t)$.

Hint: Determine an expression for $e(t)$, then $w(t)$, then $y(t)$



$$e(t) = x(t) - h_4(t) * y(t)$$

$$\begin{aligned} w(t) &= x(t) * h_1(t) + e(t) * h_2(t) \\ &= x(t) * h_1(t) + [x(t) - h_4(t) * y(t)] * h_2(t) \\ &= x(t) * h_1(t) + x(t) * h_2(t) - y(t) * h_2(t) * h_4(t) \end{aligned}$$

$$y(t) = w(t) * h_3(t) = x(t) * h_1(t) * h_3(t) + x(t) * h_2(t) * h_3(t) - y(t) * h_2(t) * h_3(t) * h_4(t)$$

$$y(t) * \underbrace{\left[\delta(t) + h_2(t) * h_3(t) * h_4(t)\right]}_{A(t)} = x(t) * \underbrace{\left[h_1(t) * h_3(t) + h_2(t) * h_3(t)\right]}_{B(t)}$$

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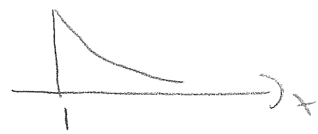
6) (20 points) Consider a linear time invariant system with impulse response given by

$$h(t) = e^{-(t-1)}u(t-1)$$



$h(1) = h(t-\lambda)$

$1 = t - \lambda$

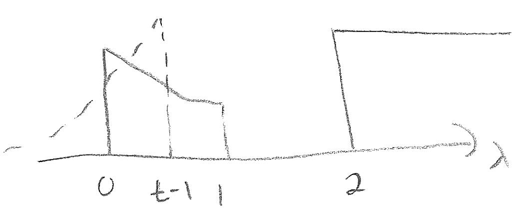
$\lambda = t - 1$

The input to the system is given by

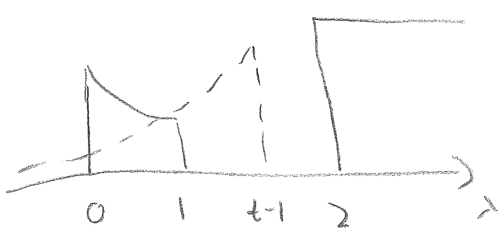
$$x(t) = e^{-t}[u(t) - u(t-1)] + 2u(t-2)$$

Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

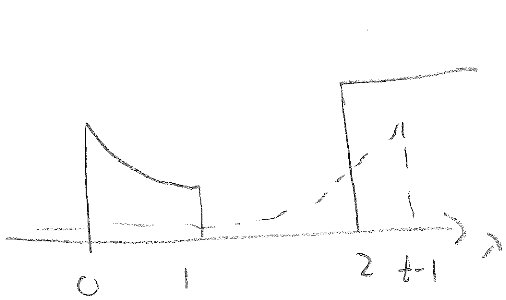
- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***



$1 \le t \le 2 \quad y(t) = \int_0^{t-1} e^{-(t-\lambda-1)} e^{-\lambda} \, d\lambda$



$2 \le t \le 3 \quad y(t) = \int_0^{1} e^{-(t-\lambda-1)} e^{-\lambda} \, d\lambda$



$3 \le t \quad y(t) = \int_0^{1} e^{-(t-\lambda-1)} e^{-\lambda} \, d\lambda + \int_2^{t-1} e^{-(t-\lambda-1)} (2) \, d\lambda$

$y(t) = 0 \quad t \le 1$