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ECE-205

Exam 1

Winter 2012

Calculators can only be used for simple calculations. Solving integrals, differential equations, systems of equations, etc. does not count as a simple calculation.

You must show your work to receive credit.

Problem 1	________	**/15**
Problem 2	________	**/20**
Problem 3	________	**/10**
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Problem 5	________	**/15**
Problem 6-11	________	**/24**

Total ____________

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1) (15 points) Assume we have a first order system with the governing differential equation

$$0.01\dot{y}(t) + y(t) = 2x(t)$$

The system has the initial value of 0.1, so $y(0) = 0.1$. The input to this system is

$$x(t) = \begin{cases} 0 & t < 0 \\ 0.2 & 0 \le t < 0.02 \\ -0.6 & 0.02 \le t \end{cases}$$

Determine the output of the system in each of the above time intervals. *Simplify your final answer as much as possible and box it. Be sure to include the correct initial value in the first interval!*

$\tau = 0.01$ $K = 2$ $y(t) = [y(t_0) - y(\infty)]e^{-(t-t_0)/\tau} + y(\infty)$ $y(\infty) = KA$

① $0 \le t < 0.02$ $y(0) = 0.1$ $y(\infty) = K(0.2) = 0.4$

$y(t) = [0.1 - 0.4]e^{-(t-0)/0.01} + 0.4$

$$\boxed{y(t) = -0.3e^{-t/0.01} + 0.4}$$

② $t \ge 0.02$ $y(0.02) = -0.3e^{-2} + 0.4 = 0.36$ $y(\infty) = K(-0.6) = -1.2$

$y(t) = [0.36 - (-1.2)]e^{-(t-0.02)/0.01} - 1.2$

$$\boxed{y(t) = 1.56\,e^{-(t-0.02)/0.01} - 1.2}$$

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2) (20 points) For the following three differential equations, assume the input is $x(t) = u(t)$ (the input is equal to one for time greater than zero), and the initial conditions are $y(0) = \dot{y}(0) = 0$

Determine the solution to each of the following differential equations and put your final answer in a box. Be sure to use the initial conditions to solve for all unknowns. You must show all your work to receive credit.

a) $\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = 2x(t)$

$2y_f = 2 \quad y_f = 1 \quad r^2 + 3r + 2 = 0 = (r+1)(r+2)$

$y(t) = 1 + c_1 e^{-t} + c_2 e^{-2t}$

$\dot{y}(t) = -c_1 e^{-t} - 2c_2 e^{-2t}$

$\left.\begin{array}{l} y(0) = 0 = 1 + c_1 + c_2 \\ \dot{y}(0) = 0 = -c_1 - 2c_2 \end{array}\right\}$ adding $0 = 1 - c_2 \quad c_2 = 1$

$c_1 = -2c_2 = -2$

$\boxed{y(t) = 1 - 2e^{-t} + e^{-2t}}$

b) $\ddot{y}(t) + 2\dot{y}(t) + y(t) = 2x(t)$

$y_f = 2 \quad r^2 + 2r + 1 = 0 = (r+1)^2$

$y(t) = 2 + c_1 e^{-t} + c_2 t e^{-t}$

$y(0) = 0 = 2 + c_1 \quad c_1 = -2$

$\dot{y}(t) = -c_1 e^{-t} + c_2 e^{-t} - c_2 t e^{-t}$

$\dot{y}(0) = 0 = -c_1 + c_2 \quad c_1 = c_2 = -2$

$\boxed{y(t) = 2 - 2e^{-t} - 2te^{-t}}$

c) $\ddot{y}(t) + 4\dot{y}(t) + 16y(t) = 4x(t)$

$16y_f = 4 \quad y_f = 4/16 = 0.25 \quad r^2 + 4r + 16 = 0$

$r = -2 \pm j3.46$

$y(t) = 0.25 + ce^{-2t}\sin(3.46t + \theta)$

$y(0) = 0 = 0.25 + c\sin(\theta) \quad c = \frac{-0.25}{\sin(\theta)}$

$\dot{y}(t) = -2ce^{-2t}\sin(3.46t + \theta) + 3.46ce^{-2t}\cos(3.46t + \theta)$

$\dot{y}(0) = 0 = -2\sin(\theta) + 3.46\cos(\theta) \quad \frac{\sin(\theta)}{\cos(\theta)} = \frac{3.46}{2} = \tan(\theta) \quad \theta = 60°$

$c = \frac{-0.25}{\sin(60°)} = -0.288$

$\boxed{y(t) = 0.25 - 0.288e^{-2t}\sin(3.46t + 60°)}$

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3) (10 points) For the following first order differential equation,

$$\dot{y}(t) + 2ty(t) = \cos(t)x(t)$$

determine an expression for the output assuming $t_0 = 0$ and $y(t_0) = y(0) = 1$.

$$\frac{d}{dt}\left[y(t)e^{t^2}\right] = e^{t^2}\cos(t)\,x(t)$$

$$y(t)e^{t^2} - y(t_0)e^{t_0^2} = \int_{t_0}^{t} e^{\lambda^2}\cos(\lambda)\,x(\lambda)\,d\lambda \qquad t_0 = 0 \quad y(t_0) = 1$$

$$y(t)e^{t^2} = 1 + \int_0^t e^{\lambda^2}\cos(\lambda)\,x(\lambda)\,d\lambda$$

$$\boxed{y(t) = e^{-t^2} + e^{-t^2}\int_0^t e^{\lambda^2}\cos(\lambda)\,x(\lambda)\,d\lambda}$$

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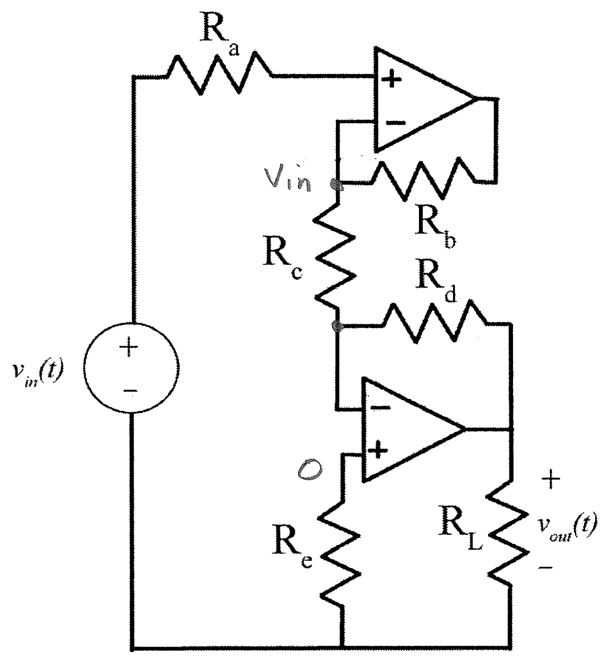
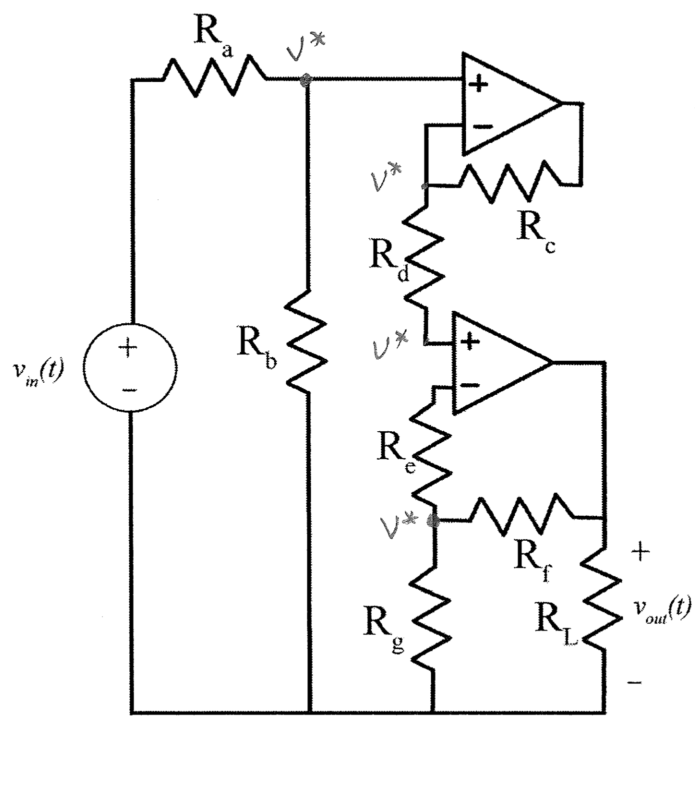
4) (16 points) For the following two op-amps circuits, we can write $v_{out}(t) = G\, v_{in}(t)$. Determine the value of G for each circuit.



$$\frac{V_{in}}{R_c} + \frac{V_{out}}{R_d} = 0$$

$$\boxed{V_{out} = \left(\frac{-R_d}{R_c}\right) V_{in}}$$



$$V^* = \frac{V_{in} R_b}{R_a + R_b}$$

$$\frac{V_{out} R_g}{R_g + R_F} = V^* = \frac{V_{in} R_b}{R_a + R_b}$$

$$\boxed{V_{out} = \left(\frac{R_g + R_F}{R_g} \frac{R_b}{R_a + R_b}\right) V_{in}}$$

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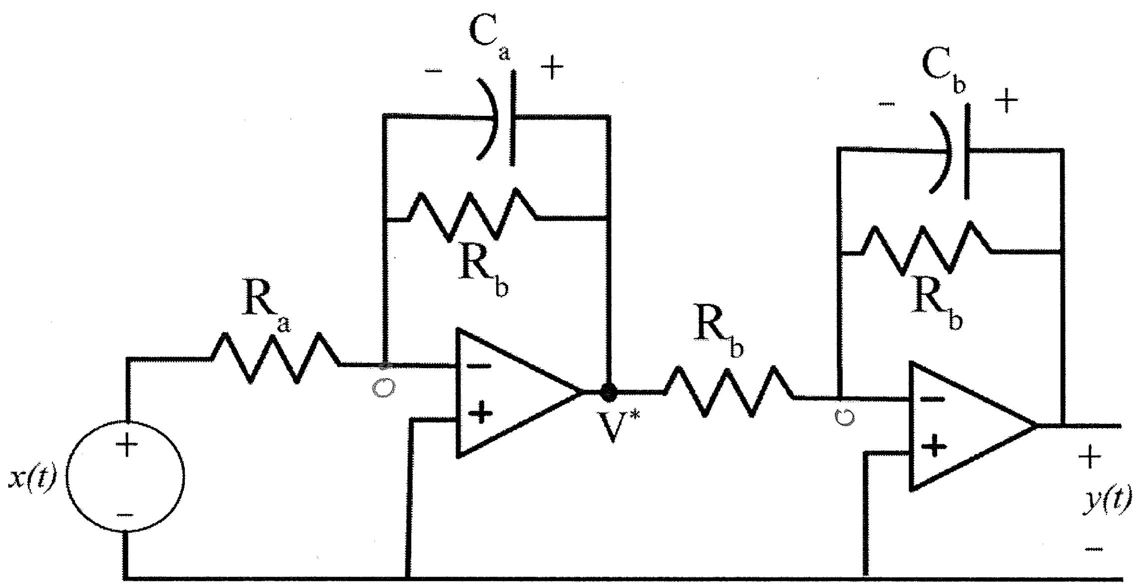
5) **(15 points)** For the second order circuit below, derive the governing second order differential equation for the output $y(t)$ and input $x(t)$. You do not need to put it into a standard form, but it must be simplified as much as possible.



Hint: Write the equations for each op amp in terms of V^*, and then eliminate this node voltage.

$$C_a \frac{dV^*}{dt} + \frac{V^*}{R_b} + \frac{x}{R_a} = 0$$

$$\frac{V^*}{R_b} + C_b \frac{dy}{dt} + \frac{y}{R_b} = 0$$

$$V^* = -y - R_b C_b \dot{y}$$

$$C_a \frac{d}{dt}\left[-y - R_b C_b \dot{y}\right] + \frac{1}{R_b}\left[-y - R_b C_b \dot{y}\right] + \frac{x}{R_a} = 0$$

$$R_a C_a \dot{y} + R_a C_a R_b C_b \ddot{y} + \frac{R_a}{R_b} y + R_a C_b \dot{y} = x$$

$$\boxed{(R_a C_a R_b C_b)\ddot{y} + R_a (C_a + C_b)\dot{y} + \frac{R_a}{R_b} y = x}$$

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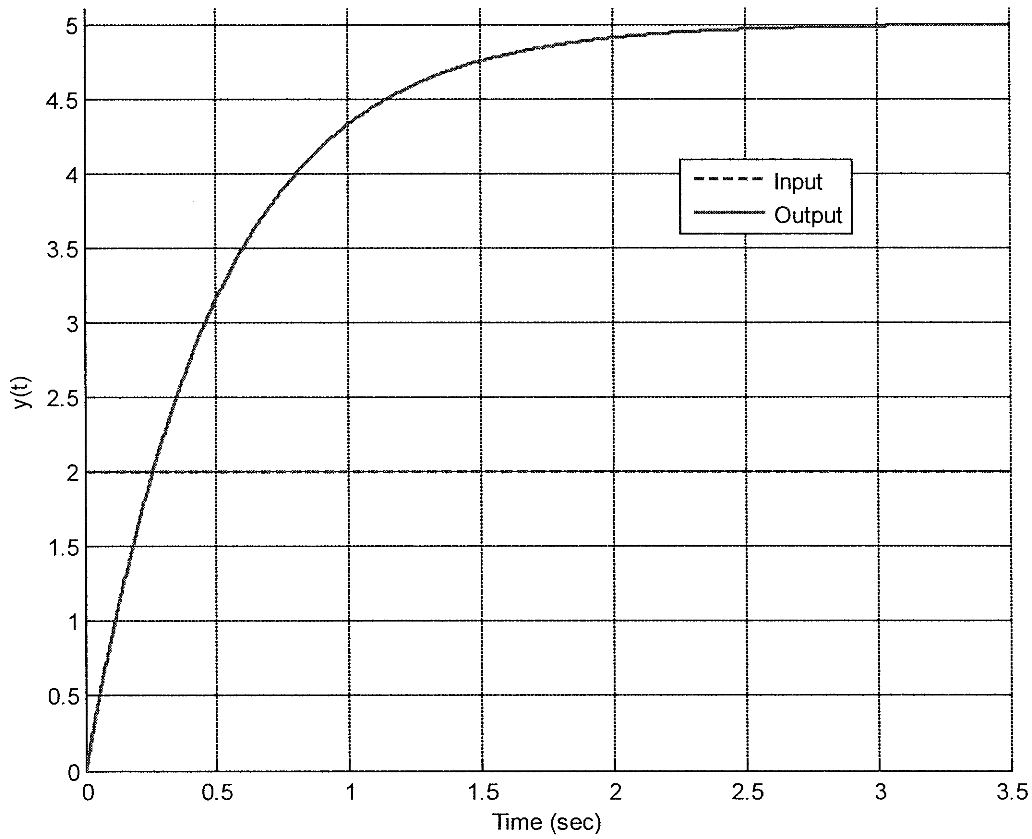
Problems 6-10, 4 points each (24 points)

For problems 6 and 7, refer to the following graph showing the input and output of a first order system. For this system the input is a step of amplitude 2.



6) What is the static gain? $K(2) = 5$ $\boxed{K = 2.5}$

7) What is the time constant? $0.98(5) = 4.9$

$y(4\tau) = 4.9$ $4\tau = 2$ $\boxed{\tau = 0.5 \text{ sec}}$

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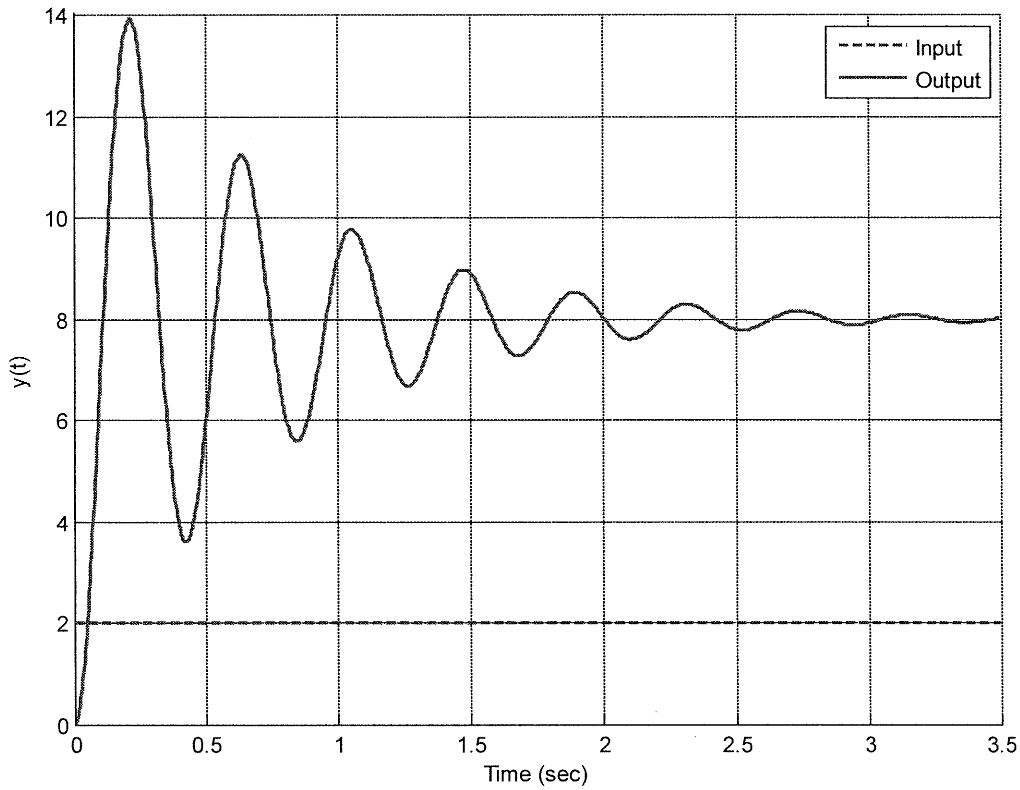
For problems 8 and 9, refer to the following graph showing the input and output of a second order system. For this system the input is a step of amplitude 2.



8) What is the static gain of the system? $K \cdot 2 = 8$ $\boxed{K = 4}$

9) What is the percent overshoot? $\frac{14-8}{8} \times 100\% = \frac{6}{8} \times 100\% = \boxed{75\%}$

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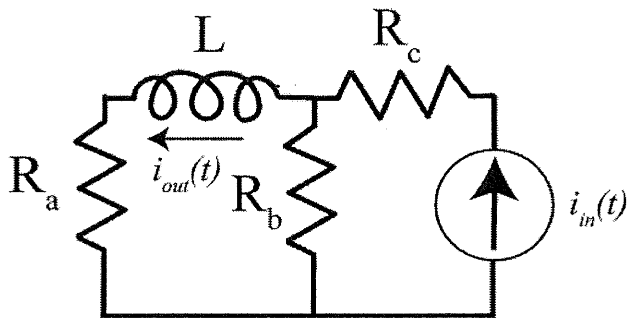
Problems 10 and 11 refer to the following first order circuit



10) Determine an expression for the time constant (you do not need to simplify it).

$\tau = L/R_{th}$ $R_{th} = R_a + R_b$

11) Determine an expression for the static gain (you do not need to simplify it).

$\frac{i_{in} R_b}{R_a + R_b} = i_{out}$ (in steady state)

$$\boxed{K = \frac{R_b}{R_a + R_b}}$$

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