Name Solutions CM ____________

ECE-205

Exam 3

Winter 2011

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 _________/25

Problem 2 _________/25

Problem 3 _________/25

Problem 4 _________/25

Total ______________

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1) (25 points) For the following impulse responses and inputs, compute the system output using Laplace transforms. Specifically, compute $H(s)$, $X(s)$, $Y(s)$, and then $y(t)$.

a) $h(t) = e^{-t}u(t)$, $x(t) = u(t)$

b) $h(t) = e^{-t}u(t)$, $x(t) = 2\delta(t-1)$

c) $h(t) = e^{-t}u(t)$, $x(t) = u(t-2)$

d) $h(t) = e^{-t}u(t)$, $x(t) = e^{-(t-2)}u(t-2)$

(a) $H(s) = \frac{1}{s+1}$ $X(s) = \frac{1}{s}$ $Y(s) = \frac{1}{s(s+1)} = \frac{A}{s} + \frac{B}{s+1}$ $A = 1$ $B = -1$

$\boxed{y(t) = [1 - e^{-t}]u(t)}$

(b) $H(s) = \frac{1}{s+1}$ $X(s) = 2e^{-s}$ $Y(s) = \frac{2e^{-s}}{s+1}$ $\boxed{y(t) = 2e^{-(t-1)}u(t-1)}$

(c) $H(s) = \frac{1}{s+1}$ $X(s) = \frac{e^{-2s}}{s}$ $Y(s) = \frac{e^{-2s}}{s(s+1)} = \boxed{[1 - e^{-(t-2)}]u(t-2) = y(t)}$

(d) $H(s) = \frac{1}{s+1}$ $X(s) = \frac{e^{-2s}}{s+1}$ $Y(s) = \frac{e^{-2s}}{(s+1)^2}$

for $G(s) = \frac{1}{(s+1)^2}$ $g(t) = te^{-t}u(t)$

$Y(s) = e^{-2s}G(s)$ $y(t) = g(t-2)$

$\boxed{y(t) = (t-2)e^{-(t-2)}u(t-2)}$

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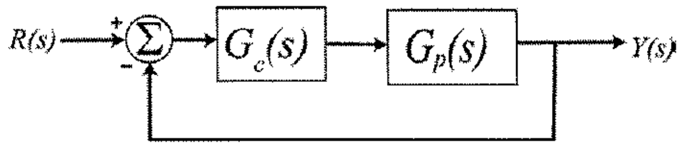
2) (25 points) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \dfrac{4}{s+2}$



a) Determine the settling time of the plant alone (assuming there is no feedback)

$$T_s = \frac{4}{2} = \boxed{2 = T_s}$$

b) Determine the steady state error for plant alone assuming the input is a unit step (simplify your answer)

$$e_{ss} = 1 - G_p(0) = 1 - 2 = \boxed{-1 = e_{ss}}$$

c) For a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function $G_0(s)$

$$G_0(s) = \frac{\frac{4}{s+2} k_p}{1 + \frac{4}{s+2} k_p} = \boxed{\frac{4k_p}{s+2+4k_p} = G_0(s)}$$

d) Determine the settling time of the closed loop system in terms of k_p

$$\boxed{T_s = \frac{4}{2+4k_p}}$$

e) Determine the steady state error of the closed loop system for a unit step, in terms of k_p (simplify your answer as much as possible)

$$e_{ss} = 1 - G_0(0) = 1 - \frac{4k_p}{2+4k_p} = \boxed{\frac{2}{2+4k_p} = e_{ss}}$$

f) For and integral controller, $G_c(s) = \dfrac{k_i}{s}$, determine the maximum positive value of k_i that produces purely real poles.

$$G_0(s) = \frac{\frac{4}{s+2} \frac{k_i}{s}}{1 + \frac{4}{s+2} \frac{k_i}{s}} = \frac{4k_i}{s^2 + 2s + 4k_i}$$

poles at $\dfrac{-2 \pm \sqrt{2^2 - 4(4k_p)}}{2}$

real if $4 - 16k_p > 0$

$4 > 16k_p$

$\frac{1}{4} > k_p$

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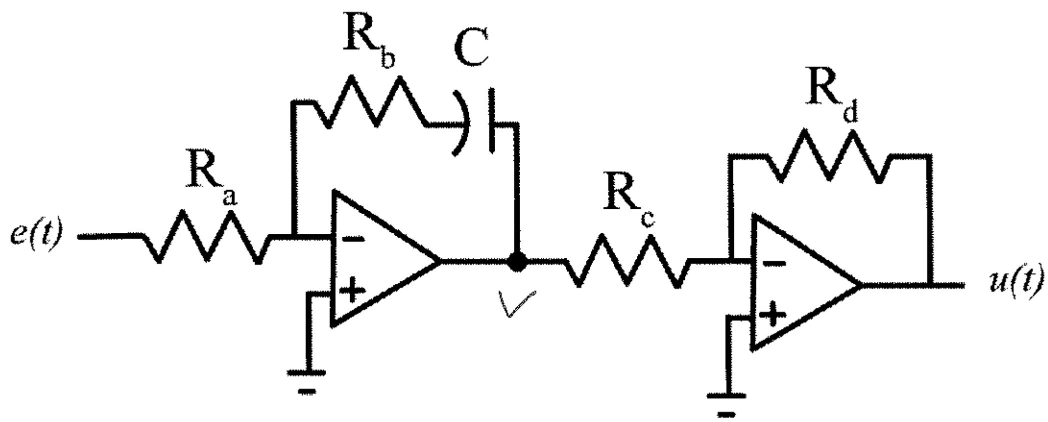
3) (25 points)

a) The following circuit can be used to implement the PI controller

$$G_c(s) = \frac{U(s)}{E(s)} = k_p + \frac{k_i}{s}$$

Determine expressions for k_p and k_i in terms of the parameters given in the circuit.



$$\frac{E}{R_a} + \frac{V}{R_b + \frac{1}{Cs}} = 0$$

$$\frac{E}{R_a} = \frac{-V}{R_b + \frac{1}{Cs}} = \frac{-V Cs}{R_b Cs + 1}$$

$$V = \frac{-E(R_b Cs + 1)}{R_a Cs}$$

$$\frac{V}{R_c} + \frac{U}{R_d} = 0$$

$$\frac{U}{R_d} = -\frac{1}{R_c} V = \frac{1}{R_c}\left[\frac{E(R_b Cs + 1)}{R_a Cs}\right]$$

$$\frac{U}{E} = \frac{R_d}{R_c}\left[\frac{R_b Cs + 1}{R_a Cs}\right]$$

$$= \underbrace{\left[\frac{R_d R_b}{R_c R_a}\right]}_{k_p} + \underbrace{\left[\frac{R_d}{R_c R_a C}\right]}_{k_i} \frac{1}{s}$$

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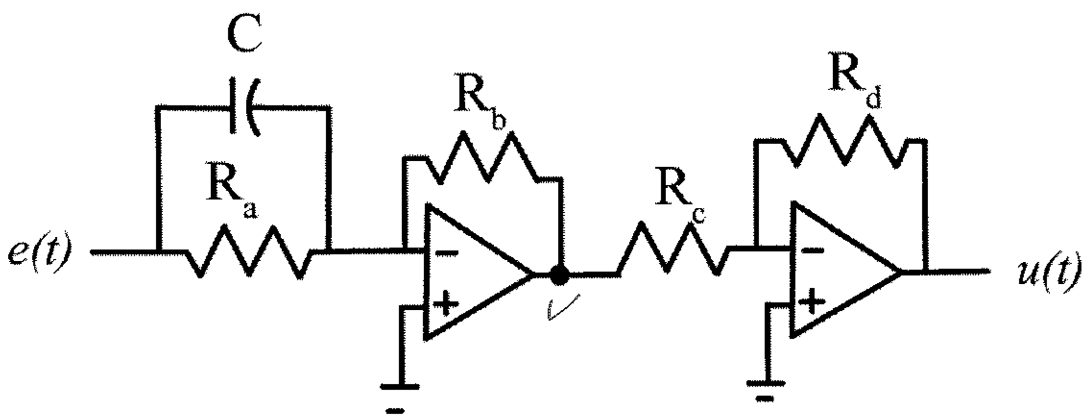
b) The following circuit can be used to implement the PD controller $G_c(s) = \frac{U(s)}{E(s)} = k_p + k_d s$

Determine expressions for k_p and k_d in terms of the parameters given in the circuit.



$$\frac{E}{R_a \| \frac{1}{Cs}} + \frac{V}{R_b} = 0 \qquad R_a \| \frac{1}{Cs} = \frac{R_a \frac{1}{Cs}}{R_a + \frac{1}{Cs}} = \frac{R_a}{R_aCs+1} \qquad \frac{V}{R_c} + \frac{u}{R_d} = 0$$

$$\frac{E(R_aCs+1)}{R_a} = -\frac{V}{R_b} \qquad V = \frac{-R_b}{R_a}(R_aCs+1)E \qquad u = -\frac{R_c}{R_d}V$$

$$u = \frac{R_c}{R_d}\frac{R_b}{R_a}(R_aCs+1)E$$

$$\frac{u}{E} = \left(\frac{R_cR_bC}{R_d}\right)s + \left(\frac{R_cR_b}{R_dR_a}\right)$$

k_d $\qquad$ k_p

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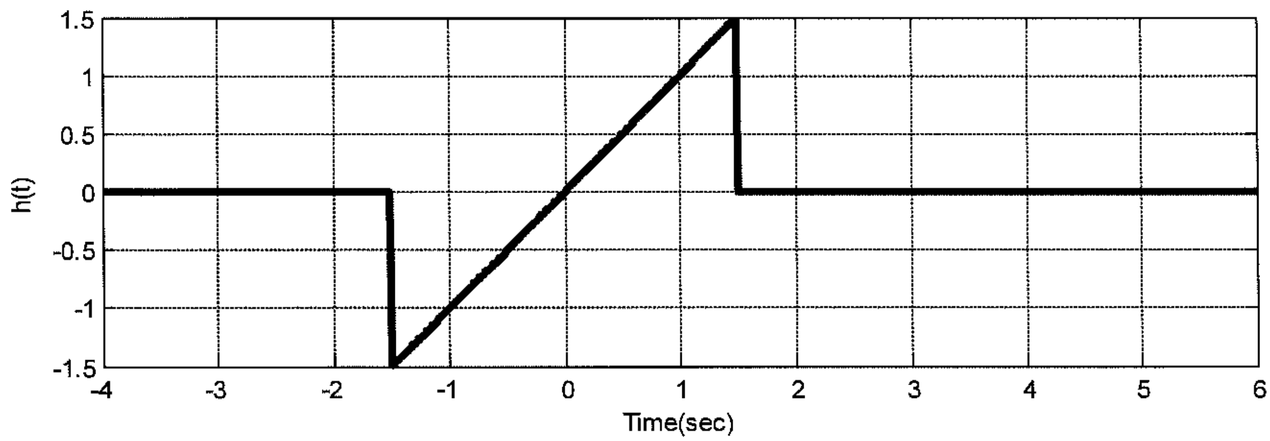
4) (25 points) Consider a linear time invariant system with impulse response given by

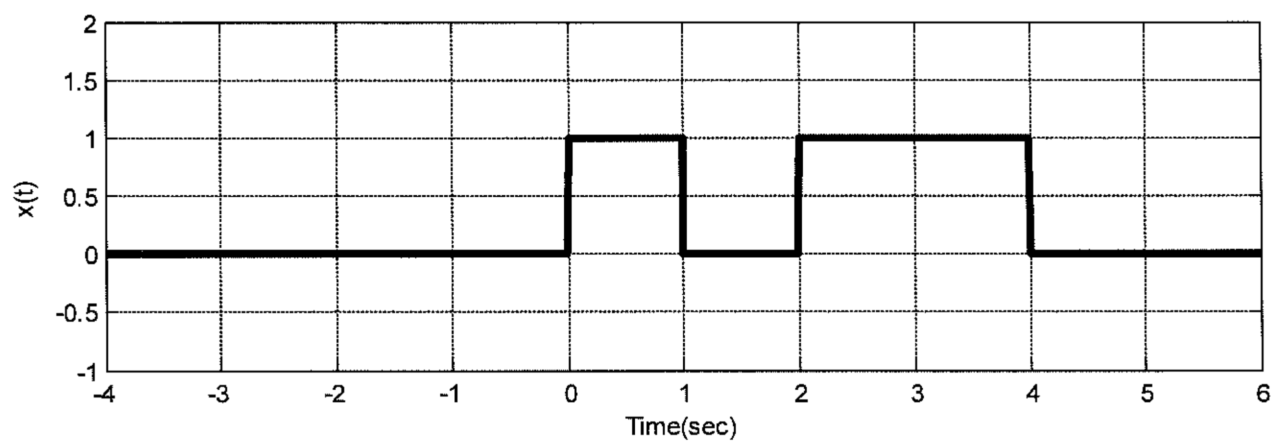
$$h(t) = t[(u(t+1.5) - u(t-1.5)]$$

The input to the system is given by

$$x(t) = [u(t) - u(t-1)] + [u(t-2) - u(t-4)]$$

The impulse response and input are shown below:





Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

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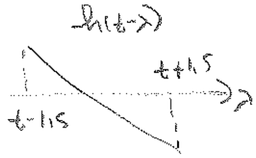
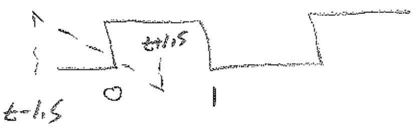


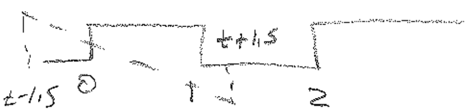
$h(t-\lambda) = (t-\lambda)$



$0 < t+1.5 < 1$

$\boxed{-1.5 < t < -0.5}$ ①

$$\boxed{y(t) = \int_0^{t+1.5} (t-\lambda)\,d\lambda}$$

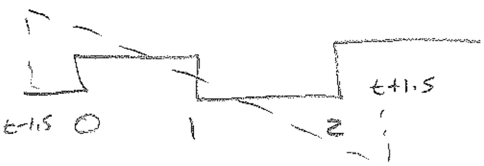


$t-1.5 < 0 \quad t < 1.5$

$t+1.5 < 2 \quad t < 0.5$

$\boxed{-0.5 < t < 0.5}$ ②

$$\boxed{y(t) = \int_0^{1} (t-\lambda)\,d\lambda}$$

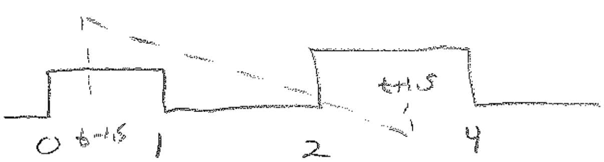


$t-1.5 < 0 \quad t < 1.5$

$t+1.5 > 2 \quad t > 0.5$

$\boxed{0.5 < t < 1.5}$ ③

$$\boxed{y(t) = \int_0^{1} (t-\lambda)\,d\lambda + \int_2^{t+1.5} (t-\lambda)\,d\lambda}$$

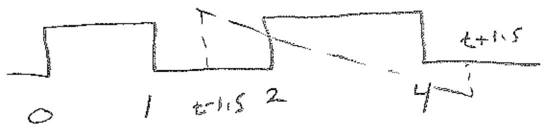


$0 < t-1.5 < 1 \qquad 2 < t+1.5 < 4$

$1.5 < t < 2.5 \qquad 0.5 < t < 2.5$

$\boxed{1.5 < t < 2.5}$ ④

$$\boxed{y(t) = \int_{t-1.5}^{1} (t-\lambda)\,d\lambda + \int_2^{t+1.5} (t-\lambda)\,d\lambda}$$

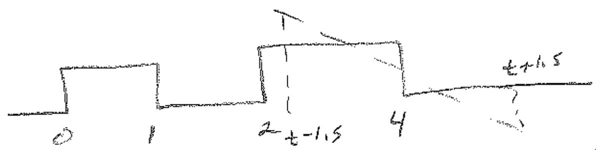


$t-1.5 < 2 \quad t < 3.5$

$t+1.5 > 4 \quad t > 2.5$

$\boxed{2.5 < t < 3.5}$ ⑤

$$\boxed{y(t) = \int_2^{4} (t-\lambda)\,d\lambda}$$



$2 < t-1.5 < 4$

$\boxed{3.5 < t < 5.5}$ ⑥

$$\boxed{y(t) = \int_{t-1.5}^{4} (t-\lambda)\,d\lambda}$$

$$\boxed{y(t) = 0 \begin{cases} t < -1.5 \\ t > 5.5 \end{cases}}$$