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ECE-205

Exam 2

Winter 2011

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 ________/18

Problem 2 ________/20

Problem 3 ________/15

Problem 4 ________/18

Problem 5 ________/20

Problem 6 ________/9

Total ____________

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1) (18 points) Fill in the non-shaded part of the following table.

	Linear? (Y/N)	Time Invariant? (Y/N)	BIBO Stable? (Y/N)
$y(t) = 2x(t) + 3$	No	Yes	
$\dot{y}(t) - \cos(t)y(t) = x(t)$	Yes	No	
$y(t) = x(1-t)$	Yes	No	
$y(t) = \int_{-\infty}^{t} e^{(t-\lambda)} x(\lambda) d\lambda$			No
$y(t) = tx(t)$			No
$y(t) = \cos\left(\frac{1}{x(t)}\right)$			Yes

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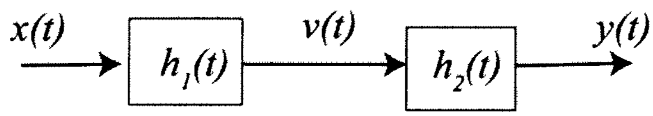
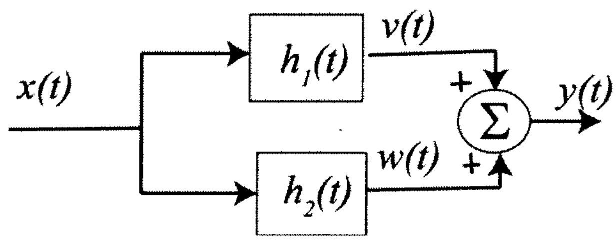
2) (20 points) For the following interconnected systems,

i) determine the overall impulse response (the impulse response between input *x(t)* and output *y(t)*) and

ii) determine if the system is causal.





a) $h_1(t) = \delta(t-2),\ h_2(t) = \delta(t+1)$

Parallel Connection: $\boxed{h(t) = \delta(t-2) + \delta(t+1)}$ $\boxed{\text{not causal}}$

Series Connections: $h(t) = \int_{-\infty}^{\infty} h_1(t-\lambda)\, h_2(\lambda)\, d\lambda = \int_{-\infty}^{\infty} \delta(t-\lambda-2)\, \delta(\lambda+1)\, d\lambda = \delta(t-1)$

$\boxed{h(t) = \delta(t-1)}$ $\boxed{\text{causal}}$

b) $h_1(t) = e^{-(t-1)}u(t-1),\ h_2(t) = u(t)$

Parallel Connection: $\boxed{h(t) = e^{-(t-1)}u(t-1) + u(t)}$ $\boxed{\text{causal}}$

Series Connections: $h(t) = \int_{-\infty}^{\infty} h_1(\lambda)\, h_2(t-\lambda)\, d\lambda = \int_{-\infty}^{\infty} e^{-(\lambda-1)} u(\lambda-1)\, u(t-\lambda)\, d\lambda$

$= \int_1^t e^1 e^{-\lambda}\, d\lambda = e^1 \left[-e^{-\lambda} \Big|_1^t \right] u(t-1)$

$= e^1 \left[-e^{-t} + e^{-1} \right] u(t-1)$

$\boxed{h(t) = \left[1 - e^{-(t-1)} \right] u(t-1)}$ $\boxed{\text{causal}}$

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3) (18 Points) Determine the impulse response for the following systems. Don't forget any necessary unit step functions.

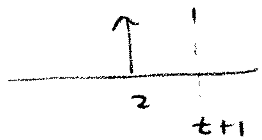
a) $y(t) = x(t-1) + x(t+1)$

b) $y(t) = \int_{-\infty}^{t+1} e^{-(t-\lambda)} x(\lambda - 2) d\lambda$

c) $3\dot{y}(t) - y(t) = 2x(t-1)$

a) $\boxed{h(t) = \delta(t-1) + \delta(t+1)}$

b) $h(t) = \int_{-\infty}^{t+1} e^{-(t-\lambda)} \delta(\lambda - 2) d\lambda$



$t+1 > 2$

$t-1 > 0$

$\boxed{h(t) = e^{-(t-2)} u(t-1)}$

c) $3\dot{h}(t) - h(t) = 2\delta(t-1)$

$\dot{h}(t) - \frac{1}{3} h(t) = \frac{2}{3}\delta(t-1)$

$\frac{d}{dt}\left[h(t) e^{-t/3}\right] = \frac{2}{3} e^{-t/3} \delta(t-1) = \frac{2}{3} e^{-1/3} \delta(t-1)$

$h(t) e^{-t/3} = \frac{2}{3} e^{-1/3} u(t-1)$

$\boxed{h(t) = \frac{2}{3} e^{(t-1)/3} u(t-1)}$

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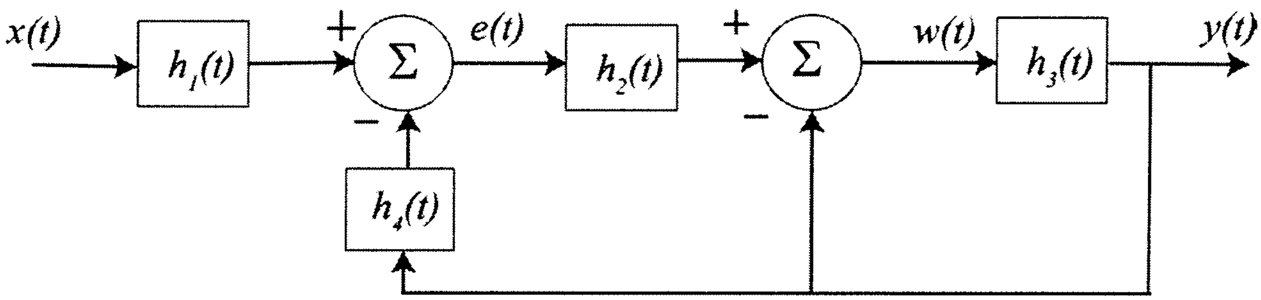
4) (15 points) The input-output relationship for the following system can be written as

$$y(t) * A(t) = x(t) * B(t)$$

Determine $A(t)$ and $B(t)$



$e(t) = x(t) * h_1(t) - y(t) * h_4(t)$ $\quad w(t) = e(t) * h_2(t) - y(t)$

$$y(t) = w(t) * h_3(t)$$
$$= [e(t) * h_2(t) - y(t)] * h_3(t)$$
$$= e(t) * h_2(t) * h_3(t) - y(t) * h_3(t)$$
$$= [x(t) * h_1(t) - y(t) * h_4(t)] * h_2(t) * h_3(t) - y(t) * h_3(t)$$
$$= x(t) * h_1(t) * h_2(t) * h_3(t) - y(t) * h_2(t) * h_3(t) * h_4(t) - y(t) * h_3(t)$$

$$y(t) + y(t) * h_3(t) + y(t) * h_2(t) * h_3(t) * h_4(t) = x(t) * h_1(t) * h_2(t) * h_3(t)$$

$$y(t) * \underbrace{[\delta(t) + h_3(t) + h_2(t) * h_3(t) * h_4(t)]}_{A(t)} = x(t) * \underbrace{[h_1(t) * h_2(t) * h_3(t)]}_{B(t)}$$

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5) (20 points) Consider a linear time invariant system with impulse response given by

$$h(t) = e^{-(t-1)}u(t-1)$$

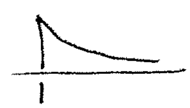
The input to the system is given by

$$x(t) = [u(t-1) - u(t-2)] + u(t-3)$$

Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

$h(t)$

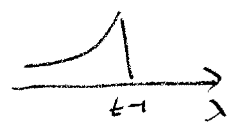


$h(1) = h(t-\lambda)$

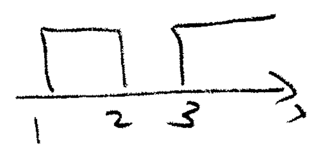
$1 = t-\lambda$

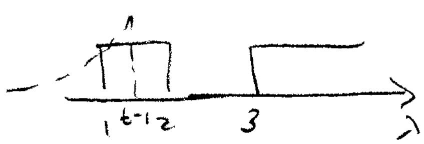
$\lambda = t-1$

$h(t-\lambda)$



$x(\lambda)$

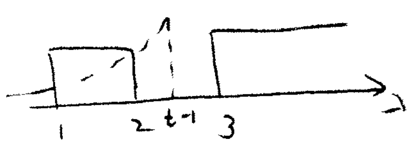




$1 < t-1 < 2$

$\boxed{2 < t < 3}$

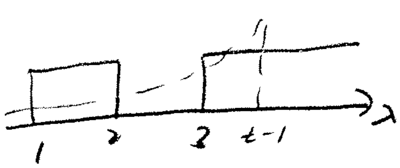
$$\boxed{y(t) = \int_1^{t-1} e^{-(t-\lambda-1)}\,d\lambda}$$



$2 < t-1 < 3$

$\boxed{3 < t < 4}$

$$\boxed{y(t) = \int_1^{2} e^{-(t-\lambda-1)}\,d\lambda}$$



$t-1 > 3$

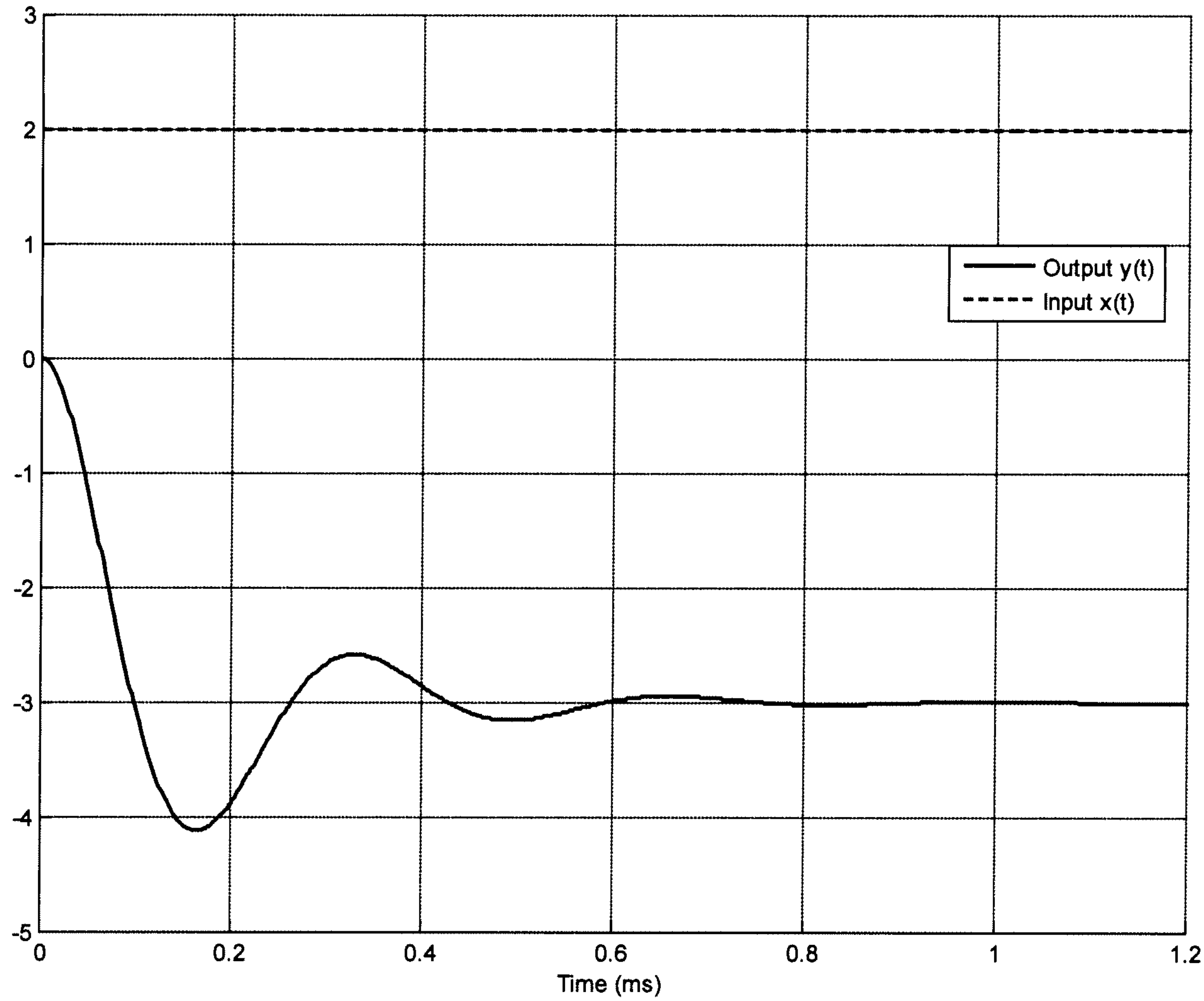
$\boxed{t > 4}$

$$\boxed{y(t) = \int_1^{2} e^{-(t-\lambda-1)}\,d\lambda + \int_3^{t-1} e^{-(t-\lambda-1)}\,d\lambda}$$

$$\boxed{y(t) = 0 \quad t < 2}$$

Name ______________________________ Mailbox ______________

6) (9 points) This problem refers the following graph showing the response of a second order system to a step input.



a) The percent overshoot for this system is best estimated as

$\frac{-4-(-3)}{-3} = \frac{1}{3}$

a) 400% b) -400 % c) 300% d) -300 % e) -33% f) 33%

b) The (2%) settling time for this system is best estimated as

a) 0.3 ms b) 0.6 ms c) 1.0 ms d) 1.2 ms

c) The static gain for this system is best estimated as

$K2 = -3$

$K = \frac{-3}{2}$

a) 1.5 b) 3 c) -1.5 d) -3

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