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ECE-205

Exam 2

Winter 2010-2011

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 __________/20

Problem 2 __________/30

Problem 3 __________/15

Problem 4 __________/20

Problem 5 _________/15

Total ________________

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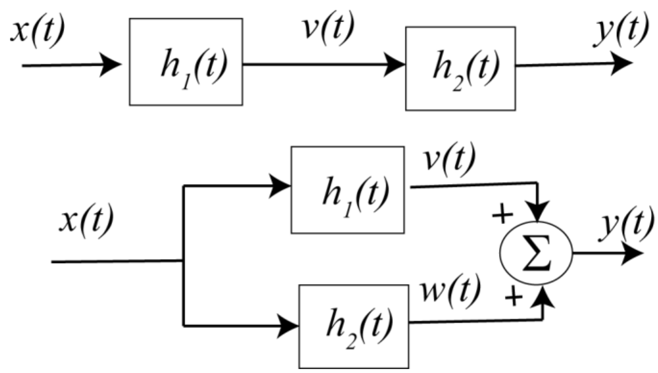
1) (20 points) For the following interconnected systems,

i) determine the overall impulse response (the impulse response between input *x(t)* and output *y(t)*)

ii) determine if the system is causal.



a) $h_1(t) = \mathrm{u}(t+2),\ h_2(t) = \delta(t-1)$

Systems in series:

Systems in parallel:

b) $h_1(t) = \mathrm{e}^{-(t-3)}u(t-3),\ h_2(t) = e^{(t+2)}u(t+2)$

Systems in series:

Systems in parallel:

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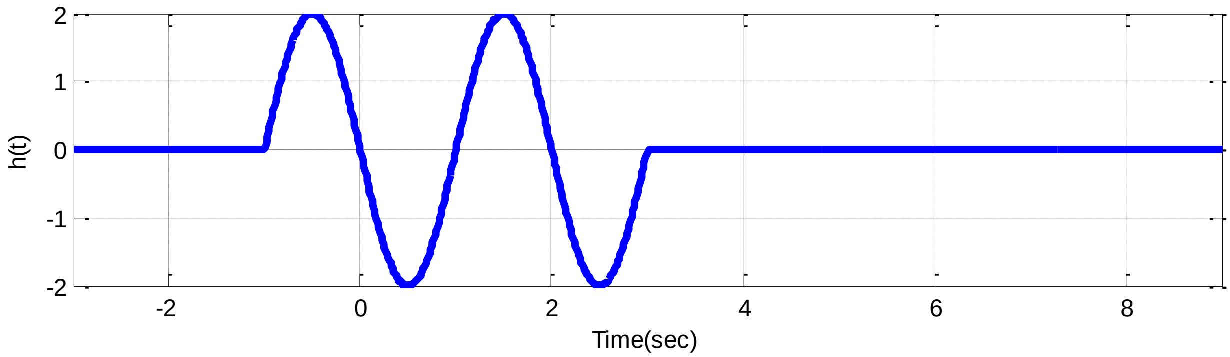
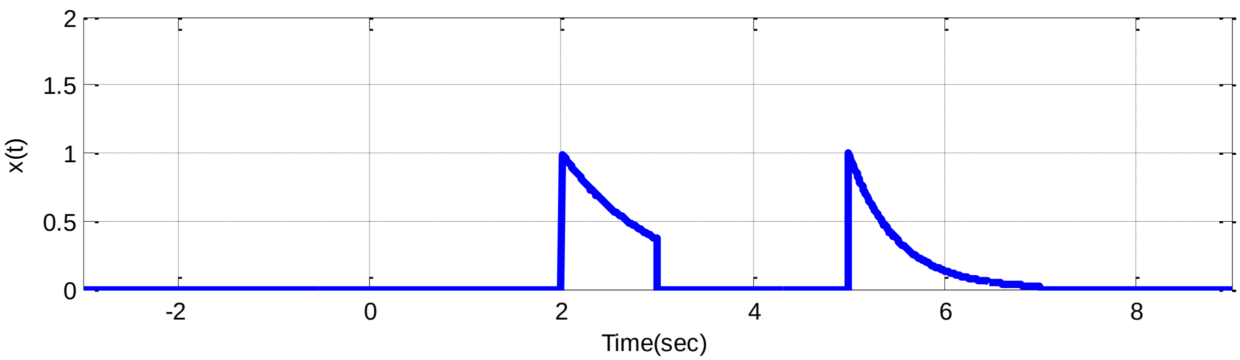
2) (30 points) Consider a linear time invariant system with impulse response given by

$$h(t) = 2\sin(\pi t)[u(t+1) - u(t-3)]$$

The input to the system is given by $x(t) = e^{-(t-2)}[u(t-2) - u(t-3)] - e^{-2(t-5)}[u(t-5) - u(t-7)]$





Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

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3) **(15 points)** Determine the **impulse response** for the following LTI systems. Do not forget any necessary unit step functions.

a) $y(t) = 2x(t) + \int_{-\infty}^{t+1} e^{-(t-\lambda)} x(\lambda - 2)\, d\lambda$

b) $2\dot{y}(t) + y(t) = 3x(t-1)$

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4) (20 points) Fill in the following table with a Y (yes) or N (no) for each of the system models given. Assume $-\infty < t < \infty$ for all of the systems and all initial conditions are zero.

System	System Model	Linear?	Time-Invariant?	Causal?	Memoryless?
1	$y(t) = x(-t)$				
2	$y(t) = \frac{1}{1 + x(t)}$				
3	$y(t) = x(t-1) + x(t-2)$				
4	$y(t) = \int_{-\infty}^{t} \lambda x(\lambda + 1)$				
5	$\dot{y}(t) + \sin(t) y(t) = e^{t+1} x(t)$				

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5) (15 points) In this problem you will derive some important relationships for second order systems that we derived in class.

Consider the system described by the second order differential equation

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = K\omega_n^2 x(t)$$

If the system is initially at rest and the is input $x(t) = Au(t)$, the output can be written as

$$y(t) = KA\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi)\right]$$

where $\tan(\phi) = \frac{\sqrt{1-\zeta^2}}{\zeta}$ and $\omega_d = \omega_n\sqrt{1-\zeta^2}$

a) Time to Peak (Tp). To determine the time to peak, take the derivative of $y(t)$ and determine the time at which the derivative is zero. Remember that the tangent is periodic with period π.

You should get $T_p = \frac{\pi}{\omega_d}$. Even if you don't, use this value in part b.

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b) Percent Overshoot (PO). The percent overshoot is defined as the value at the peak $y(T_p)$ minus the steady state value $y(\infty)$ divided by the steady state value, and then multiplied by 100%,

$$PO = \frac{y(T_p) - y(\infty)}{y(\infty)} \times 100\%$$

i) Determine the steady state value $y(\infty)$

ii) Using your knowledge that $\tan(\phi) = \frac{\sqrt{1-\zeta^2}}{\zeta}$, draw a triangle and determine an expression for $\sin(\phi)$

iii) Using the answer to ii and the fact that $\sin(\pi + \phi) = -\sin(\phi)$, determine $y(T_p)$

iv) Determine an expression for the percent overshoot. You should get $PO = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\%$

c) Settling Time (Ts). Determine the time constant for the system, and then the 2% settling time (4 time constants)