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ECE-205

Exam 1

Winter 2010-2011

Calculators can only be used for simple calculations. Solving integrals, differential equations, systems of equations, etc. does not count as a simple calculation.

You must show your work to receive credit.

Problem 1 __________/20

Problem 2 __________/30

Problem 3 __________/18

Problem 4-11 __________/32

Total ______________

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1) (20 points) Assume we have a first order system with the governing differential equation

$$0.6\dot{y}(t) + y(t) = 2x(t)$$

The system has the initial value of 2, so $y(0) = 2$. The input to this system is

$$x(t) = \begin{cases} 0 & t < 0 \\ 1 & 0 \le t < 1 \\ -2 & 1 \le t < 3 \\ 3 & 3 < t \end{cases}$$

Determine the output of the system in each of the above time intervals. *Simplify your final answer as much as possible and box it. Be sure to include the correct initial value in the first interval!*

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2) (30 points) For the following three differential equations, assume the input is $x(t) = 4u(t)$ (the input is equal to four for time greater than zero), and the initial conditions are $y(0) = \dot{y}(0) = 0$

Determine the solution to each of the following differential equations and put your final answer in a box. Be sure to use the initial conditions to solve for all unknowns. You must show all your work to receive credit.

a) $\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = x(t)$

b) $\ddot{y}(t) + 4\dot{y}(t) + 4y(t) = 8x(t)$

c) $\ddot{y}(t) + 4\dot{y}(t) + 16y(t) = 4x(t)$

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3) (18 points) The form of the under damped ($0<\zeta<1$) solution to the second order differential equation

$$\ddot{y}(t)+2\zeta\omega_n\dot{y}(t)+\omega_n^2 y(t)=\mathrm{K}\ \omega_n^2 x(t)$$

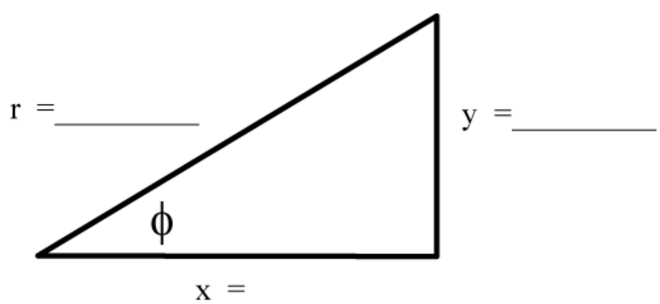
for a step input $x(t)=Au(t)$ is

$$y(t)=KA+ce^{-\zeta\omega_n t}\sin(\omega_d t+\phi)$$

where c and ϕ are constants to be determined and the damped frequency $\omega_d=\omega_n\sqrt{1-\zeta^2}$

a) Using the initial condition $\dot{y}(0)=0$ show that $\tan(\phi)=\dfrac{\sqrt{1-\zeta^2}}{\zeta}$

b) We can express the relationship in part a using the following triangle. Fill in the blanks and then use this triangle determine an expression for $\sin(\phi)$.



c) Use your answer to part b, and the initial condition $y(0)=0$ to determine the remaining unknown constant, and write out the complete solution for $y(t)$.

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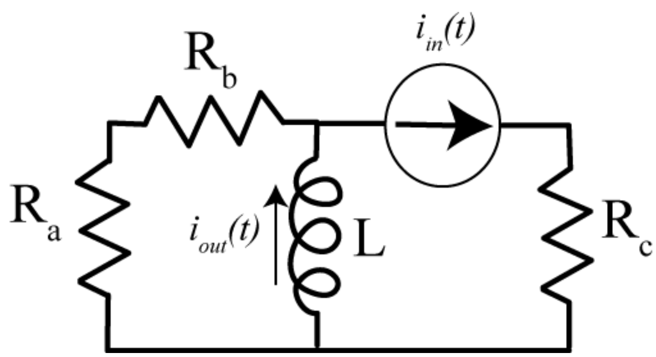
Problems 4-11, 4 points each, no partial credit (32 points)

Problems 4 and 5 refer to the following circuit



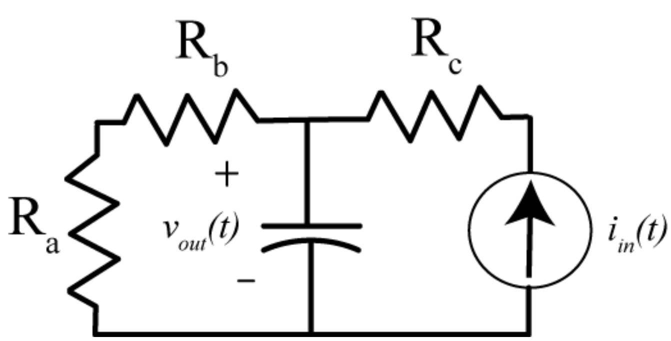
4) The Thevenin resistance seen from the ports of the inductor is

a) $R_{th} = R_c \,||\, (R_a + R_b)$ b) $R_{th} = R_c$ c) $R_{th} = R_a + R_b$ d) $R_{th} = R_a + R_b + R_c$ e) none of these

5) The static gain for the system is

a) $K = 1$ b) $K = \dfrac{R_a + R_b}{R_a + R_b + R_c}$ c) $K = \dfrac{R_c}{R_a + R_b + R_c}$ d) $K = \dfrac{R_c}{R_a + R_b}$ e) none of these

Problems 6 and 7 refer to the following circuit



6) The Thevenin resistance seen from the ports of the capacitor is

a) $R_{th} = R_a + R_b$ b) $R_{th} = R_c$ c) $R_{th} = R_c \,||\, (R_a + R_b)$ d) $R_{th} = R_a + R_b + R_c$ e) none of these

7) The static gain for the system is

a) $K = 1$ b) $K = R_c$ c) $K = R_a + R_b$ d) $K = R_c \,||\, (R_a + R_b)$ e) none of these

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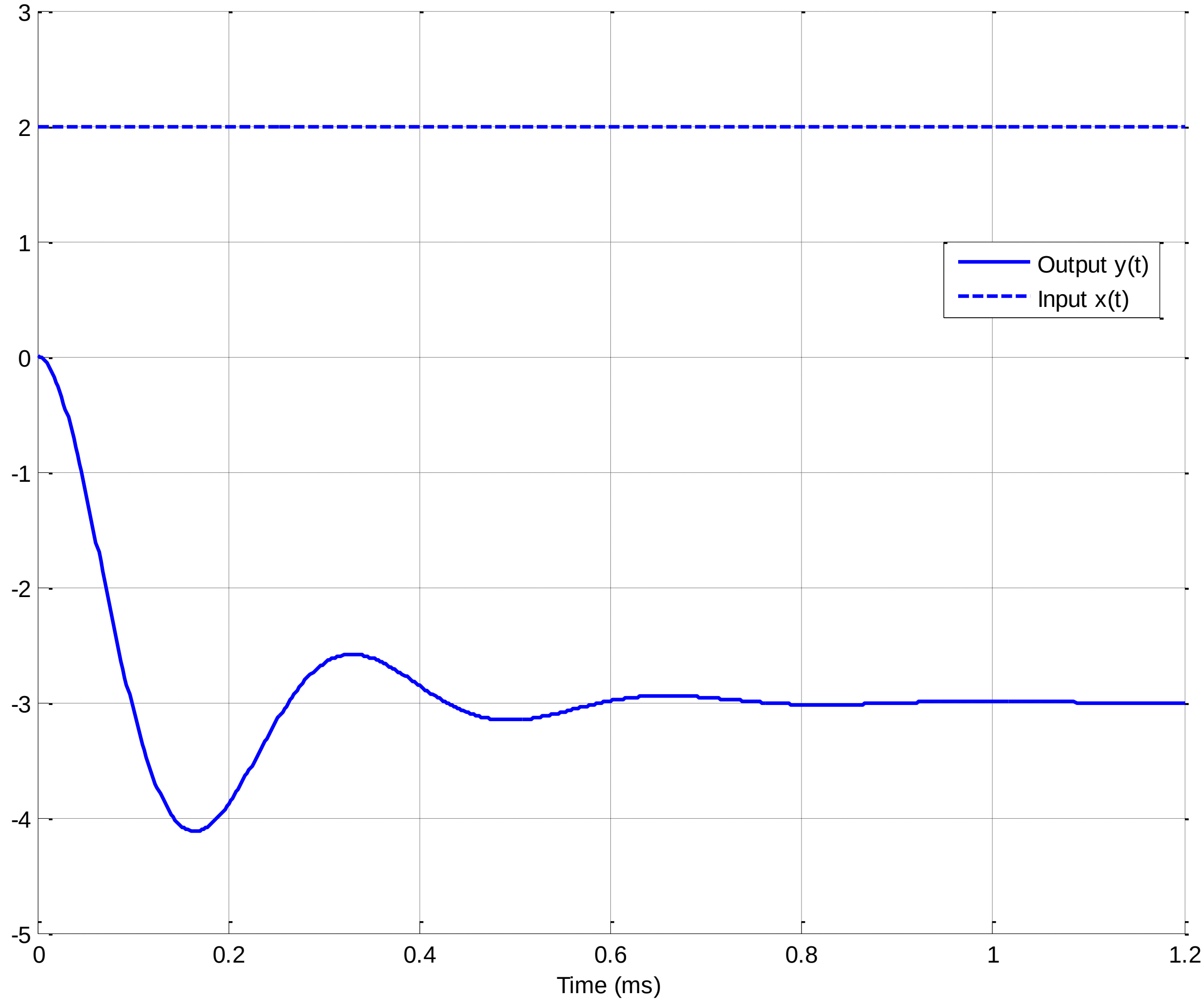
Problems 8 and 9 refer the following graph showing the response of a second order system to a step input.



8) The percent overshoot for this system is best estimated as

a) 400% b) -400 % c) 300% d) -300 % e) -33% f) 33%

9) The static gain for this system is best estimated as

a) 1.5 b) 3 c) -1.5 d) -3

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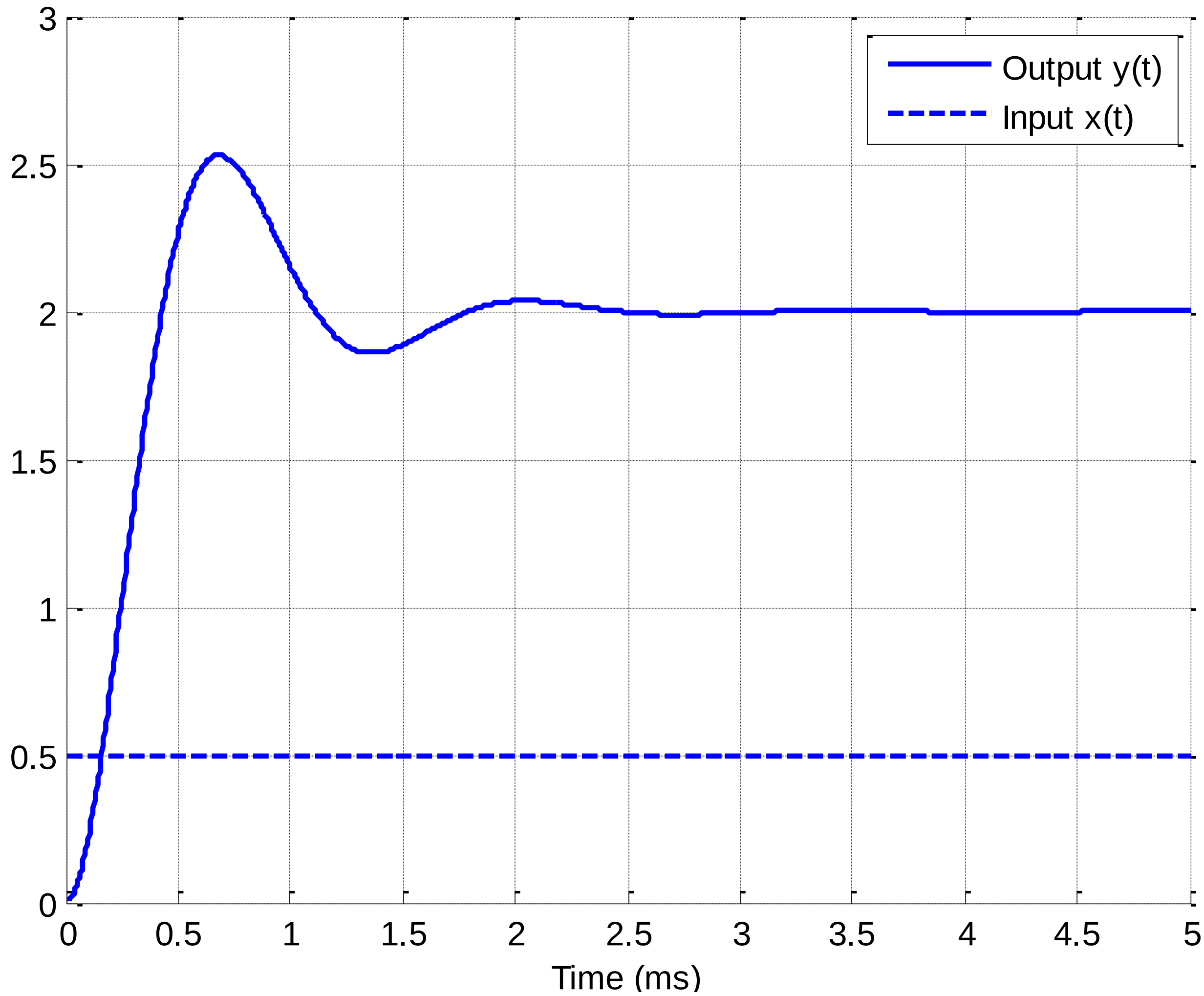
Problems 10-11 refer the following graph showing the response of a second order system to a step input.



10) The percent overshoot for this system is best estimated as

a) 400% b) 250 % c) 200% d) 150 % e) 100 % f) 25%

11) The static gain for this system is best estimated as

a) 1 b) 2 c) 3 d) 4

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