ECE-205 : Dynamical Systems

Homework #6

Due : <u>Friday</u> January 28 at noon

1) In this problem we will derive some useful properties of Laplace transforms starting from the basic relationship

$$X(s) = \int_{0^-}^{\infty} x(t)e^{-st}dt$$

a) Let's assume $x(t)$ is a causal signal (it is zero for $t < 0$). We can then write $x(t) = x(t)u(t)$ to emphasize the fact that $x(t)$ is zero before time zero. If there is a delay in the signal and it starts at time t_0, then we can write the signal as $x(t-t_0) = x(t-t_0)u(t-t_0)$ to emphasize the fact that the signal is zero before time t_0.

Using the definition of the Laplace transform and a simple change of variable in the integral, show that $\mathcal{L}\left\{x(t-t_0)u(t-t_0)\right\} = X(s)e^{-st_0}$

b) Using the results from part **a**, determine the inverse Laplace transform of $X(s) = \dfrac{e^{-3s}}{(s+2)(s+4)}$

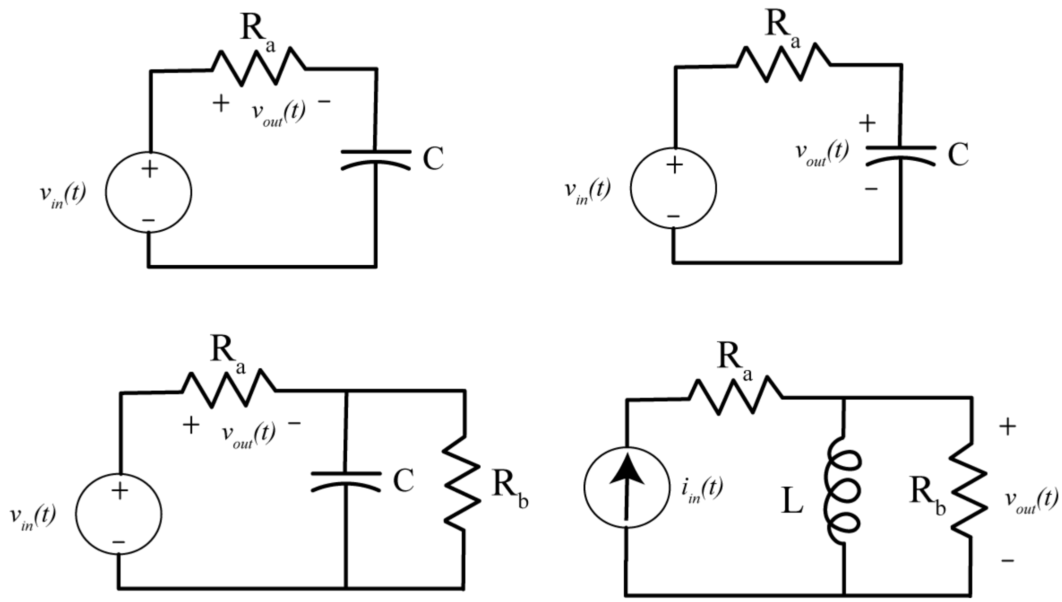
Answer: $x(t) = \dfrac{1}{2}\left[e^{-2(t-3)} - e^{-4(t-3)}\right]u(t-3)$

c) Starting from the definition of the Laplace transform, show that $\mathcal{L}\left\{tx(t)\right\} = -\dfrac{dX(s)}{ds}$.

d) Using the result from part c, and the transform pair $x(t) = e^{-at}u(t) \leftrightarrow X(s) = \dfrac{1}{s+a}$, and some simple calculus, show that

$$\mathcal{L}\left\{te^{-at}u(t)\right\} = \frac{1}{(s+a)^2},\ \mathcal{L}\left\{t^2e^{-at}u(t)\right\} = \frac{2}{(s+a)^3},\ \mathcal{L}\left\{t^3e^{-at}u(t)\right\} = \frac{6}{(s+a)^4}$$

2) For the following circuits, determine the transfer function and the corresponding impulse responses.



Scrambled Answers:

$$h(t) = R_b\delta(t) - \frac{R_b^2}{L}e^{-tR_b/L}u(t),\ h(t) = \delta(t) - \frac{1}{R_aC}e^{-t/R_aC}u(t),\ h(t) = \frac{1}{R_aC}e^{-t/RaC}u(t), h(t) = [\delta(t) - \frac{1}{R_aC}e^{-t\frac{R_a+R_b}{CR_aR_b}}]u(t)$$

3) For the following impulse responses and inputs, compute the system output using transfer functions.

a) $h(t) = e^{-t}u(t),\ x(t) = u(t)$ **b)** $h(t) = e^{-2t}u(t),\ x(t) = \delta(t)$ **c)** $h(t) = e^{-2(t-1)}u(t-1),\ x(t) = e^{-2t}u(t)$

d) $h(t) = e^{-t}u(t),\ x(t) = (t-1)u(t-1)$ **e)** $h(t) = e^{-2t}u(t),\ x(t) = u(t) - u(t-1)$

f) $h(t) = e^{-2(t-1)}u(t-1),\ x(t) = te^{-3t}u(t)$

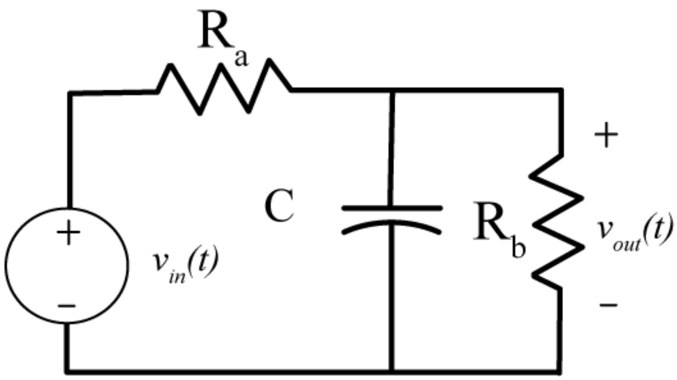
Scrambled Answers :

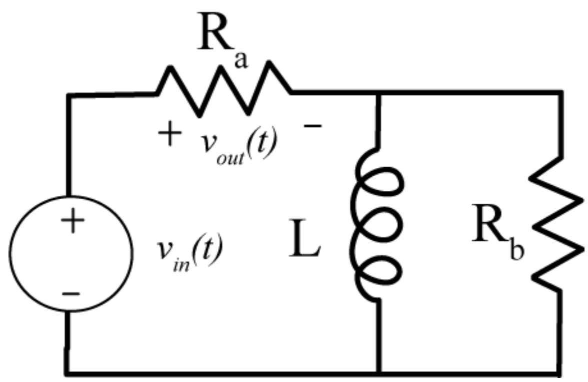
$$y(t) = \frac{1}{2}\left[1 - e^{-2t}\right]u(t) - \frac{1}{2}\left[1 - e^{-2(t-1)}\right]u(t-1),\ y(t) = \left(1 - e^{-t}\right)u(t),\ y(t) = (t-1)e^{-2(t-1)}u(t-1)$$

$$y(t) = e^{-2t}u(t),\ y(t) = [-1 + (t-1) + e^{-(t-1)}]u(t-1),$$

$$y(t) = [e^{-2(t-1)} - e^{-3(t-1)} - (t-1)e^{-3(t-1)}]u(t-1)$$

4) For the following circuits, determine and expression for the output $V_{out}(s)$ in terms of the ZSR and ZIR. Do not assume the initial conditions are zero. Also determine the system transfer function.





Answers:

$$V_{out}(s) = \left[\frac{R_b}{R_a R_b C s + R_a + R_b}\right] V_{in}(s) + \left[\frac{R_a R_b C}{R_a R_b C s + R_a + R_b}\right] v(0^-)$$

$$V_{out}(s) = \left[\frac{R_a(R_b + Ls)}{(R_a + R_b)Ls + R_a R_b}\right] V_{in}(s) + \left[\frac{R_a R_b L}{(R_a + R_b)Ls + R_a R_b}\right] i(0^-)$$

5) For the following transfer functions

$$H(s) = \frac{2}{s^2 + 2s + 2} \quad H(s) = \frac{3}{s^2 + 4s + 6} \quad H(s) = \frac{5}{s^2 + 6s + 10}$$

$$H(s) = \frac{4}{s^2 - 4s + 7} \quad H(s) = \frac{1}{s^2 + 4}$$

By computing the inverse Laplace transform show that the step responses are given by

$$y(t) = \left[1 - e^{-t}\cos(t) - e^{-t}\sin(t)\right]u(t) \quad y(t) = \left[\frac{1}{2} - \frac{1}{\sqrt{2}}e^{-2t}\sin(\sqrt{2}t) - \frac{1}{2}e^{-2t}\cos(\sqrt{2}t)\right]u(t)$$

$$y(t) = \left[\frac{1}{2} - \frac{3}{2}e^{-3t}\sin(t) - \frac{1}{2}e^{-3t}\cos(t)\right]u(t) \quad y(t) = \left[\frac{4}{7} + \frac{8\sqrt{3}}{21}e^{2t}\sin(\sqrt{3}t) - \frac{4}{7}e^{2t}\cos(\sqrt{3}t)\right]u(t)$$

$$y(t) = \left[\frac{1}{4} - \frac{1}{4}\cos(2t)\right]u(t)$$