

ECE-205 : Dynamical Systems

Homework #4

Due : Friday January 7th at noon

1) For each of the following mathematical description of a system, determine if the system is linear, time invariant, causal, and memoryless and fill in the following table. Unless the system does not meet the homogeneity condition, you must use a formal technique to show the system is linear. If it does not meet the homogeneity condition, you must give an example. For the memoryless and causal you can just say it's obvious (assuming it is).

System	Linea (Y/N)	Time Invariant (Y/N)	Memoryless (Y/N)	Causal (Y/N)
$\dot{y}(t) = x(t)$				
$y(t) = \frac{1}{1+x(t)}$				
$\ddot{y}(t) + e^{-t}y(t) = \cos(t)x(t)$				
$y(t) = \int_{-\infty}^t e^{t+\lambda} x(\lambda) d\lambda$				
$y(t) = \int_{-\infty}^t e^{t-\lambda} x(\lambda+1) d\lambda$				
$y(t) = \int_{-\infty}^t e^{-\lambda} \lambda x(\lambda) d\lambda$				
$\dot{y}(t) + y(t)x(t) = x(t)$				

Answers: 5 are L, 4 are TI, 1 is memoryless, 6 are causal

2) For each of the following mathematical descriptions of a system, determine if the system is BIBO stable.

a) $y(t) = \frac{1}{1+x(t)}$ b) $y(t) = e^{-x(t)}$ c) $y(t) = \cos(x(t))$

d) $y(t) = \int_{-\infty}^t e^{-2(t-\lambda)} x(\lambda) d\lambda$ e) $y(t) = \int_{-\infty}^t e^{2(t-\lambda)} x(\lambda) d\lambda$ f) $y(t) = tx(t)$

Answers: 3 are BIBO stable

3) Assume we have an LTI system with the input-output relationship $x(t) = u(t) \rightarrow y(t) = e^{-2(t-1)}u(t-1)$. This means that if the input is a unit step starting at zero, the output is a decaying exponential starting at $t = 1$. Determine an expression for output the following inputs:

a) $x(t) = u(t-2)$ b) $x(t) = 6u(t+2)$ c) $x(t) = 2[u(t-1) - u(t-2)]$

The answer to c is $y(t) = 2e^{-2(t-2)}u(t-2) - 2e^{-2(t-3)}u(t-3)$

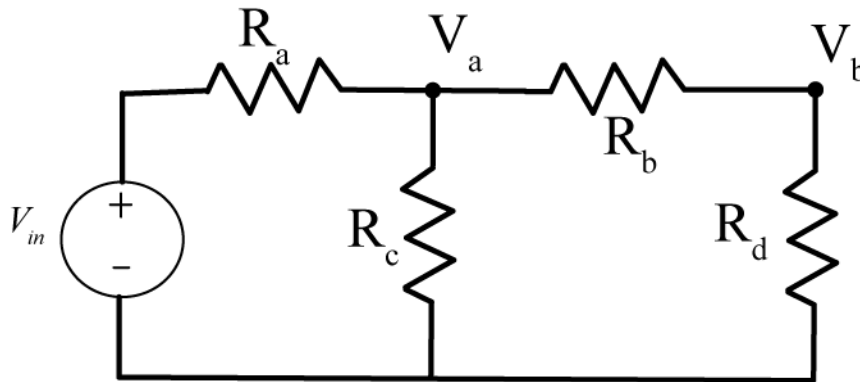
4) Matlab Problem

Read the Appendix at the end of this homework and then do the following:

a) Create an m-files to set up and solve matrix equations for the node values for the following circuits. Use **both** methods from the Appendix to set up the system of equations.

Assume $R_a = 1000\Omega$, $R_b = 5000\Omega$, $R_c = 2000\Omega$, $R_d = 1500\Omega$ and $V_{in} = 5V$.

Answer: $V_a = 3.023V$, $V_b = 0.698V$

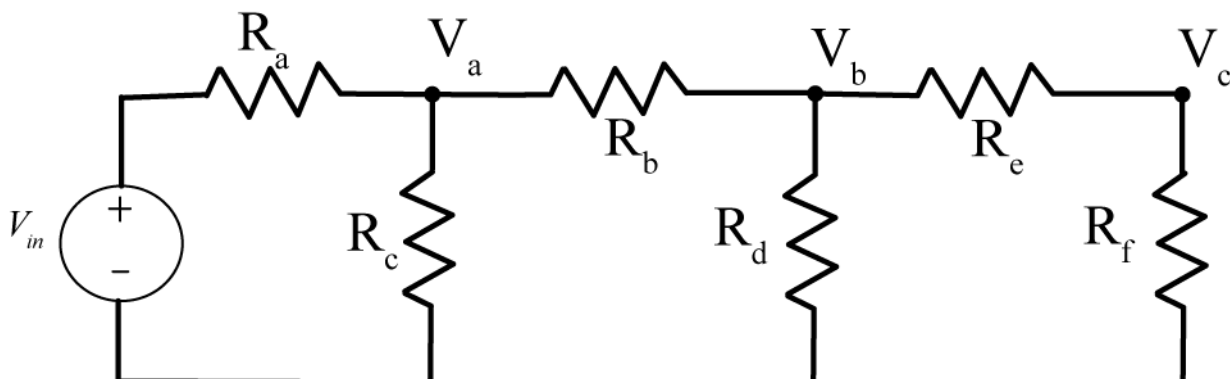


b) Create m-files to set up and solve matrix equations for the node values for the following circuit. Use **both** methods from the Appendix to set up the system of equations.

Assume $R_a = 1000\Omega$, $R_b = 5000\Omega$, $R_c = 2000\Omega$, $R_d = 1500\Omega$, $R_e = 500\Omega$, $R_f = 5000\Omega$ and $V_{in} = 5V$.

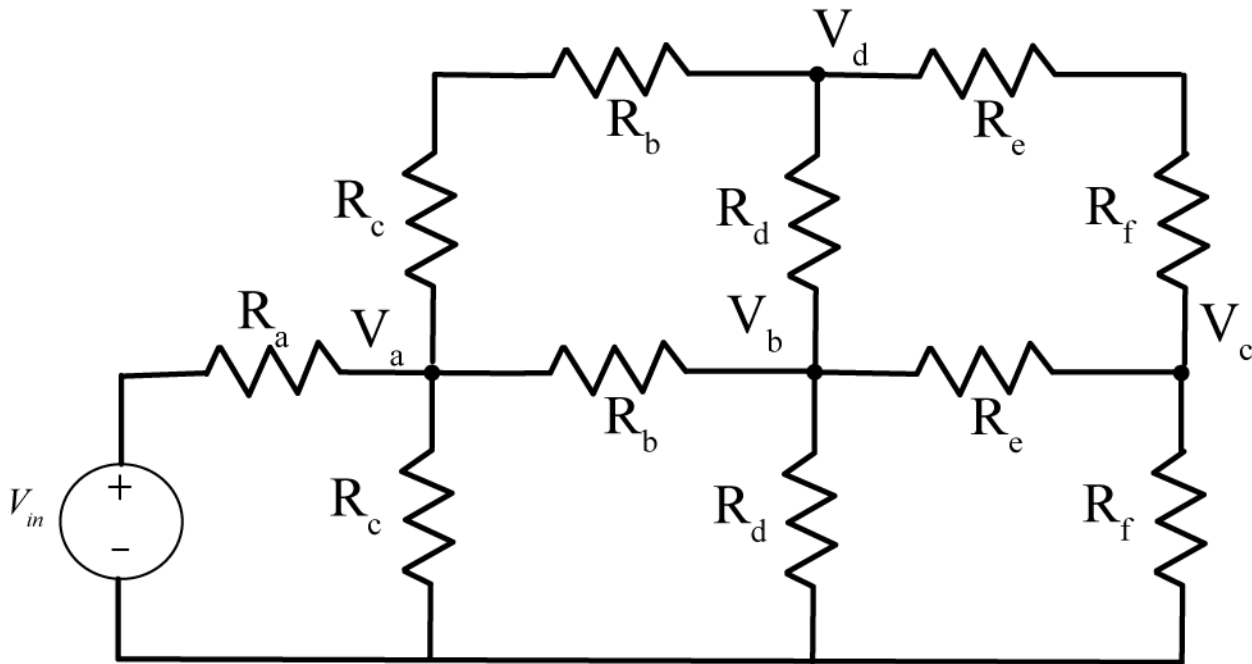
Answer: $V_a = 3.0087V$, $V_b = 0.5739V$, $V_c = 0.5217V$

Turn in your m-files and write the node voltages obtained on your code.



c) Create m-files to set up and solve matrix equations for the node values for the following circuit. Use ***both*** methods from the Appendix to set up the system of equations.

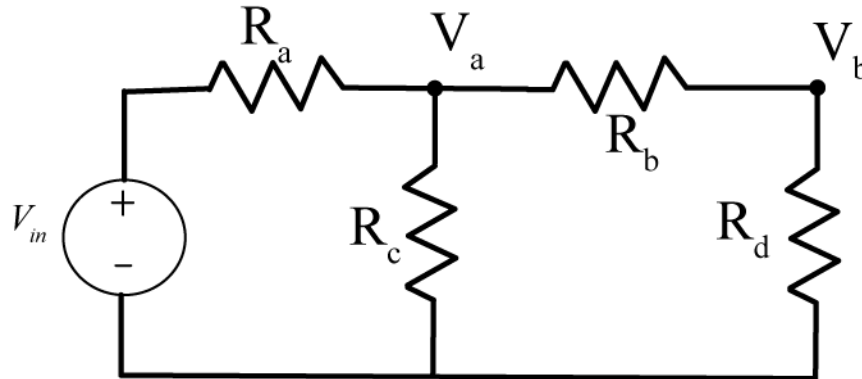
Assume $R_a = 1000\Omega$, $R_b = 5000\Omega$, $R_c = 2000\Omega$, $R_d = 1500\Omega$, $R_e = 500\Omega$, $R_f = 5000\Omega$ and $V_{in} = 5V$. Turn in your m-files and write the node voltages obtained on your code.



Appendix

In addition to working well with vectors, Matlab is also very good at matrices. In this Appendix we will show you how to set up and solve matrix equations in Matlab.

Consider the following circuit, with two nodes V_a and V_b .



We can write node equations at these two nodes as

$$\frac{V_a - V_{in}}{R_a} + \frac{V_a}{R_c} + \frac{V_a - V_b}{R_b} = 0$$
$$\frac{V_b - V_a}{R_b} + \frac{V_b}{R_d} = 0$$

Rearranging these we get

$$\left[\frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_c} \right] V_a + \left[-\frac{1}{R_b} \right] V_b = \left[\frac{1}{R_a} \right] V_{in}$$
$$\left[-\frac{1}{R_b} \right] V_a + \left[\frac{1}{R_b} + \frac{1}{R_d} \right] V_b = 0$$

We can then write these in standard matrix form, $Ax = b$, as

$$\begin{bmatrix} \frac{1}{R_a} + \frac{1}{R_b} + \frac{1}{R_c} & -\frac{1}{R_b} \\ -\frac{1}{R_b} & \frac{1}{R_b} + \frac{1}{R_d} \end{bmatrix} \begin{bmatrix} V_a \\ V_b \end{bmatrix} = \frac{1}{R_a} \begin{bmatrix} V_{in} \\ 0 \end{bmatrix}$$

Before we put this into Matlab, we need to assign some values to the resistors. Let's assume we have the values

$R_a = 1000\Omega$, $R_b = 5000\Omega$, $R_c = 2000\Omega$, $R_d = 1500\Omega$ and $V_{in} = 5V$. We can now construct our matrices in two different ways in Matlab, as shown by the following code segment

```

%
% circuit parameters
%
Ra = 1000; Rb = 5000; Rc=2000;
Rd = 1500; Re = 500; Rf = 5000;
Vin = 5;
%
% Method 1
%
% create matrices and vectors that are all zeros, then fill them in
%
A = zeros(2,2); % construct a 2x2 matrix filled with zeros
b = zeros(2,1); % construct a 2x1 vector filled with zeros
%
% fill in the values
%
A(1,1) = 1/Ra + 1/Rb + 1/Rc;
A(1,2) = -1/Rb;
A(2,1) = -1/Rb;
A(2,2) = 1/Rb + 1/Rd;
b(1) = (1/Ra)*Vin;
%
% solving the matrix equation
%
x = A\b
%
% Method 2
%
% construct the matrices and vectors directly
%
% note that the columns are separated by spaces, and each row
% ends with a semicolon (;)
%
A = [(1/Ra+1/Rb+1/Rc) -1/Rb; -1/Rb (1/Rb + 1/Rd)];
b = [(1/Ra)*Vin; 0]
%
% solving the matrix equation
%
x = A\b
%
% check your answer
%
A*x
b
%
% extract the node voltages
%
Va = x(1)
Vb = x(2)

```