ECE-205 Solutions

Exam 3

Winter 2010-2011

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 _________/30

Problem 2 _________/20

Problem 3 _________/20

Problem 4 _________/15

Problems 5-9_______/15 (3 points each)

Total _____________

1) (30 points) For the following transfer functions, determine the <u>unit step response</u> of the system. *Do not forget any necessary unit step functions.*

a) $H(s) = \dfrac{1}{(s+1)(s+2)}$

b) $H(s) = \dfrac{e^{-3s}}{(s+1)^2}$

c) $H(s) = \dfrac{13}{s^2+4s+13}$

(a) $Y(s) = \dfrac{1}{s(s+1)(s+2)} = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{C}{s+2}$ $\quad A = \frac{1}{2} \quad B = -1 \quad C = \frac{1}{2}$

$$\boxed{y(t) = \left(\frac{1}{2} - e^{-t} + \frac{1}{2}e^{-2t}\right)u(t)}$$

(b) $Y(s) = e^{-3s}\,G(s) \qquad G(s) = \dfrac{1}{s(s+1)^2} = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{C}{(s+1)^2}$ $\quad A = 1 \quad C = -1$

$\times s$ let $s \to \infty$ $\quad 0 = A + B$

$B = -1$

$g(t) = (1 - e^{-t} - te^{-t})u(t)$

$$\boxed{y(t) = \left[1 - e^{-(t-3)} - (t-3)e^{-(t-3)}\right]u(t-3)}$$

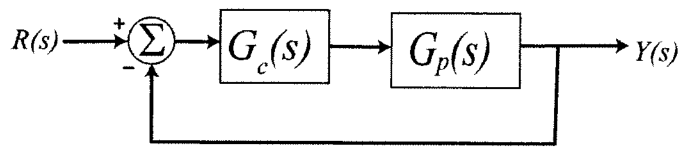
(c) $Y(s) = \dfrac{13}{s(s^2+4s+13)} = \dfrac{13}{s\left[(s+2)^2+3^2\right]} = \dfrac{A}{s} + \dfrac{B\,3}{(s+2)^2+9} + \dfrac{C(s+2)}{(s+2)^2+9}$

$A = 1$ $\quad \times s$, let $s \to \infty$ $\quad 0 = A + C \quad C = -1$

let $s = -2$ $\quad \dfrac{13}{(-2)(9)} = \dfrac{1}{-2} + \dfrac{3B}{9} \qquad 13 = 9 - 6B \qquad 4 = -6B \quad B = -2/3$

$$\boxed{y(t) = \left[1 - \frac{2}{3}e^{-2t}\sin(3t) - e^{-2t}\cos(3t)\right]u(t)}$$

2) (20 points) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \dfrac{5}{s+2}$



a) Determine the settling time of the plant alone (assuming there is no feedback)

b) For a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function $G_0(s)$ and then

  i) the settling time, in terms of k_p

  ii) the steady state error for a unit step, in terms of k_p

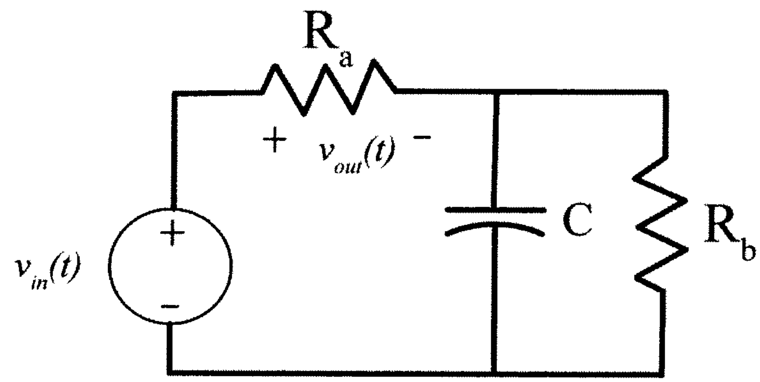
c) For and integral controller, $G_c(s) = \dfrac{k_i}{s}$, determine the closed loop transfer function $G_0(s)$ and the steady state error for a unit step in terms of k_i

(a) $T_s = \dfrac{4}{2} = \boxed{2 = T_s}$

(b) $\boxed{G_o(s) = \dfrac{5k_p}{s+2+5k_p}}$ $\boxed{T_s = \dfrac{4}{2+5k_p}}$ $\boxed{e_{ss} = \dfrac{2}{2+5k_p}}$

(c) $G_o(s) = \dfrac{\frac{k_i}{s}\frac{5}{s+2}}{1+\frac{k_i}{s}\frac{5}{s+2}} = \boxed{\dfrac{5k_i}{s^2+2s+5k_i} = G_o(s)}$ $\boxed{e_{ss} = 0}$ since $G_o(0) = 1$

3) (20 points) For the following circuit determine the *transfer function* and the corresponding *impulse response*.



$$\frac{V_{out}(s)}{R_a} = \frac{V_{in}(s) - V_{out}(s)}{1/Cs} + \frac{V_{in}(s) - V_{out}(s)}{R_b}$$

$$V_{out}(s)\left[\frac{1}{R_a} + \frac{1}{R_b} + Cs\right] = V_{in}(s)\left[Cs + 1/R_b\right]$$

$$V_{out}(s)\left[\frac{R_b + R_a + R_aR_bCs}{R_aR_b}\right] = V_{in}(s)\left[\frac{R_bCs+1}{R_b}\right]$$

$$\frac{V_{out}(s)}{V_{in}(s)} = H(s) = \left[\frac{R_bCs+1}{R_b}\right]\left[\frac{R_aR_b}{R_a+R_b+R_aR_bCs}\right] = \boxed{\frac{R_aR_bCs+R_a}{R_aR_bCs+R_a+R_b} = H(s)}$$

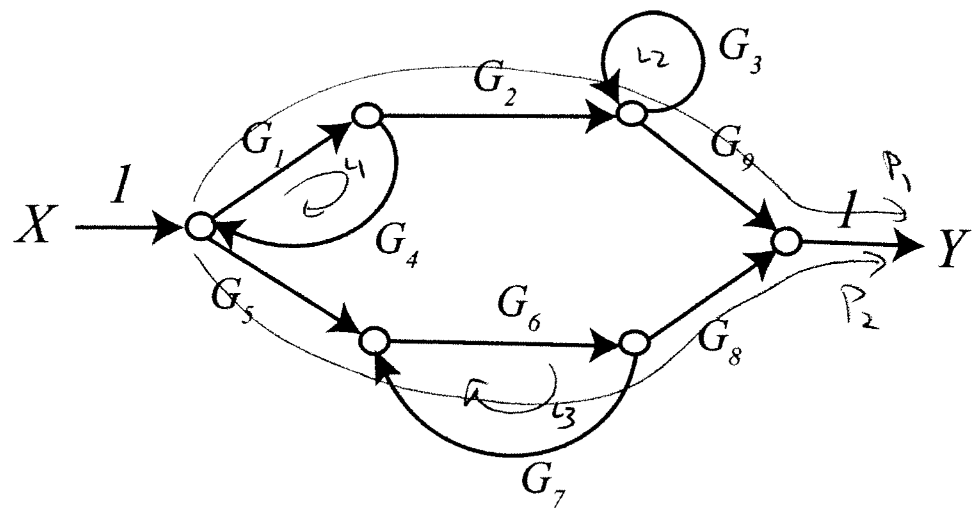
$$\begin{array}{r|l} & \quad 1 \\ \hline R_aR_bCs+R_a+R_b & R_aR_bCs+R_a \\ & R_aR_bCs+R_a+R_b \\ \hline & \qquad -R_b \end{array}$$

$$H(s) = 1 - \frac{R_b}{R_aR_bCs+R_a+R_b} = 1 - \frac{R_b}{R_aR_bC\left(s + \frac{R_a+R_b}{R_aR_bC}\right)} = 1 - \frac{1}{R_aC\left(s + \frac{R_a+R_b}{R_aR_bC}\right)}$$

$$\boxed{h(t) = \delta(t) - \frac{1}{R_aC}e^{-\left(\frac{R_a+R_b}{R_aR_bC}\right)t}u(t)}$$

15

4) (~~20~~ points) For the following signal flow diagram



Determine the system transfer function using Mason's gain rule. You must clearly indicate all of the paths, the loops, the determinant and the cofactors, but <u>*you do not need to simplify your final answer*</u> (it can be written in terms of the P_i, L_i, and Δ_i)

1 1 1 1 1

$P_1 = G_1 G_2 G_9$ $\quad$ $P_2 = G_5 G_6 G_8$ $\quad$ $L_1 = G_1 G_4$ $\quad$ $L_2 = G_3$ $\quad$ $L_3 = G_6 G_7$

3 3

$\Delta = 1 - (L_1 + L_2 + L_3) + (L_1 L_2 + L_1 L_3 + L_2 L_3) - (L_1 L_2 L_3)$

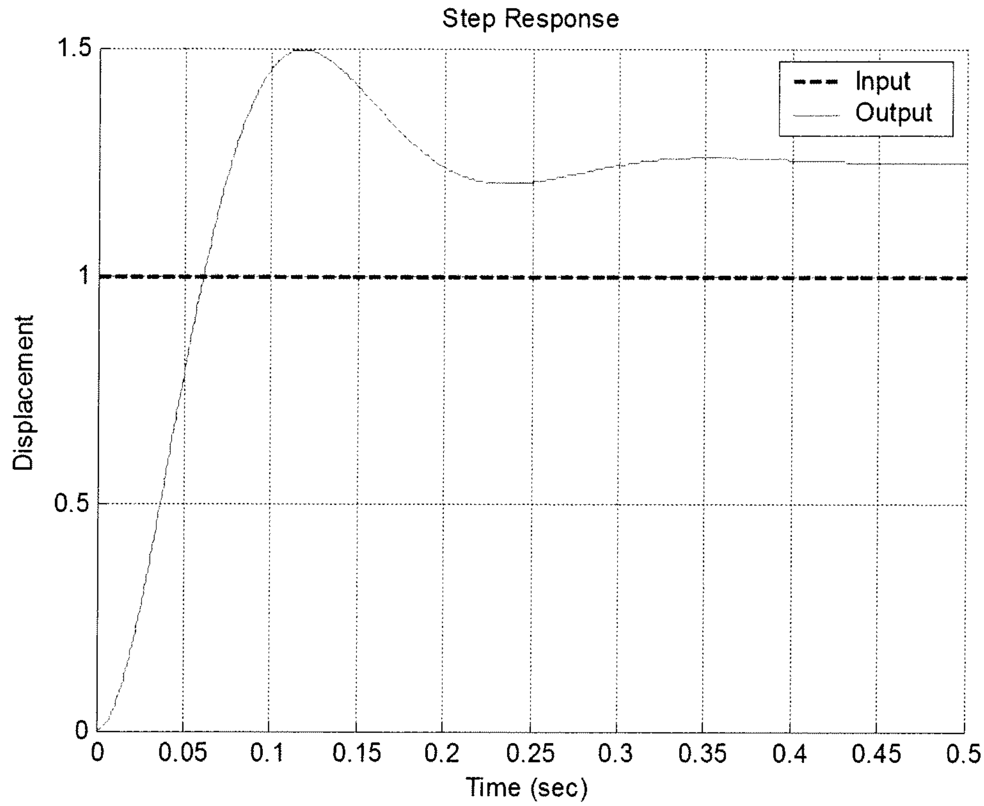
$\Delta_1 = 1 - L_3$ $\quad$ $\Delta_2 = 1 - L_2$

2 2

$$G_0 = \frac{Y}{X} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

Name ______________________________ Mailbox ______________

Problems 5 and 6 refer to the **unit step response** of a system, shown below



5) The best estimate of the **steady state error** for a **unit step input** is

a) 0.50 b) 0.25 c) -0.25 d) 0.0 e) impossible to determine

6) The best estimate of the **percent overshoot** is a) 20% b) 50% c) 25% d) 150%

$$\frac{1.5-1.25}{1.25} = \frac{0.25}{1.25} = \frac{1}{5} = 20\%$$

Name ______________________________ Mailbox ______________

7) How many of the following transfer functions represents (asymptotically) **stable** systems?

$G_a(s) = \dfrac{s+1}{s-1}$ $\quad G_b(s) = \dfrac{1}{s(s+1)}$ $\quad G_c(s) = \dfrac{s}{s^2-1}$

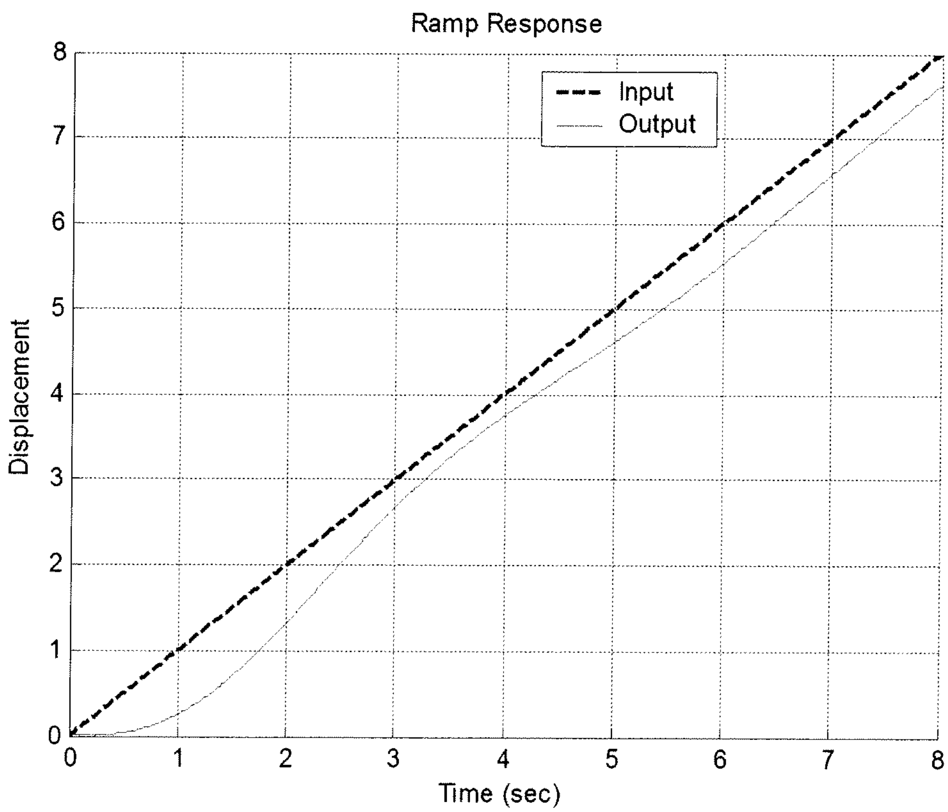
$G_d(s) = \dfrac{s+1}{(s+1+j)(s+1-j)}$ $\quad G_e(s) = \dfrac{(s-1-j)(s-1+j)}{s}$ $\quad G_f(s) = \dfrac{(s-1-j)(s-1+j)}{(s+1-j)(s+1+j)}$

a) 0 b) 1 c) 2 d) 3 e) 4 f) 5 g) 6

8) For the **unit ramp response** of a system shown below, the best estimate of the **steady state error** is

8 − 7.5 = 0.5

a) 0.8 b) -0.8 c) 0.5 d) -0.5



9) For a plant with transfer function $G_p(s) = \dfrac{5}{(s+4)(s+2)}$ The (2%) settling time for this plant is

a) 1 seconds b) 2 seconds c) 3 seconds d) 4 seconds e) none of these

$\frac{4}{4}, \frac{4}{2} \to 2$