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ECE-205

Exam 2

Winter 2010-2011

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 ________/20

Problem 2 ________/30

Problem 3 ________/15

Problem 4 ________/20

Problem 5 ________/15

Total Solutions

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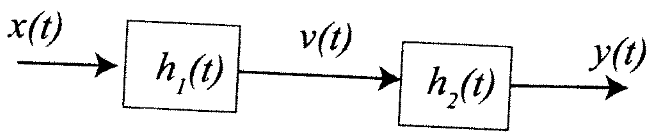
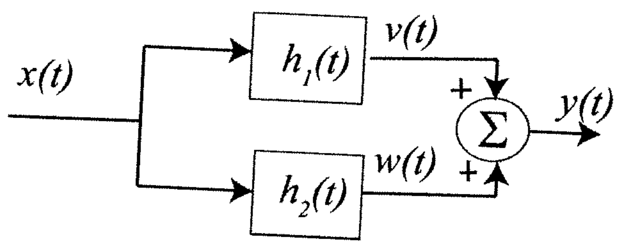
1) (20 points) For the following interconnected systems,

i) determine the overall impulse response (the impulse response between input $x(t)$ and output $y(t)$)

ii) determine if the system is causal.





a) $h_1(t) = u(t+2)$, $h_2(t) = \delta(t-1)$

Systems in series: $h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(\lambda) h_2(t-\lambda)\,d\lambda = \int_{-\infty}^{\infty} u(\lambda+2)\,\delta(t-\lambda-1)\,d\lambda$

$\boxed{h(t) = u(t+1) \text{ not causal}}$

Systems in parallel: $h(t) = h_1(t) + h_2(t) = \boxed{u(t+2) + \delta(t-1) = h(t) \text{ not causal}}$

b) $h_1(t) = e^{-(t-3)}u(t-3)$, $h_2(t) = e^{(t+2)}u(t+2)$

Systems in series: $h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(t-\lambda) h_2(\lambda)\,d\lambda$

$t-3 > -2$

$t > 1$

$= \int_{-\infty}^{\infty} e^{-(t-\lambda-3)} u(t-\lambda-3)\, e^{(\lambda+2)} u(\lambda+2)\,d\lambda = \int_{-2}^{t-3} e^{-t} e^{\lambda} e^{3} e^{\lambda} e^{2}\,d\lambda$

$= e^{-t} e^{5} \int_{-2}^{t-3} e^{2\lambda}\,d\lambda = \boxed{\frac{1}{2} e^{-t} e^{5} \left[e^{2(t-3)} - e^{-4} \right] u(t-1) = h(t) \text{ causal}}$

Systems in parallel: $h(t) = h_1(t) + h_2(t) = \boxed{e^{-(t-3)} u(t-3) + e^{(t+2)} u(t+2) = h(t) \text{ not causal}}$

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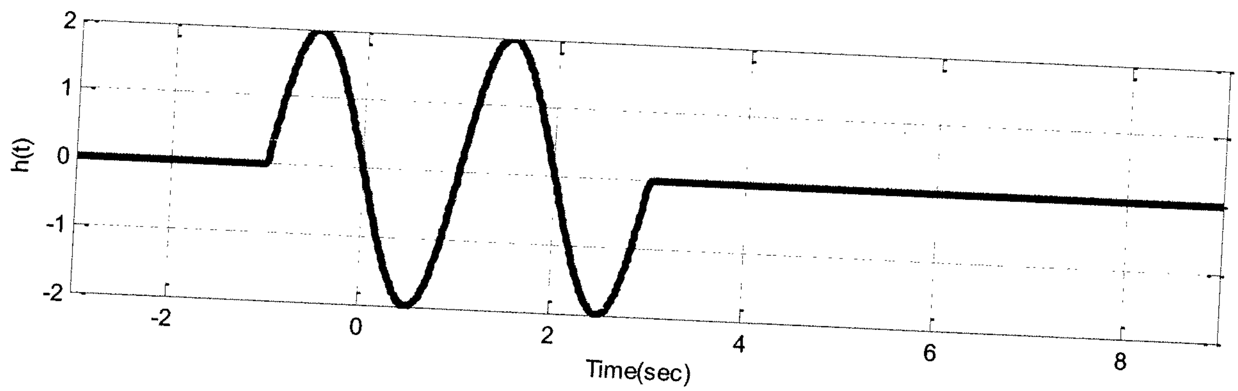
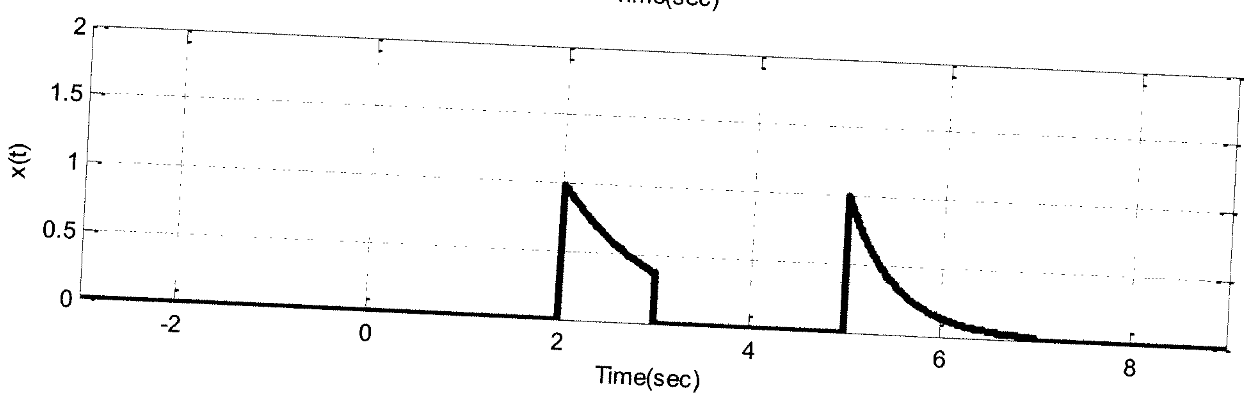
2) (30 points) Consider a linear time invariant system with impulse response given by

$$h(t) = 2\sin(\pi t)[u(t+1) - u(t-3)]$$

The input to the system is given by $x(t) = e^{-(t-2)}[u(t-2) - u(t-3)] - e^{-2(t-5)}[u(t-5) - u(t-7)]$





Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

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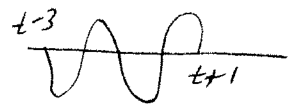
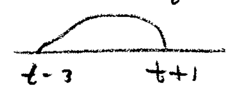
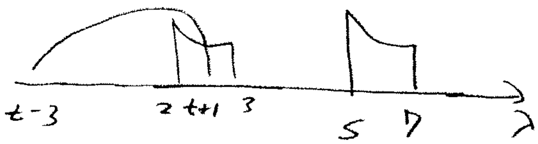
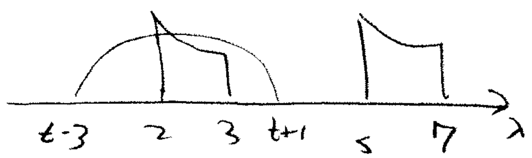
$h(t-\lambda)$ 

I can't draw this very well, so I will just draw $h(t-\lambda)$ as 

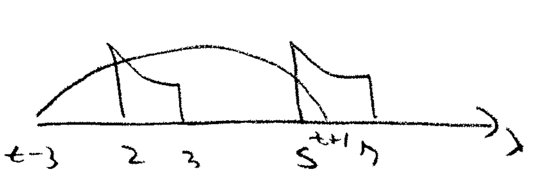
$t \le 1 \quad y(t) = 0$



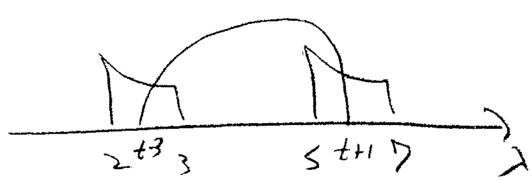
$$1 \le t \le 2 \quad y(t) = \int_{2}^{t+1} 2\sin(\pi(t-\lambda))e^{-(\lambda-2)}\,d\lambda$$



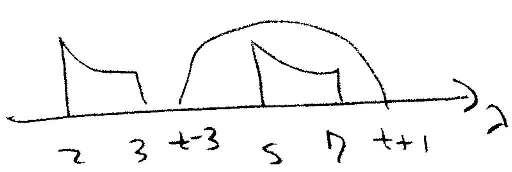
$$2 \le t \le 4 \quad y(t) = \int_{2}^{3} 2\sin(\pi(t-\lambda))e^{-(\lambda-2)}\,d\lambda$$



$$4 \le t \le 5 \quad y(t) = \int_{2}^{3} 2\sin(\pi(t-\lambda))e^{-(\lambda-2)}\,d\lambda + \int_{5}^{t+1} 2\sin(\pi(t-\lambda))e^{-2(\lambda-5)}\,d\lambda$$

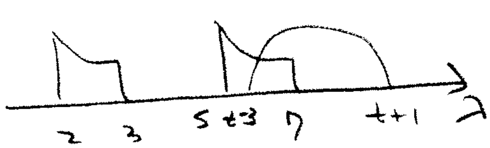


$$5 \le t \le 6 \quad y(t) = \int_{t+3}^{3} 2\sin(\pi(t-\lambda))e^{-(\lambda-2)}\,d\lambda + \int_{5}^{t+1} 2\sin(\pi(t-\lambda))e^{-2(\lambda-5)}\,d\lambda$$



$$6 \le t \le 8 \quad y(t) = \int_{5}^{7} 2\sin(\pi(t-\lambda))e^{-2(\lambda-5)}\,d\lambda$$

$$8 \le t \le 10 \quad y(t) = \int_{t-3}^{7} 2\sin(\pi(t-\lambda))e^{-2(\lambda-5)}\,d\lambda$$



$t \ge 10 \quad y(t) = 0$

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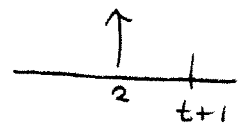
3) (15 points) Determine the **impulse response** for the following LTI systems. Do not forget any necessary unit step functions.

a) $y(t) = 2x(t) + \int_{-\infty}^{t+1} e^{-(t-\lambda)} x(\lambda - 2)\, d\lambda$

$$h(t) = 2\delta(t) + \int_{-\infty}^{t+1} e^{-(t-\lambda)}\,\delta(\lambda - 2)\, d\lambda$$



$t+1 > 2$

$t > +1$

$$\boxed{h(t) = 2\delta(t) + e^{-(t-2)}u(t-1)}$$

b) $2\dot{y}(t) + y(t) = 3x(t-1)$

$$\dot{h}(t) + \frac{1}{2}h(t) = \frac{3}{2}\delta(t-1)$$

$$\frac{d}{dt}\left[h(t)e^{t/2}\right] = \frac{3}{2}e^{t/2}\delta(t-1) = \frac{3}{2}e^{1/2}\delta(t-1)$$

$$h(t)e^{t/2} = \int_{-\infty}^{t} \frac{3}{2}e^{1/2}\,\delta(\lambda - 1)\, d\lambda = \frac{3}{2}e^{1/2}u(t-1)$$

$$\boxed{h(t) = \frac{3}{2}e^{-\frac{1}{2}(t-1)}u(t-1)}$$

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4) **(20 points)** Fill in the following table with a Y (yes) or N (no) for each of the system models given. Assume $-\infty < t < \infty$ for all of the systems and all initial conditions are zero.

System	System Model	Linear?	Time-Invariant?	Causal?	Memoryless?
1	$y(t) = x(-t)$	Y	N	N	N
2	$y(t) = \dfrac{1}{1+x(t)}$	N	Y	Y	Y
3	$y(t) = x(t-1) + x(t-2)$	Y	Y	Y	N
4	$y(t) = \int_{-\infty}^{t} \lambda x(\lambda+1)$	Y	N	N	N
5	$\dot{y}(t) + \sin(t)y(t) = e^{t+1}x(t)$	Y	N	Y	N

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5) (15 points) In this problem you will derive some important relationships for second order systems that we derived in class.

Consider the system described by the second order differential equation

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = K\omega_n^2 x(t)$$

If the system is initially at rest and the is input $x(t) = Au(t)$, the output can be written as

$$y(t) = KA\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi)\right]$$

where $\tan(\phi) = \frac{\sqrt{1-\zeta^2}}{\zeta}$ and $\omega_d = \omega_n\sqrt{1-\zeta^2}$

a) Time to Peak (Tp). To determine the time to peak, take the derivative of $y(t)$ and determine the time at which the derivative is zero. Remember that the tangent is periodic with period π. You should get $T_p = \frac{\pi}{\omega_d}$. Even if you don't, use this value in part b.

$$\dot{y}(t) = KA(\zeta\omega_n)\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi) - KA(\omega_d)\frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\cos(\omega_d t + \phi) = 0$$

$$\frac{\sin(\omega_d t + \phi)}{\cos(\omega_d t + \phi)} = \tan(\omega_d t + \phi) = \frac{\omega_d}{\zeta\omega_n} = \frac{\omega_n\sqrt{1-\zeta^2}}{\omega_n\zeta} = \frac{\sqrt{1-\zeta^2}}{\zeta}$$

$$\omega_d t + \phi = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{\zeta}\right) = \phi \Rightarrow \omega_d T_p = \pi$$

$$\boxed{T_p = \frac{\pi}{\omega_d}}$$

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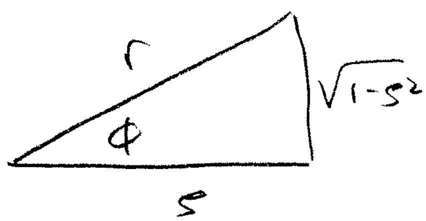
b) Percent Overshoot (PO). The percent overshoot is defined as the value at the peak $y(T_p)$ minus the steady state value $y(\infty)$ divided by the steady state value, and then multiplied by 100%,

$$PO = \frac{y(T_p) - y(\infty)}{y(\infty)} \times 100\%$$

i) Determine the steady state value $y(\infty)$ $y(\infty) = \lim_{t \to \infty} y(t) = \boxed{KA = y(\infty)}$

ii) Using your knowledge that $\tan(\phi) = \frac{\sqrt{1-\zeta^2}}{\zeta}$, draw a triangle and determine an expression for $\sin(\phi)$



$r^2 = \zeta^2 + \left(\sqrt{1-\zeta^2}\right)^2 = 1 \quad r = 1$

$\sin(\phi) = \frac{\sqrt{1-\zeta^2}}{r} = \boxed{\sqrt{1-\zeta^2} = \sin(\phi)}$

iii) Using the answer to ii and the fact that $\sin(\pi + \phi) = -\sin(\phi)$, determine $y(T_p)$

$$y(T_p) = KA\left[1 + e^{-\zeta\omega_n\pi/\omega_d}\right] = \boxed{KA\left[1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}}\right] = y(T_p)}$$

iv) Determine an expression for the percent overshoot. You should get $PO = e^{-\frac{\zeta\pi}{\sqrt{1-\zeta^2}}} \times 100\%$

$$P.O. = \left(\frac{KA\left[1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}}\right] - KA}{KA}\right) \times 100\%$$

$$\boxed{P.O. = e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\%}$$

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c) Settling Time (Ts). Determine the time constant for the system, and then the 2% settling time (4 time constants)

$$\tau = \frac{1}{\zeta\omega_n} \qquad T_s = 4\tau = \boxed{\frac{4}{\zeta\omega_n} = T_s}$$

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