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ECE-205

Exam 1

Winter 2009

Calculators can only be used for simple calculations. Solving integrals, differential equations, systems of equations, etc. does not count as a simple calculation.

You must show your work to receive credit.

Problem 1 __________

Problem 2 __________

Problem 3 __________

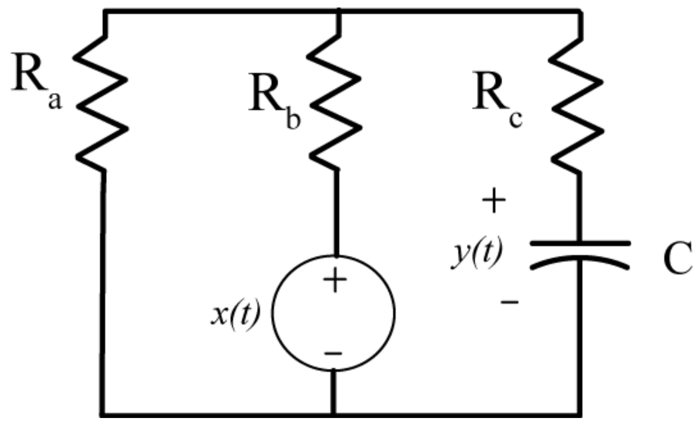
Problem 4 __________

Problem 5 __________

Total ________________

Name ____ Mailbox __________________

1) (15 points) Derive the governing differential equation for the following first order circuit. You can use any method you want (except for copying). You do not need to put your answer in a standard form.



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2) (30 points) Assume we have a first order system with the governing differential equation

$$2\dot{y}(t) + 4y(t) = 8x(t)$$

The system is initially at rest, so $y(0) = 0$. The input to this system is

$$x(t) = \begin{cases} 0 & t \le 0 \\ 3 & 0 < t \le 1 \\ -2 & 1 < t \le 1.5 \\ 1 & 1.5 < t \end{cases}$$

Determine the output of the system in each of the above time intervals. *Simplify your final answer as much as possible and box it.*

Name ____ Mailbox ________________

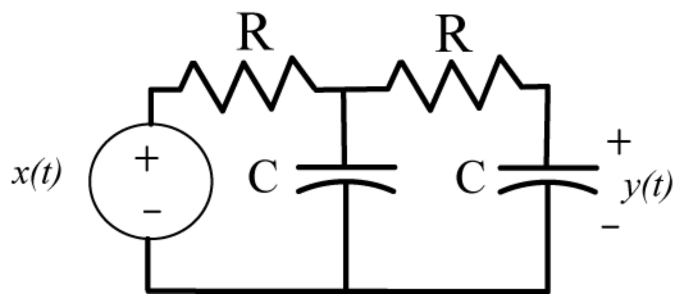
3) (15 points) Assume we have a first order system with the governing differential equation

$$2\dot{y}(t) + 3y(t) = e^{-t}x(t-1)$$

The initial time is $t_0 = 1$ and initial value $y(1) = 2$. Use the method of integrating factors to determine the output $y(t)$ as a function of the (unknown) input $x(t)$. *Simplify your answer as much as possible and box it.*

Name ____ Mailbox ___________________

4) (20 points) For the second order circuit below, derive the governing second order differential equation for the output $y(t)$ and input $x(t)$. You do not need to put it into a standard form.



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5) (20 points) The form of the under damped ($0 < \zeta < 1$) solution to the second order differential equation

$$\ddot{y}(t) + 2\zeta\omega_n \dot{y}(t) + \omega_n^2 y(t) = \mathrm{K}\ \omega_n^2 x(t)$$

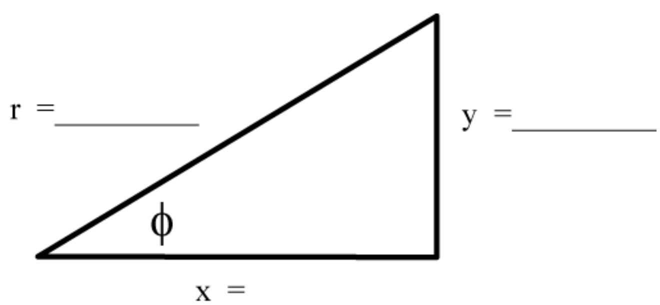
for a step input $x(t) = Au(t)$ is

$$y(t) = KA + ce^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$$

where c and ϕ are constants to be determined and the damped frequency $\omega_d = \omega_n\sqrt{1-\zeta^2}$

a) Using the initial condition $\dot{y}(0) = 0$ show that $\tan(\phi) = \frac{\sqrt{1-\zeta^2}}{\zeta}$

b) We can express the relationship in part a using the following triangle. Fill in the blanks and then use this triangle determine an expression for $\sin(\phi)$.



c) Use your answer to part b, and the initial condition $y(0) = 0$ to determine the remaining unknown constant, and write out the complete solution for $y(t)$.

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