ECE-205 : Dynamical Systems

Homework #8

Due : **Thursday** February 4 at the beginning of class

1) For the following transfer functions

$$H(s)=\frac{2}{s^2+2s+2} \quad H(s)=\frac{3}{s^2+4s+6} \quad H(s)=\frac{5}{s^2+6s+10}$$
$$H(s)=\frac{4}{s^2-4s+7} \quad H(s)=\frac{1}{s^2+4}$$

By computing the inverse Laplace transform show that the step responses are given by

$$y(t)=\left[1-e^{-t}\cos(t)-e^{-t}\sin(t)\right]u(t) \quad y(t)=\left[\frac{1}{2}-\frac{1}{\sqrt{2}}e^{-2t}\sin(\sqrt{2}t)-\frac{1}{2}e^{-2t}\cos(\sqrt{2}t)\right]u(t)$$
$$y(t)=\left[\frac{1}{2}-\frac{3}{2}e^{-3t}\sin(t)-\frac{1}{2}e^{-3t}\cos(t)\right]u(t) \quad y(t)=\left[\frac{4}{7}+\frac{8\sqrt{3}}{21}e^{2t}\sin(\sqrt{3}t)-\frac{4}{7}e^{2t}\cos(\sqrt{3}t)\right]u(t)$$
$$y(t)=\left[\frac{1}{4}-\frac{1}{4}\cos(2t)\right]u(t)$$

2) For the following transfer functions, determine

- the characteristic polynomial
- the characteristic modes
- if the system is (asymptotically) stable, unstable, or marginally stable

a) $H(s)=\dfrac{s-1}{s(s+2)(s+10)}$ b) $H(s)=\dfrac{s(s-1)}{(s+1)^2(s^2+s+1)}$ c) $H(s)=\dfrac{1}{s^2(s+1)}$

d) $H(s)=\dfrac{s^2-1}{(s-1)(s+2)(s^2+1)}$ e) $H(s)=\dfrac{1}{(s^2+2)(s+1)}$

Partial Answer: 1 stable, 2 unstable, 2 marginally stable

3) For a system with the following pole locations, estimate the settling time and determine the dominant poles

a) -1,-2,-4,-5 b) -4, -6, -7, -8

c) -1+j, -1-j, -2, -3 d) -3-2j, -3+2j, -4+j, -4-j

Scrambled Answers: 4/3, 4, 4, 1

4) An ideal second order system has the transfer function $G_o(s)$. The system specifications for a step input are as follows:

a) Percent Overshoot < 5%

b) Settling Time < 4 seconds (2% criteria)

c) Peak Time < 1 second

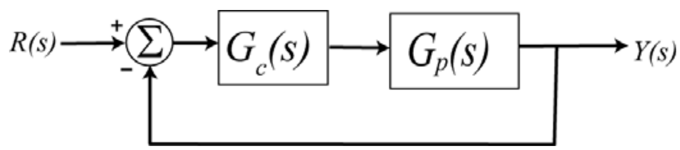
Show the permissible area for the poles of $G_o(s)$ in order to achieve the desired response.

5) Determine the static gain for the systems represented by the following transfer functions, and then the steady state output for an input step of amplitude 3:

$$H(s)=\frac{s+2}{s^2+s+1}, \quad H(s)=\frac{1}{s^2+4s+4}, \quad H(s)=\frac{s-4}{s^2+s+1}$$

Answers: 2, 0.25, -4, 6, 0.75, -12 (this should be very easy)

6) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s)=\dfrac{3}{s+2}$ and the controller is a proportional controller, so $G_c(s)=k_p$.



a) Determine the settling time of the plant alone (assuming there is no feedback)

b) Determine the closed loop transfer function, $G_0(s)$

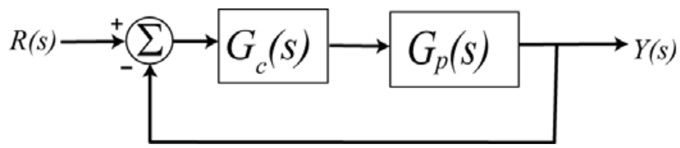
c) Determine the value of k_p so the settling time of the system is 0.5 seconds.

d) If the input to the system is a unit step, determine the output of the system.

e) The steady state error is the difference between the input and the output as $t \to \infty$. Determine the steady state error for this system.

Partial Answer: $y(t)=\frac{3}{4}\left[1-e^{-8t}\right]u(t), \quad e_{ss}=0.25$

7) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \dfrac{3}{s+2}$ and the controller is an integral controller, so $G_c(s) = \dfrac{k_i}{s}$.



a) Determine the closed loop transfer function, $G_0(s)$

b) Determine the poles of value of $G_0(s)$ and show they are only real if $0 < k_i < \dfrac{1}{3}$. Note that the best possible setting time is 4 seconds. Use $k_i = \dfrac{1}{3}$ for the remainder of this problem.

c) If the input to the system is a unit step, determine the output of the system.

d) The steady state error is the difference between the input and the output as $t \to \infty$. Determine the steady state error for this system.

Partial Answer: $y(t) = \left[1 - e^{-t} - te^{-t}\right]u(t), \quad e_{ss} = 0$

8) Show that the following circuit can be used to implement the PI controller

$$G_c(s) = \frac{U(s)}{E(s)} = k_p + k_i \frac{1}{s} = \frac{R_4 R_2}{R_3 R_1} + \frac{R_4}{R_3 R_1 C_2} \frac{1}{s}$$