ECE-205 : Dynamical Systems

Homework #6

Due : <u>Tuesday</u> January 19 at the beginning of class

Exam #2, Thursday Jan 21

1) For LTI systems with the following impulse responses, determine if the system is BIBO stable.

a) $h(t)=\delta(t)$ b) $h(t)=u(t)$ c) $h(t)=e^{-t}u(t)$ d) $h(t)=e^{-t^2}u(t)$

Answers: 3 are stable, one is unstable

2) Consider and LTI system with impulse response $h(t)$. We assume that the impulse response begins at some time t_h. That is, the impulse response is zero before this time. Hence we can write $h(t)=\tilde{h}(t)u(t-t_h)$. Next assume we have an input that starts (is zero before) some time t_x, so we can write $x(t)=\tilde{x}(t)u(t-t_x)$. Note that we do not really care what the functions look like here, only their initial times. In doing the graphical convolution, just make them blobs.

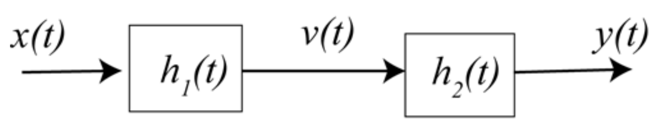
a) Determine, using graphical convolution, the first possible time at which the output of the system, $y(t)$, can be non-zero, and call this time t_y. This time should be a function of t_h and t_x

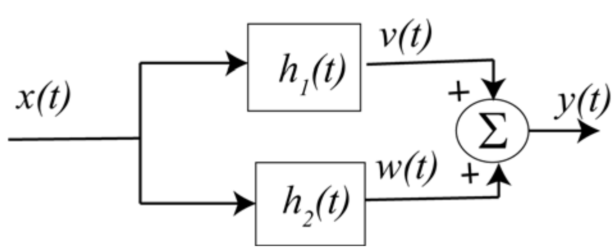
b) Show that, in order for the system to be causal, $t_h \ge 0$. That is, the impulse response must be zero for negative times.

3) For the following interconnected systems,

i) determine the overall impulse response (the impulse response between input *x(t)* and output *y(t)*) and

ii) determine if the system is causal.





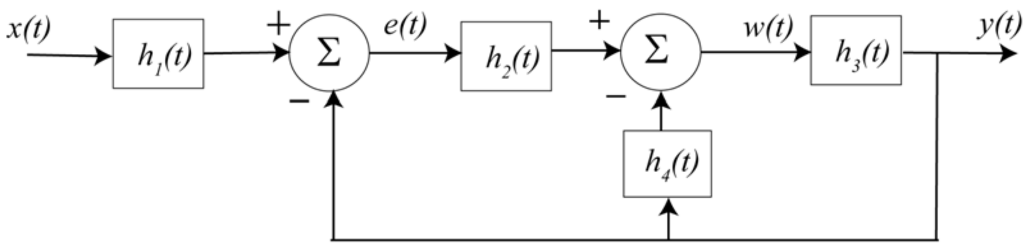
a) $h_1(t)=u(t-1),\ h_2(t)=u(t+1)$ **b)** $h_1(t)=u(t-1),\ h_2(t)=\delta(t+2)$ **c)** $h_1(t)=e^{-(t-1)}u(t-1),\ h_2(t)=\delta(t)+u(t)$

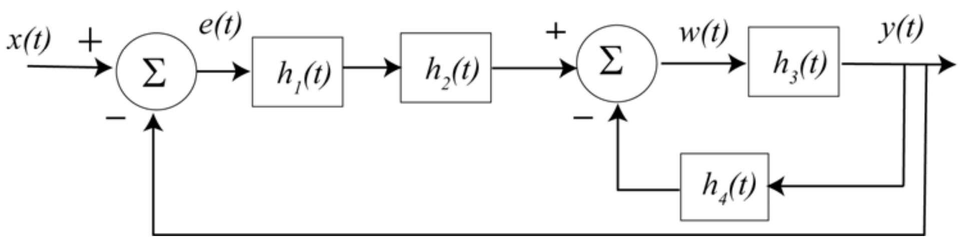
Scrambled Answers:

$h(t)=u(t-1),\ h(t)=tu(t),\ h(t)=u(t+1),\ h(t)=u(t+1)+u(t-1),\ h(t)=\delta(t)+u(t)+e^{-(t-1)}u(t-1),\ h(t)=u(t-1)+\delta(t+2)$

Three systems not causal

4) For the following systems, determine the relationship between the input and the output



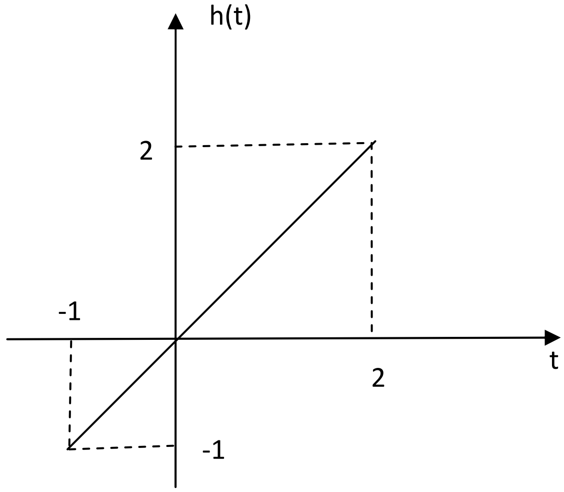


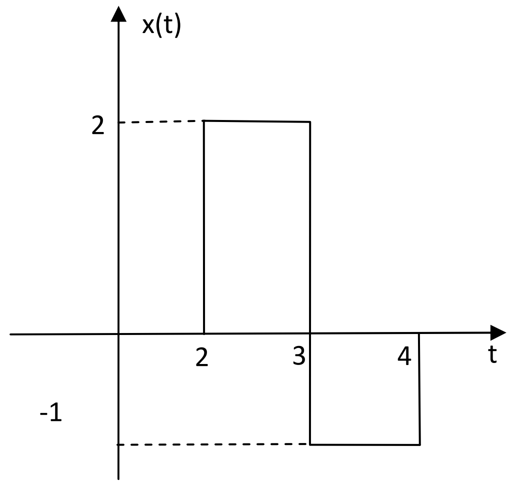
Answers:

$$y(t)*[\delta(t)+h_2(t)*h_3(t)+h_4(t)*h_3(t)]=x(t)*\left[h_1(t)*h_2(t)*h_3(t)\right]$$
$$y(t)*[\delta(t)+h_1(t)*h_2(t)*h_3(t)+h_3(t)*h_4(t)]=x*[h_1(t)*h_2(t)*h_3(t)]$$

5) Consider a linear time invariant system with impulse response given by

$h(t) = t\,[u(t+1) - u(t-2)]$ and input $x(t) = 2u(t-2) - 3u(t-3) + u(t-4)$, shown below





Using ***graphical convolution***, determine the output $y(t) = h(t) * x(t)$

Specifically, you must

a) Flip and slide $h(t)$, ***NOT*** $x(t)$

b) Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest

c) Determine the range of t for which each part of your solution is valid

d) Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.

e) ***DO NOT EVALUATE THE INTEGRALS!!***

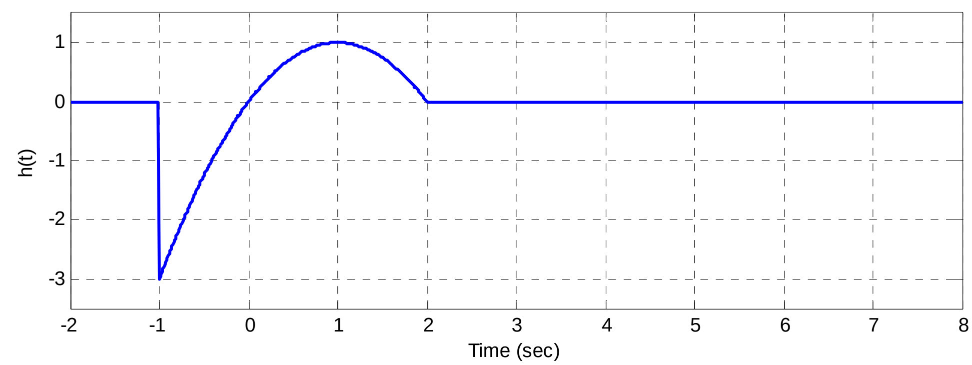
(The answer is at the end of the homework)

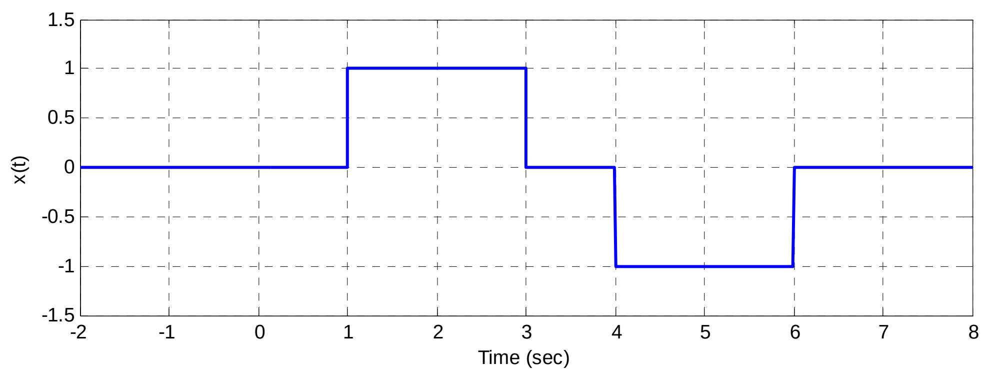
6) Consider a noncausal linear time invariant system with impulse response given by

$$h(t) = [1-(t-1)^2][u(t+1)-u(t-2)]$$

The input to the system is given by

$$x(t) = u(t-1) - u(t-3) - u(t-4) + u(t-6)$$





Using ***graphical convolution***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

(The answer is at the end of the homework)

Answer to problem 5:

$$y(t)=\begin{cases} 0 & t\le 1 \\ \int_{2}^{t+1}(t-\lambda)(2)d\lambda & 1\le t\le 2 \\ \int_{2}^{3}(t-\lambda)(2)d\lambda+\int_{3}^{t+1}(t-\lambda)(-1)d\lambda & 2\le t\le 3 \\ \int_{2}^{3}(t-\lambda)(2)d\lambda+\int_{3}^{4}(t-\lambda)(-1)d\lambda & 3\le t\le 4 \\ \int_{t-2}^{3}(t-\lambda)(2)d\lambda+\int_{3}^{4}(t-\lambda)(-1)d\lambda & 4\le t\le 5 \\ \int_{t-2}^{4}(t-\lambda)(-1)d\lambda & 5\le t\le 6 \\ 0 & t\ge 6 \end{cases}$$

Answer to problem 6:

$$y(t)=\begin{cases} 0 & t\le 0 \\ \int_{1}^{t+1}[1-(t-\lambda-1)^2](1)d\lambda & 0\le t\le 2 \\ \int_{1}^{3}[1-(t-\lambda-1)^2](1)d\lambda & 2\le t\le 3 \\ \int_{t-2}^{3}[1-(t-\lambda-1)^2](1)d\lambda+\int_{4}^{t+1}[1-(t-\lambda-1)^2](-1)d\lambda & 3\le t\le 5 \\ \int_{4}^{6}[1-(t-\lambda-1)^2](-1)d\lambda & 5\le t\le 6 \\ \int_{t-2}^{6}[1-(t-\lambda-1)^2](-1)d\lambda & 6\le t\le 8 \\ 0 & t\ge 8 \end{cases}$$