

## ECE-205 : Dynamical Systems

### Homework #10

**Due : Friday** February 19 at the beginning of lab

1) For the following systems, determine

i) the settling time based on the pole locations

ii) the magnitude and phase of  $H(j\omega_0)$  from the pole-zero diagram (graphically)

iii) the steady state output of the system

a)  $H(s) = \frac{s+2}{(s+5+j)(s+5-j)}$ ,  $x(t) = 4\cos(3t)u(t)$

b)  $H(s) = \frac{s+5}{(s+2+6j)(s+2-6j)}$ ,  $x(t) = 5\sin(4t+30^\circ)u(t)$

c)  $H(s) = \frac{30}{(s+1)(s+4+4j)(s+4-4j)}$ ,  $x(t) = 10\cos(2t-45^\circ)u(t)$

Answers:

$$T_s = 0.8, y_{ss}(t) = 0.42\cos(3t - 4.15^\circ)$$

$$T_s = 2, y_{ss}(t) = 1.11\sin(4t + 34.97^\circ)$$

$$T_s = 4, y_{ss}(t) = 4.16\cos(2t - 138.17^\circ)$$

2) For the following transfer functions, using the provided Bode plots

i) determine the correct form for each building block, i.e.  $K, (\tau s + 1)^n, \left(\frac{1}{\omega_n^2} s^2 + \frac{2\zeta}{\omega_n} s + 1\right)^n$

ii) sketch and label each building block (magnitude and phase)

iii) sketch the straight line approximation to the Bode plot (magnitude and phase) by adding the building block components

a)  $H(s) = \frac{100}{s+10}$

b)  $H(s) = \frac{(s+100)}{s}$

c)  $H(s) = \frac{(s+10)}{s(s+100)}$

d)  $H(s) = \frac{1000(s+10)}{(s+1000)^2}$

3) The following three figures display the magnitude of six transfer functions. All of the poles and zeros of these transfer functions are in the left half plane (these are minimum phase transfer functions). All of the magnitudes, poles, and zeros are either zero or simple powers of 10 ( $10^{-1}, 1, 10^1, 10^2$  etc). Estimate the transfer functions.

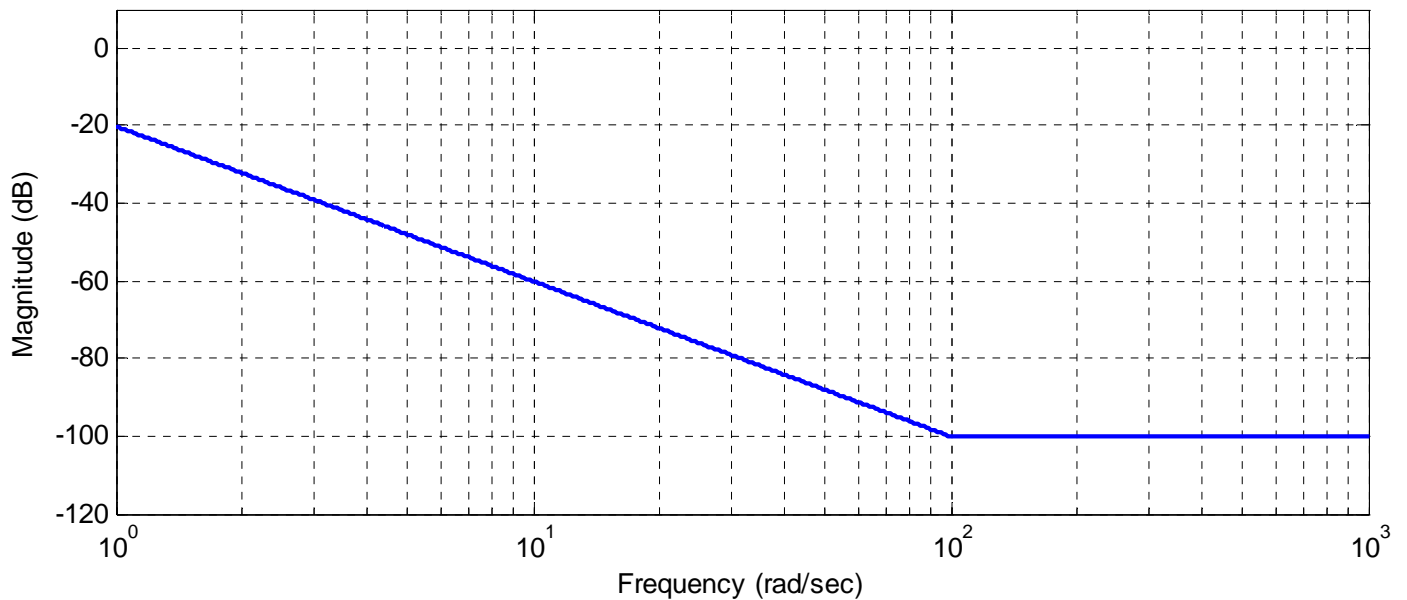
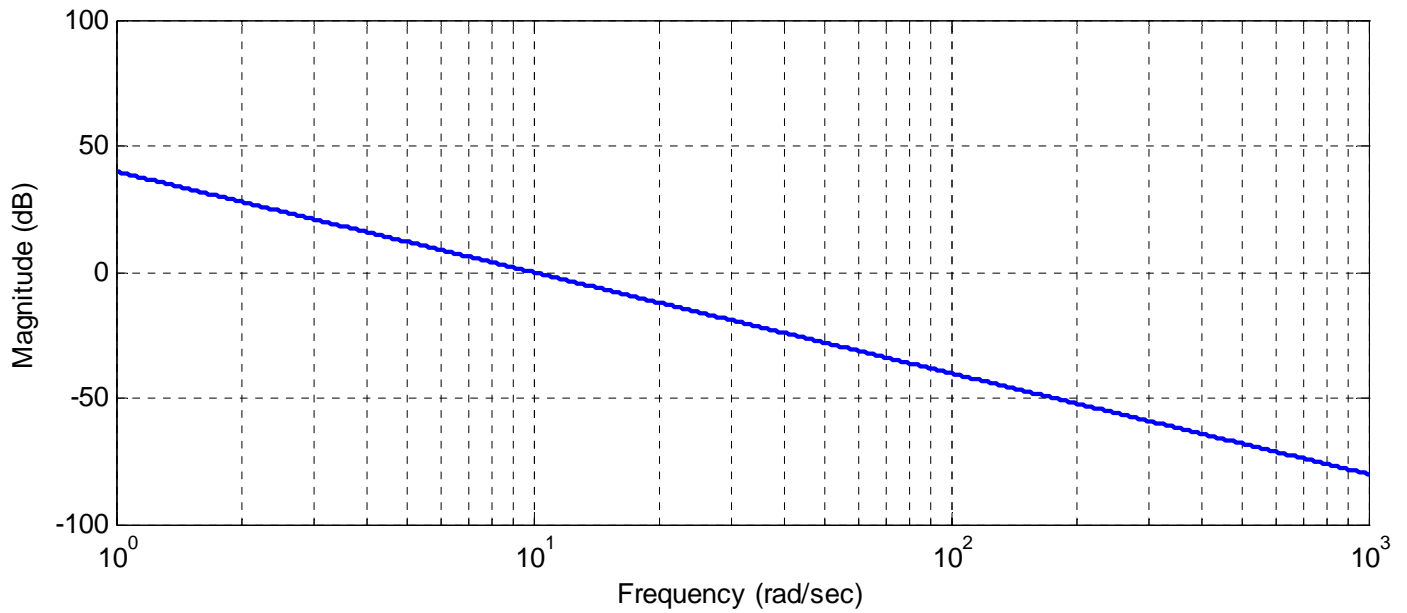
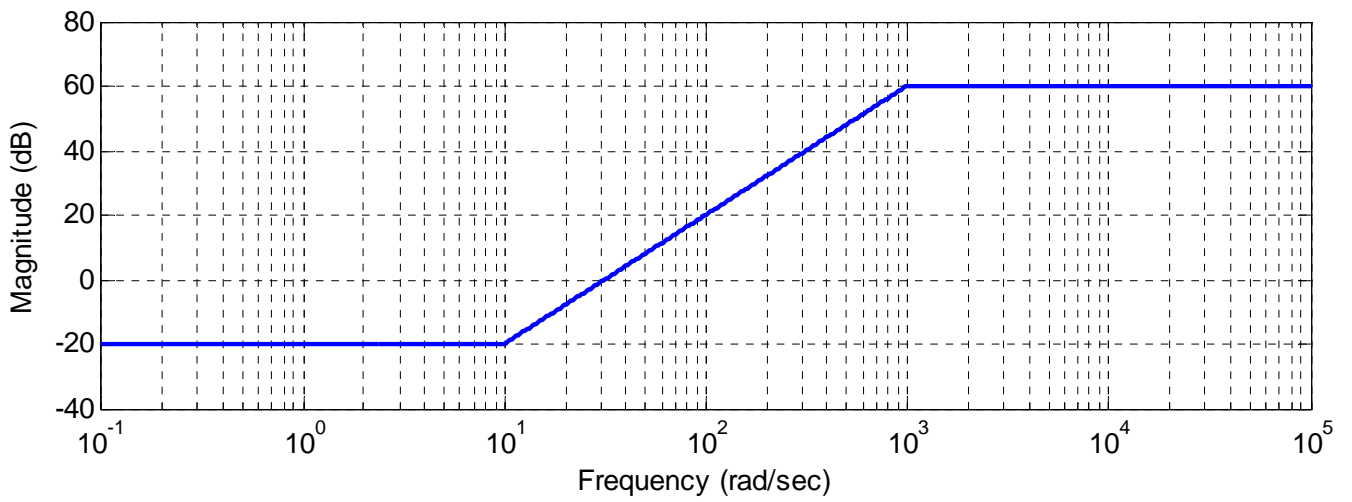
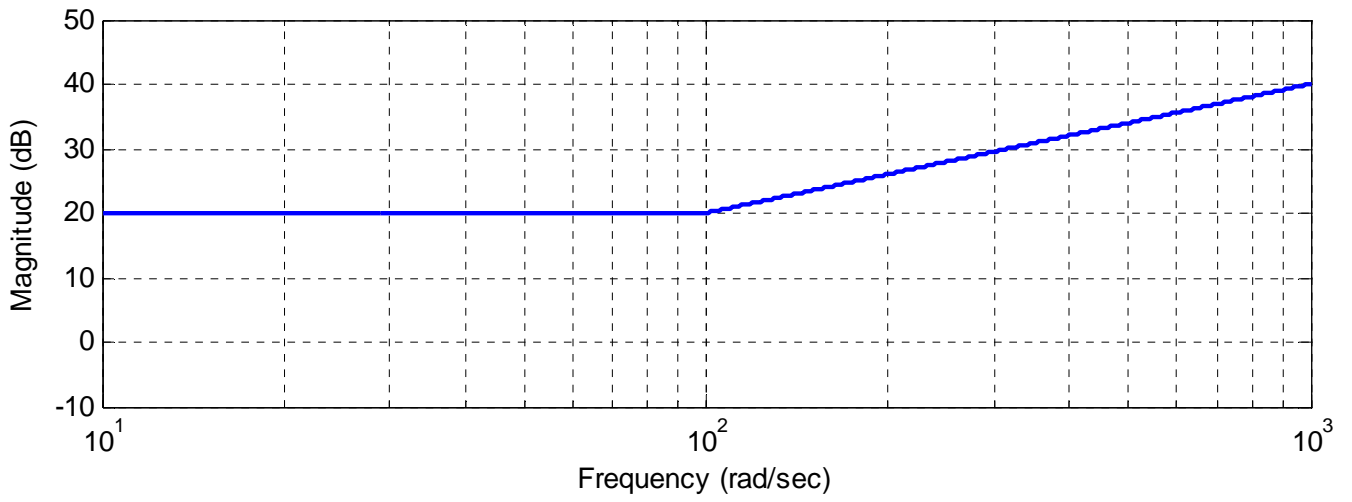
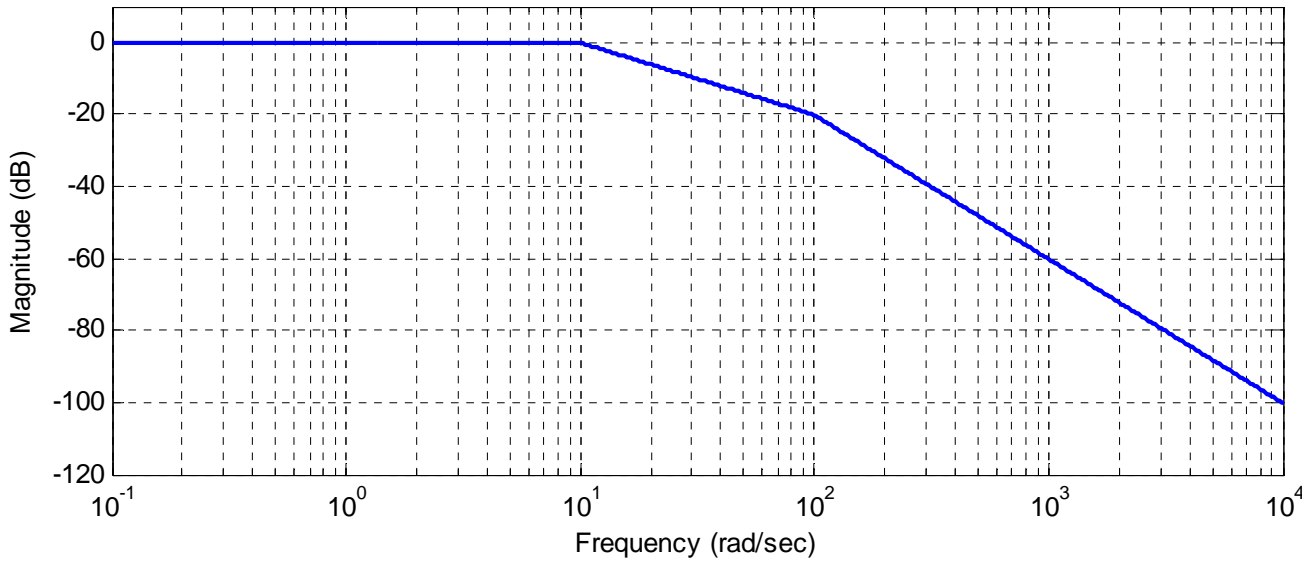
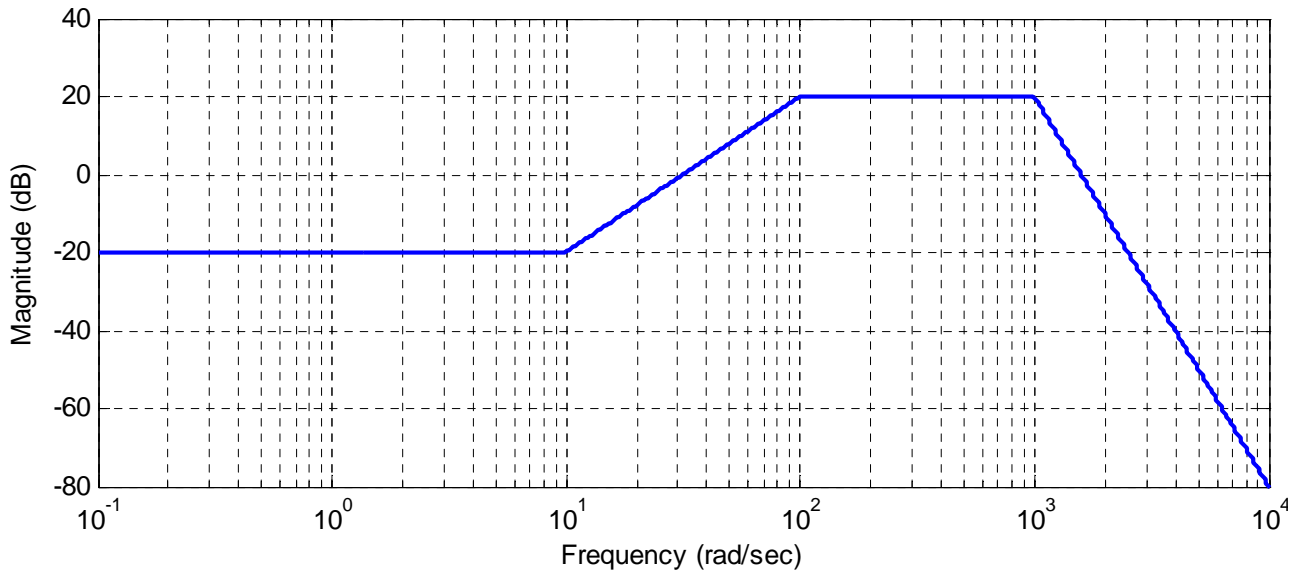


Figure 1: Problem 3, Systems a and b



**Figure 2:** Problem 3, Systems c and d



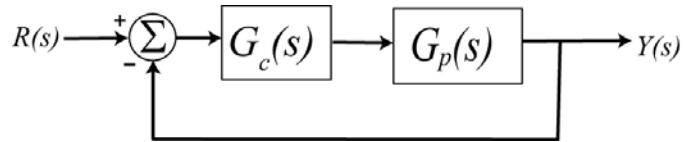
**Figure 3:** Problem 3, Systems e and f

**Answers:**

$$H(s) = \frac{100}{s^2}, \quad H(s) = \frac{0.1 \left( \frac{1}{100}s + 1 \right)^2}{s^2}, \quad H(s) = 10 \left( \frac{1}{100}s + 1 \right)$$

$$H(s) = \frac{0.1 \left( \frac{1}{10}s + 1 \right)^2}{\left( \frac{1}{100}s + 1 \right)^2}, \quad H(s) = \frac{0.1 \left( \frac{1}{10}s + 1 \right)^2}{\left( \frac{1}{100}s + 1 \right)^2 \left( \frac{1}{1000}s + 1 \right)^5}, \quad H(s) = \frac{1}{\left( \frac{1}{10}s + 1 \right) \left( \frac{1}{100}s + 1 \right)}$$

4) Consider the following simple feedback control block diagram. The plant is  $G_p(s) = \frac{2}{s+4}$ .



- a) What is the bandwidth of the plant alone (assuming there is no feedback)
- b) Assuming a proportional controller,  $G_c(s) = k_p$ , determine the closed loop transfer function,  $G_0(s)$
- c) Assuming a proportional controller,  $G_c(s) = k_p$ , determine the value of  $k_p$  so the bandwidth of the closed loop system is 24 rad/sec.
- d) Assuming the proportional controller from problem c, determine the settling time and the steady state error for a unit step.

Partial Answers: 4, 10, 1/6

5) In this problem we will relate the settling time of a system with the bandwidth of a system if there are not complex poles.

a) Consider a transfer function for a first order system,  $G(s) = \frac{B}{s+p}$ . Show that for this system, the product of the (2%) settling time and the bandwidth is equal to 4. Thus *to decrease the settling time you need to increase the bandwidth, and visa versa.*

b) Consider a transfer function for a second order system with real poles,  $G(s) = \frac{B}{(s+p_1)(s+p_2)}$ . Show that for this system, the product of the settling time and the bandwidth is equal to 4. Thus *to decrease the settling time you need to increase the bandwidth, and visa versa. Note that for two or more real poles, we actually need to assume the poles are not too close together, but the basic result holds.*

6) (Matlab Problem #1) Use Matlab to construct bode plots of the transfer functions you derived in problem 3. Print out each plot and turn it in.

Matlab Hints:

(1) If we want to plot a Bode plot of the transfer function

$$G(s) = \frac{0.1 \left[ \frac{1}{10} s + 1 \right]}{\left[ \frac{1}{100} s + 1 \right]}$$

from  $\omega = 10^{-2}$  to  $\omega = 10^5$ , then we would type the following in Matlab

`G = tf(0.1*[1/10 1],[1/100 1]);`

`Bode(G,logspace(-2,5,1000)); grid;`

(2) If we have a transfer function with repeated poles, such as

$$G(s) = \frac{\left[ \frac{1}{100} s + 1 \right]^3}{\left[ \frac{1}{10} s + 1 \right]^4}$$

we want Matlab to do the work of multiplying these out. To multiply polynomials in Matlab, we use the command `conv`, which performs a discrete-time convolution. To construct the above transfer function then, we would type something like the following:

```
np = [1/100 1];
```

```
n = conv(np,np); % raised to the second power
```

```
n = conv(n,np); % raised to the third power
```

```
dp = [1/10 1];
```

```
d = conv(dp,dp); % raised to the second power
```

```
d = conv(d,dp); % raised to the third power
```

```
d = conv(d,dp); % raised to the fourth power
```

```
G = tf(n,d);
```

**7) (Matlab Problem #2)** The Matlab routine `homework10.m` plots the magnitude of the frequency response of the transfer function based on the location of the poles and zeros entered in lines 8 and 9 of the code. This magnitude plot is not in Db and the amplitude has been scaled to have a maximum value of 1 for all of the plots. All you are asked to do in this problem is change the location of the poles and the zeros, run the code, and turn in your plots. You can save your plots to a word file and then turn that in, but do not put more than three plots in a page, and the grader must be able to read them. However, try to think about the pole-zero plots you made in problem 1 as you do the problem.

a) Run the code with poles at  $-10 \pm 10j$ ,  $-5 \pm 10j$ ,  $-1 \pm 10j$  and no zeros (leave the variable zeros as an empty array, do not put in a zero). Note that as the magnitude of the imaginary part of the pole gets significantly larger than the magnitude of the real part, we see well defined peaks in the frequency response. *Turn in all three plots.*

b) Leave the poles at  $-1 \pm 10j$  and add zeros first at 0, then at 0 and  $\pm 5j$ , then at 0 and  $\pm 15j$ . Note that at the frequency where a zero of the transfer function occurs, the output of the transfer function is ...zero! *Turn in all three plots.*