ECE-205

Exam 3

Winter 2009

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 _________/20

Problem 2 _________/20

Problem 3 _________/20

Problem 4 _________/20

Problem 5 _________/20

Total _____________

1) (20 points) For the following impulse responses and system inputs, determine the system output using Laplace transforms.

a) $h(t) = e^{-2(t-1)}u(t-1),\ x(t) = u(t-2) - u(t-4)$

b) $h(t) = te^{-3t}u(t),\ x(t) = u(t)$

Do not forget any necessary unit step functions.

a) $H(s) = \frac{e^{-s}}{s+2}$ $\quad X(s) = \frac{e^{-2s}}{s} - \frac{e^{-4s}}{s}$

$G(s) = \frac{1}{s(s+2)} = \frac{A}{s} + \frac{B}{s+2}$ $\quad A = 1/2 \quad B = -\frac{1}{2}$

$g(t) = \left(\frac{1}{2} - \frac{1}{2}e^{-2t}\right)u(t)$

$Y(s) = H(s)X(s) = G(s)e^{-3s} - G(s)e^{-5s}$

$y(t) = g(t-3) - g(t-5) = \boxed{\frac{1}{2}\left(1 - e^{-2(t-3)}\right)u(t-3) - \frac{1}{2}\left(1 - e^{-2(t-5)}\right)u(t-5) = y(t)}$

b) $H(s) = \frac{1}{(s+3)^2}$ $\quad X(s) = \frac{1}{s}$

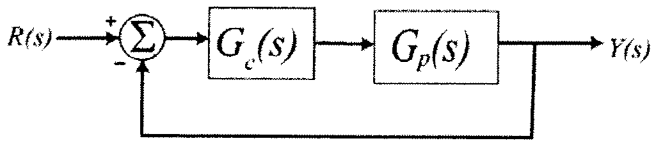
$Y(s) = H(s)X(s) = \frac{1}{s(s+3)^2} = \frac{A}{s} + \frac{B}{s+3} + \frac{C}{(s+3)^2}$ $\quad A = 1/9 \quad C = -1/3$

$\times s$, let $s \to \infty$ $\quad 0 = A + B$

$B = -A = -1/9$

$\boxed{y(t) = \left(\frac{1}{9} - \frac{1}{9}e^{-3t} - \frac{1}{3}te^{-3t}\right)u(t)}$

2) (20 points) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \dfrac{2}{s+3}$ and the controller is a proportional controller, so $G_c(s) = k_p$.



a) Determine the settling time of the plant alone (assuming there is no feedback)

b) Determine the closed loop transfer function, $G_0(s)$

c) Determine the value of k_p so the settling time of the system is 4/25 seconds.

d) Determine the value of k_p so the steady state error of the system for a unit step is 3/23.

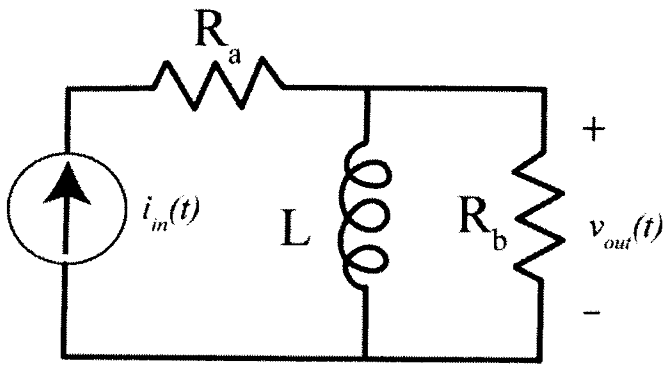
a) $T_s = \frac{4}{3}$

b) $G_o(s) = \dfrac{\frac{2K_p}{s+3}}{1+\frac{2K_p}{s+3}} = \boxed{\dfrac{2K_p}{s+3+2K_p} = G_o(s)}$

c) $T_s = \dfrac{4}{3+2K_p} = \dfrac{4}{25}$ $\boxed{K_p = 11}$

d) $1 - G_o(0) = 1 - \dfrac{2K_p}{3+2K_p} = \dfrac{3+2K_p-2K_p}{3+2K_p} = \dfrac{3}{3+2K_p} = \dfrac{3}{23}$ $\boxed{K_p = 10}$

3) (20 points) For the following circuit, determine the transfer function and the corresponding impulse response.



$$I_{in}(s) = \frac{V_{out}(s)}{Ls} + \frac{V_{out}(s)}{R_b} = V_{out}(s)\left[\frac{1}{Ls} + \frac{1}{R_b}\right] = V_{out}(s)\left[\frac{R_b + Ls}{R_b L s}\right]$$

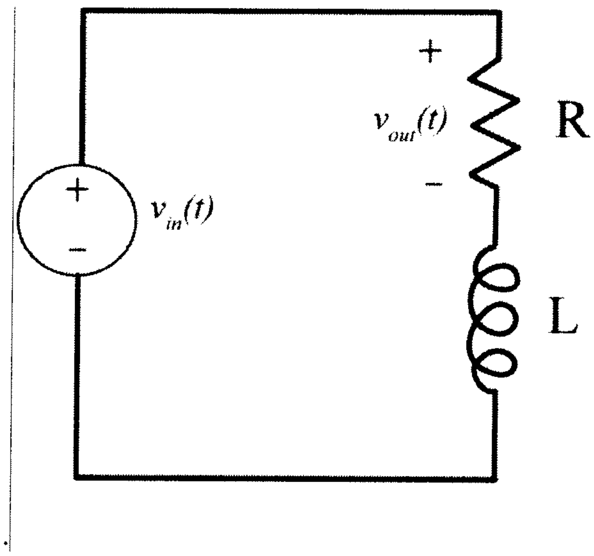
$$\frac{V_{out}(s)}{I_{in}(s)} = \boxed{H(s) = \frac{R_b L s}{Ls + R_b}}$$

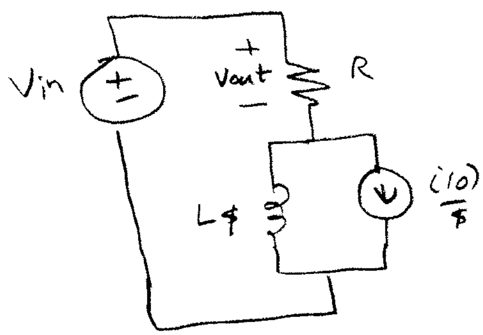
$$\begin{array}{r|l} & R_b \\ \hline Ls + R_b & R_b L s \\ & R_b L s + R_b^2 \\ \hline & -R_b^2 \end{array}$$

$$H(s) = R_b - \frac{R_b^2}{Ls + R_b} = R_b - \frac{R_b^2}{L(s + R_b/L)} = R_b - \frac{R_b^2}{L}\,\frac{1}{s + R_b/L}$$

$$\boxed{h(t) = R_b\,\delta(t) - \frac{R_b^2}{L}e^{-\frac{R_b}{L}t}u(t)}$$

4) (20 points) For the following circuit, determine and expression for the output $V_{out}(s)$ in terms of the ZSR and ZIR. Do not assume the initial conditions are zero. Also determine the system transfer function





$$\frac{V_{out}(s)}{R} = \frac{V_{in}(s) - V_{out}(s)}{Ls} + \frac{i(0)}{s}$$

$$V_{out}(s)\left[\frac{1}{R} + \frac{1}{Ls}\right] = V_{in}(s)\left[\frac{1}{Ls}\right] + \frac{i(0)}{s}$$

$$V_{out}(s)\left[\frac{Ls + R}{RLs}\right] = V_{in}(s)\left[\frac{1}{Ls}\right] + \frac{i(0)}{s}$$

$$\boxed{\begin{aligned} V_{out}(s) &= \underbrace{\left[V_{in}(s)\,\frac{R}{R+Ls}\right]}_{ZSR} + \underbrace{\left[\frac{RL}{R+Ls}\,i(0)\right]}_{ZIR} \\ H(s) &= \frac{V_{out}(s)}{V_{in}(s)} = \frac{R}{R+Ls} \end{aligned}}$$

5) (20 points) Consider the following closed loop system, with plant $G_p(s)$ and controller $G_c(s)$

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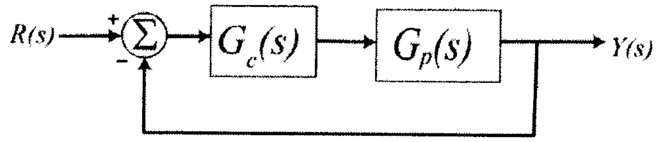

One way to choose the controller is to try and make your closed loop system match a transfer function that you choose (hence the name model matching). Let's assume that our **desired** closed loop transfer function $G_o(s)$, our plant can be written in terms of numerators and denominators as $G_o(s) = \dfrac{N_o(s)}{D_o(s)}$ $G_p(s) = \dfrac{N_p(s)}{D_p(s)}$

Determine an expression for the required controller $G_c(s)$ in terms of $N_o(s), D_o(s), N_p(s), D_p(s)$
For full credit you must simplify your answers as much as possible.

$$G_o = \frac{G_c G_p}{1 + G_c G_p}$$

$$G_o + G_o G_c G_p = G_c G_p$$

$$G_o = G_c G_p - G_c G_p G_o$$

$$= G_c G_p (1 - G_o)$$

$$G_c = \frac{G_o}{G_p(1 - G_o)} = \frac{\frac{N_o}{D_o}}{\frac{N_p}{D_p}\left(1 - \frac{N_o}{D_o}\right)}$$

$$= \frac{N_o}{D_o \frac{N_p}{D_p} - \frac{N_p}{D_p} N_o} = \boxed{\frac{N_o D_p}{N_p (D_o - N_o)} = G_c}$$