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ECE-205

Exam 2

Spring 2013

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 ________/18

Problem 2 ________/14

Problem 3 ________/15

Problem 4 ________/15

Problem 5 ________/15

Problem 6 ________/23

Total ____________

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1) (18 points) Fill in the non-shaded part of the following table. You do not need to show any work.

	Linear? (Y/N)	Time Invariant? (Y/N)	BIBO Stable? (Y/N)
$y(t)=\frac{1}{2}\left[x(t+1)+x(t-1)\right]$	Yes	Yes	
$\dot{y}(t)+e^{t}y(t)=\sin(t)x(t+1)$	Yes	No	
$y(t)=x(t-2)$	Yes	Yes	
$y(t)=\int_0^t e^{-\lambda}x(\lambda)d\lambda$			Yes
$y(t)=\int_{-\infty}^t e^{\lambda}x(\lambda)d\lambda$			No
$y(t)=t\,x(t)$			No

$$\int_0^t e^{-\lambda} N d\lambda = -e^{-\lambda}\Big|_0^t N = (1-e^{-t})N$$

$$\int_{-\infty}^{t} e^{\lambda} N d\lambda = N e^{\lambda}\Big|_{-\infty}^{t} = Ne^{t}$$

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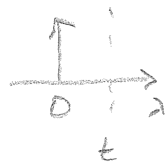
2) (14 points) Simplify the following as much as posible.

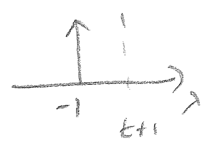
$y(t) = e^{t}\delta(t-1) = \boxed{e^{1}\,\delta(t-1)}$

$y(t) = \int_{-\infty}^{t} \delta(\lambda)d\lambda = \boxed{u(t)}$



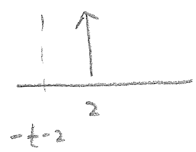
$t+1 > -1 \qquad t+2 > 0$

$y(t) = \int_{-\infty}^{t+1} \delta(\lambda+1)d\lambda = \boxed{u(t+2)}$



$-t-2 < 2 \qquad 0 < t+4$

$y(t) = \int_{-t-2}^{3} \delta(\lambda-2)d\lambda = \boxed{u(t+4)}$



For the following integrals you do not need to include any unit step functions in the answer.

$y(t) = \int_{0}^{t} e^{-(t-\lambda)}e^{-\lambda}d\lambda = e^{-t}\int_{0}^{t} d\lambda = \boxed{t e^{-t}}$

$y(t) = \int_{1}^{t} \lambda e^{-(t-\lambda)}e^{-\lambda}d\lambda = e^{-t}\int_{1}^{t}\lambda\, d\lambda = e^{-t}\frac{\lambda^2}{2}\Big|_{1}^{t} = \boxed{e^{-t}\left[\frac{t^2}{2} - \frac{1}{2}\right]}$

$y(t) = \int_{2}^{t-1} e^{-3(t-\lambda)}e^{-\lambda}d\lambda = e^{-3t}\int_{2}^{t-1} e^{2\lambda}d\lambda = \frac{e^{-3t}}{2} e^{2\lambda}\Big|_{2}^{t-1} = \frac{e^{-3t}}{2}\left[e^{2t-2} - e^{4}\right]$

$= \boxed{\frac{1}{2}\left[e^{-t-2} - e^{-3t+4}\right]}$

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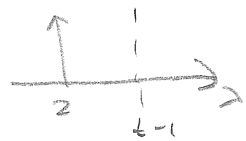
3) (15 Points) Determine the ***impulse response*** for the following systems. Don't forget any necessary unit step functions

a) $y(t) = x(t) + \int_{-\infty}^{t+1} x(\lambda+1)d\lambda$

$t+1 > -1 \quad t+2 > 0$

b) $y(t) = \int_{-\infty}^{t-1} e^{-(t-\lambda)} x(\lambda-2)d\lambda$



$t-1 > 2 \quad t-3 > 0$

c) $\dot{y}(t) + 3y(t) = 2x(t+1)$

a) $\boxed{h(t) = \delta(t) + u(t+2)}$

b) $\boxed{h(t) = e^{-(t-2)} u(t-3)}$

c) $\frac{d}{dt}\left[h(t)e^{3t}\right] = 2e^{3t}\delta(t+1) = 2e^{-3}\delta(t+1)$

$h(t)e^{3t} = 2e^{-3}u(t+1)$

$\boxed{h(t) = 2e^{-3(t+1)}u(t+1)}$

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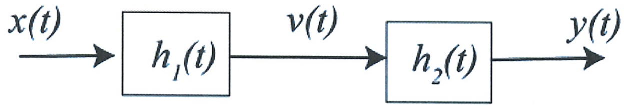
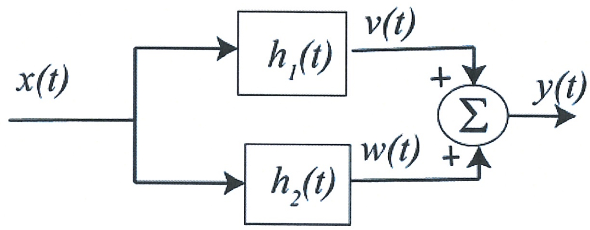
4) (15 points) For the following block diagram

For the following interconnected systems,

i) determine the overall impulse response (the impulse response between input $x(t)$ and output $y(t)$) and

ii) determine if the system is causal.





a) $h_1(t) = \delta(t),\ h_2(t) = \delta(t+2)$

b) $h_1(t) = e^{-(t+1)}u(t+1),\ h_2(t) = u(t-2) + \delta(t-2)$

Series Connections:

a) $h(t) = \int_{-\infty}^{\infty} h_1(\lambda) h_2(t-\lambda)\,d\lambda = \int_{-\infty}^{\infty} \delta(\lambda)\,\delta(t-\lambda+2)\,d\lambda = \boxed{\delta(t+2)}$ noncausal

b) $h(t) = \int_{-\infty}^{\infty} h_1(\lambda) h_2(t-\lambda)\,d\lambda = \int_{-\infty}^{\infty} e^{-(\lambda+1)} u(\lambda+1)\left[u(t-\lambda+2) + \delta(t-\lambda-2)\right] d\lambda$

$= e^{-1}\int_{-1}^{t-2} e^{-\lambda}\,d\lambda + e^{-(t-1)}u(t-1) = e^{-1}\left[e^{1} - e^{-t+2}\right]u(t-1) + e^{-(t-1)}u(t-1)$

$= \left[1 - e^{-t+1} + e^{-t+1}\right]u(t-1) = \boxed{u(t-1)}$ Causal

Parallel Connections:

a) $\boxed{h(t) = \delta(t) + \delta(t+2)}$ noncausal

b) $\boxed{h(t) = e^{-(t+1)}u(t+1) + u(t-2) + \delta(t-2)}$ noncausal

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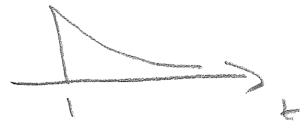
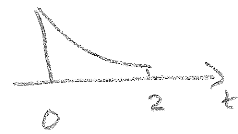
5) (15 points) Consider a linear time invariant system with impulse response given by

$$h(t) = e^{-(t-1)}u(t-1)$$



The input to the system is given by

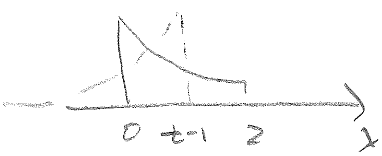
$$x(t) = e^{-t}[u(t) - u(t-2)]$$



Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

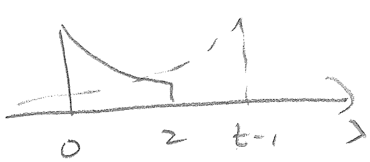
- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

$\boxed{y(t) = 0 \quad t \le 1}$



$\boxed{y(t) = \int_0^{t-1} e^{-(t-\lambda-1)} e^{-\lambda}\, d\lambda \qquad 1 \le t \le 3}$

$\boxed{y(t) = \int_0^{2} e^{-(t-\lambda-1)} e^{-\lambda}\, d\lambda \qquad t \ge 3}$



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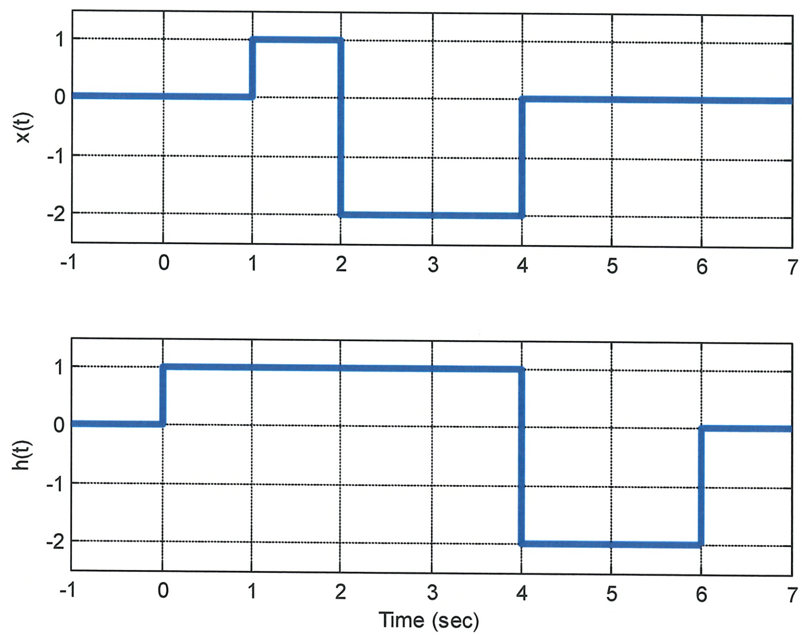
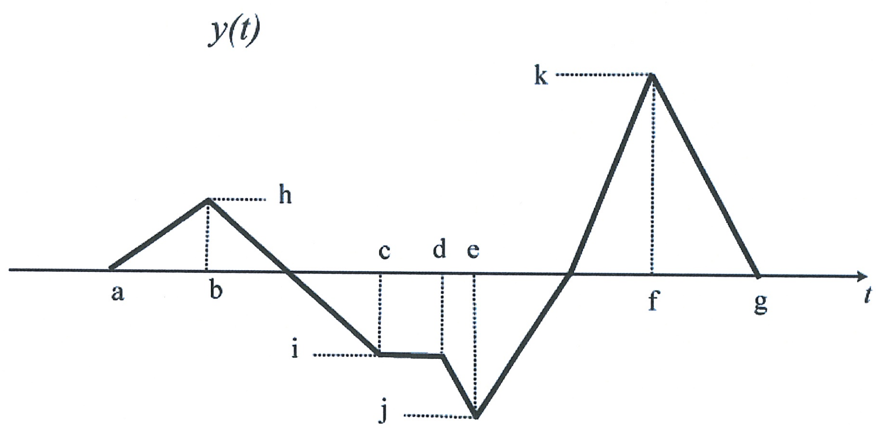
6) (23 Points) An LTI system has input, impulse response, and output as shown below. Determine numerical values for the parameters *a-k.* Note that parameters *a-g* correspond to *times*, and *h-k* correspond to *amplitudes.*





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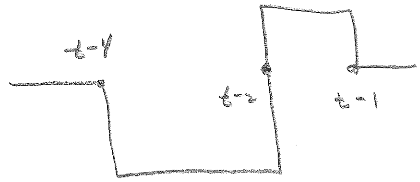
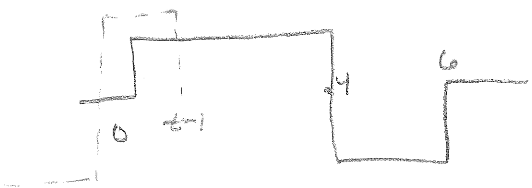
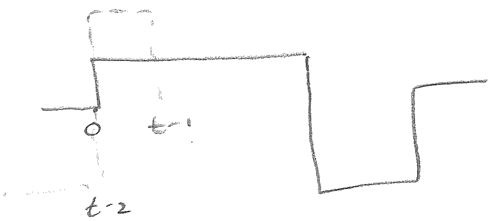
$x(t-\lambda)$



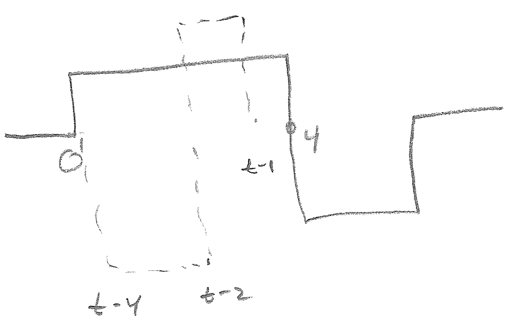


$t-1=0 \quad t=1 \quad \boxed{a=1}$



$t-2=0 \quad t=2 \quad \boxed{b=2}$

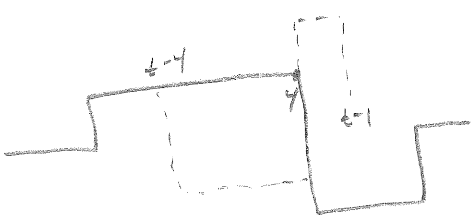
$h = 1\cdot1\cdot1 = \boxed{1=h}$



$t-4=0 \quad \boxed{c=4}$

$i = (1\cdot1\cdot1) + (-2)(1)(2) = \boxed{-3=i}$

$t-1=4 \quad t=5 \quad \boxed{d=5}$



$t-2=4 \quad t=6 \quad \boxed{e=6}$

$j = (-2)(2)(1) + (1)(-2)(1) = \boxed{-6=j}$

$t-2=6 \quad t=8 = \boxed{f=8}$

$K = (-2)(-2)(2) = \boxed{8=K}$

$t-4=6 \quad t=\boxed{10=g}$