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ECE-205

Exam 1

Spring 2013

Calculators can only be used for simple calculations. Solving integrals, differential equations, systems of equations, etc. does not count as a simple calculation.

You must show your work to receive credit.

Problem 1 ____________/16

Problem 2 ____________/16

Problem 3 ____________/14

Problem 4 ____________/10

Problem 5 ____________/12

Problem 6 ____________/16

Problem 7-10 ____________/16

Total ________________

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1) (16 points) Assume we have a first order system with the governing differential equation

$$0.2\dot{y}(t) + y(t) = 3x(t)$$

The system has the initial value of 0.4, so $y(0) = 0.4$. The input to this system is

$$x(t) = \begin{cases} 0 & t < 0 \\ 0.2 & 0 \le t < 0.6 \\ 0 & 0.6 \le t \end{cases}$$

a) Determine the output of the system in each of the above time intervals. *Simplify your final answer as much as possible and box it. Be sure to include the correct initial value in the first interval!*

b) Make a sketch of $y(t)$. (I am looking mostly at the shape of $y(t)$ from 0 to 0.6 and from 0.6 on.)

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2) (16 points) For the following three differential equations, assume the input is $x(t) = 3u(t)$ (the input is equal to one for time greater than zero), and the initial conditions are $y(0) = \dot{y}(0) = 0$

Determine the solution to each of the following differential equations and put your final answer in a box. Be sure to use the initial conditions to solve for all unknowns. You must show all your work to receive credit, ***you cannot simply use the formula from problem 3.***

a) $\ddot{y}(t) + 6\dot{y}(t) + 9y(t) = 9x(t)$

b) $\ddot{y}(t) + 4\dot{y}(t) + 13y(t) = 26x(t)$

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3) (14 points) The form of the under damped ($0 < \zeta < 1$) solution to the second order differential equation

$$\ddot{y}(t) + 2\zeta\omega_n \dot{y}(t) + \omega_n^2 y(t) = \mathrm{K}\ \omega_n^2 x(t)$$

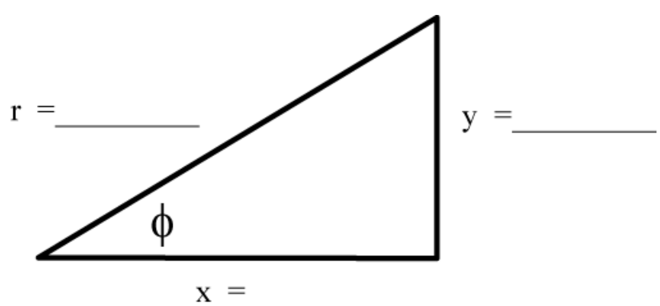
for a step input $x(t) = Au(t)$ is

$$y(t) = KA + ce^{-\zeta\omega_n t}\sin(\omega_d t + \phi)$$

where c and ϕ are constants to be determined and the damped frequency $\omega_d = \omega_n\sqrt{1-\zeta^2}$

a) Using the initial condition $\dot{y}(0) = 0$ show that $\tan(\phi) = \frac{\sqrt{1-\zeta^2}}{\zeta}$

b) We can express the relationship in part a using the following triangle. Fill in the blanks and then use this triangle determine an expression for $\sin(\phi)$.



c) Use your answer to part b, and the initial condition $y(0) = 0$ to determine the remaining unknown constant, and write out the complete solution for $y(t)$.

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4) (10 points) For the following first order differential equation,

$$3\dot{y}(t) + y(t) = x^2(t)$$

determine an expression for the output assuming $t_0 = 0$ and $y(t_0) = y(0) = 1$.

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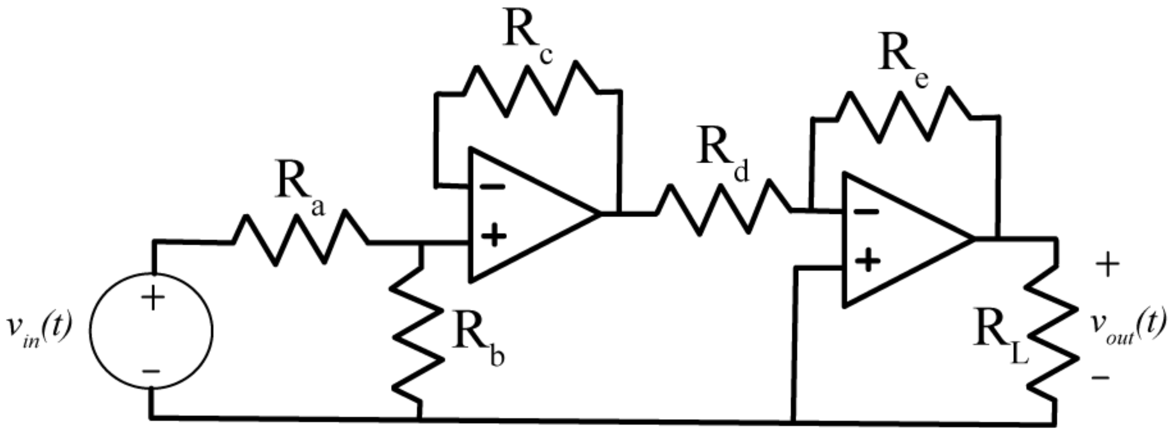
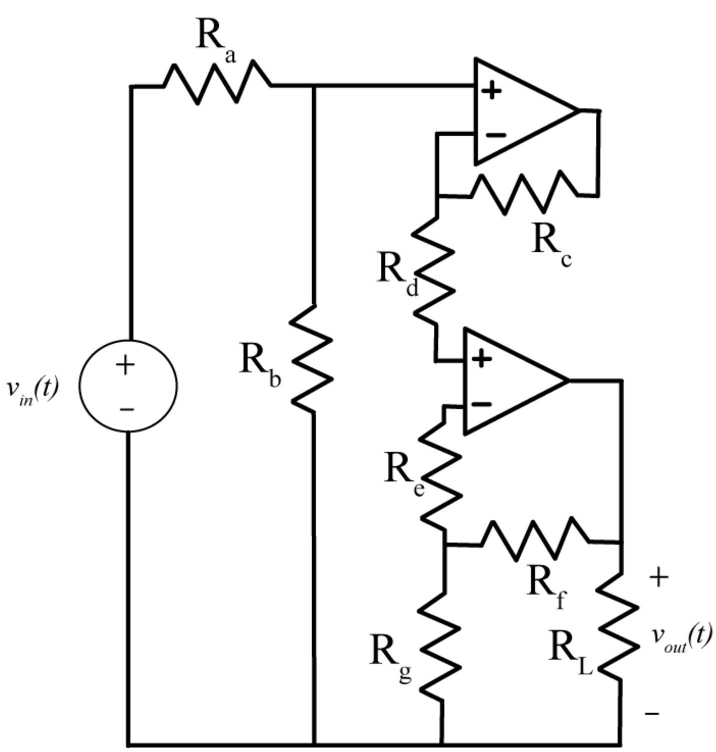
5) (12 points) For the following two op-amps circuits, we can write $v_{out}(t) = G\, v_{in}(t)$. Determine the value of G for each circuit.





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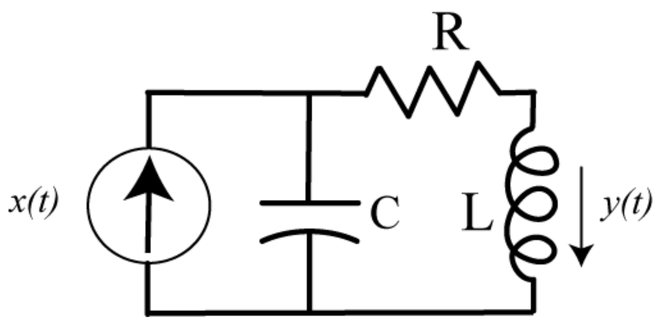
6) (16 points) For the second order circuit below,



Derive the governing second order differential equation for the output $y(t)$ and input $x(t)$. You do not need to put the equation in standard form.

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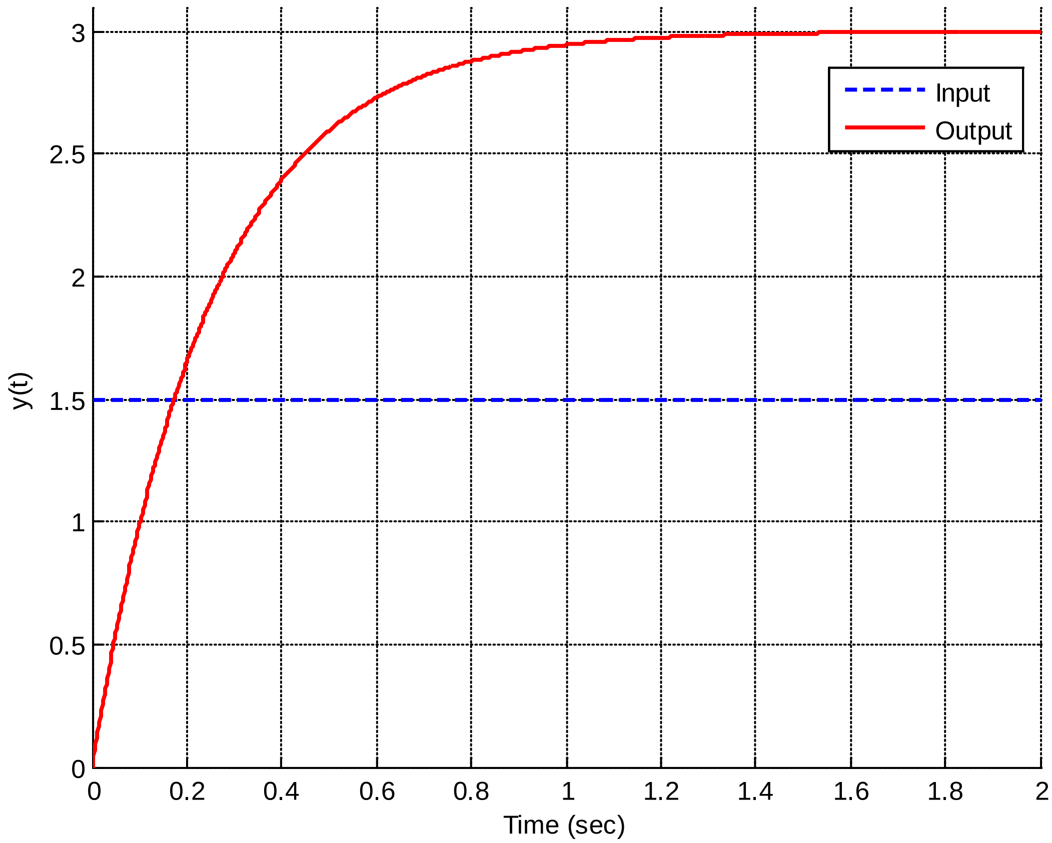
Problems 7-10, 4 points each (16 points)

For problems 7 and 8, refer to the following graph showing the input and output of a first order system. For this system the input is a step of amplitude 1.5.



7) What is the static gain?

8) What is the time constant?

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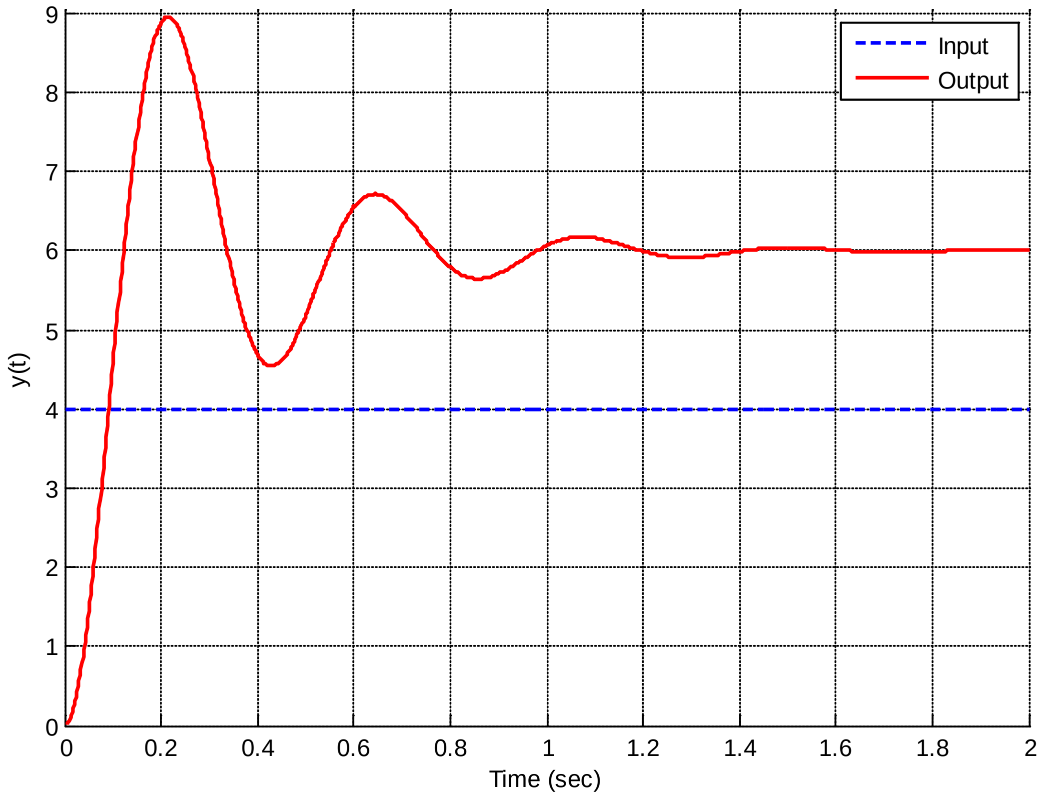
For problems 9 and 10, refer to the following graph showing the input and output of a second order system. For this system the input is a step of amplitude 4.



9) What is the static gain of the system?

10) What is the percent overshoot?