ECE205 Dynamical Systems

Midterm Exam 3
5/12/11

NAME: Solutions CM: ___________

- You must **show work** to receive partial and full credit.
- Put a box around your final answer and it must include units, if necessary.
- Time allowed : 50 minutes.

Question #	Possible Points	Awarded Points
1	10	
2	20	
3	20	
4	20	
5	20	
6	10	
Total	100	

Midterm Exam 3

1) **(10 points)** For the following multiple choice questions circle the letter next to the correct answer.

The following transfer function is for questions i, ii, and iii.

$$H(s) = \frac{1}{(s+4)\underbrace{(s^2+4s+4)}(s^2+2s+5)}$$

$[(s+2)^2][(s+1)^2+2^2]$

i) Which of the following is **not** a **characteristic mode** of the system?

a) e^{-4t} b) te^{-2t} c) e^{-2t} d) $e^{t}\cos(2t)$ e) $e^{-t}\sin(2t)$

ii) The best estimate of the **settling time** for this system is

a) 4 seconds b) 2 seconds c) 1 second d) 0.2 seconds e) 8 seconds

iii) The **dominant pole(s)** of this system are

a) -2 and -2 b) -1+2j and -1-2j c) -4 d) -20 e) 0

iv) How many of the following impulse responses represent **unstable systems**?

$h_1(t) = [t + e^{-t}]u(t)$ u

$h_2(t) = e^{-2t}u(t)$ S

$h_3(t) = [2+\sin(t)]u(t)$ m

$h_4(t) = [1 - t^3 e^{-0.1t}]u(t)$ m

$h_5(t) = [1 + t + e^{-t}]u(t)$ u

$h_6(t) = [te^{-t}\cos(5t) + e^{-2t}\sin(3t)]u(t)$ S

a) 0 b) 1 c) 2 d) 3 e)5

v) Which of the following transfer functions represents a **stable** system?

$$G_a(s) = \frac{s-1}{s+1} \qquad G_b(s) = \frac{1}{s(s+1)} \qquad G_c(s) = \frac{s}{s^2-1}$$

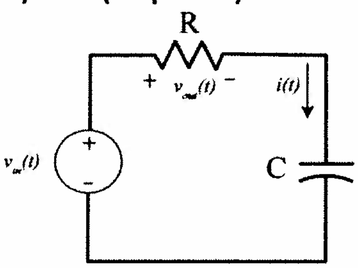
$$G_d(s) = \frac{s+1}{(s+1+j)(s+1-j)} \qquad G_e(s) = \frac{(s-1-j)(s-1+j)}{s} \qquad G_f(s) = \frac{(s-1-j)(s-1+j)}{(s+1-j)(s+1+j)}$$

a) all but G_c b) only G_a, G_b, and G_d c) only G_a, G_d, and G_f

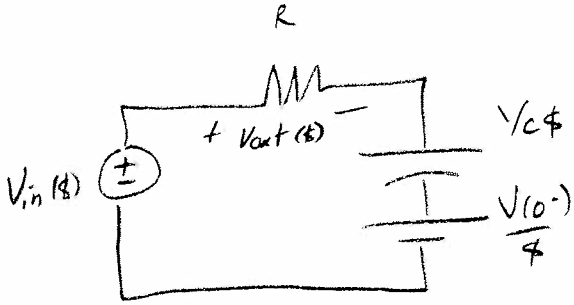
d) only G_d and G_f e) only G_a and G_d

Midterm Exam 3

2) **(20 points)** For the following ciruit,



Write the output, $V_{out}(s)$ in terms of $V_{in}(s)$, R, C and $v(0^-)$. Identify the ZSR (zero state reponse) and the ZIR (zero input response). (You can leave your answer in the s-domain)

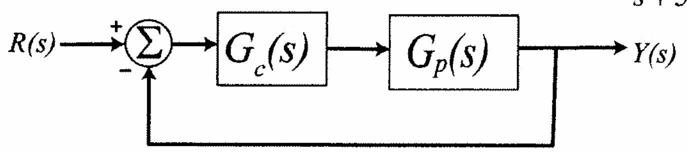


$$\frac{V_{out}(s)}{R} = \frac{V_{in}(s) - \frac{V(0^-)}{s}}{R + 1/Cs} = \frac{Cs\,V_{in}(s) - C\,V(0^-)}{RCs+1}$$

$$V_{out}(s) = \underbrace{\left[\frac{RCs\,V_{in}(s)}{RCs+1}\right]}_{ZSR} + \underbrace{\left[\frac{-RC\,V(0^-)}{RCs+1}\right]}_{ZIR}$$

Midterm Exam 3

3) **(20 points)** Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \dfrac{3}{s+5}$.



a) Determine the settling time of the plant alone (assuming there is no feedback).

$$T_s = \frac{4}{5}$$

b) Determine the steady-state error due to a unit step input of the plant alone (assuming there is no feedback).

$$e_{ss} = 1 - 3/5 = 2/5$$

c) For a proportional controller, $G_c(s) = k_p$,

i) Determine the closed loop transfer function $G_o(s)$.

$$G_o(s) = \frac{3k_p}{\cancel{s+}\,5+3k_p}$$

ii) What is the settling time in terms of k_p?

$$T_s = \frac{4}{5+3k_p}$$

iii) What is the steady state error due to a unit step input, in terms of k_p?

$$e_{ss} = 1 - \frac{3k_p}{5+3k_p} = \frac{5}{5+3k_p}$$

Midterm Exam 3

4) **(20 points)** The following figure shows three different circuits, which are subsystems for a larger system. We can write the transfer functions for these systems as

$$G_a(s) = \frac{-K_{low}\omega_{low}}{s+\omega_{low}} \qquad G_b(s) = \frac{-K_{high}s}{s+\omega_{high}} \qquad G_c(s) = -K_{ap}$$

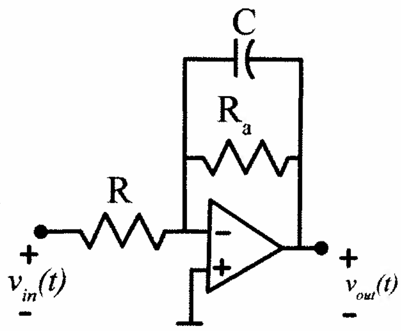
Determine the parameters $K_{low}, \omega_{low}, K_{high}, \omega_{high}$, and K_{ap} in terms of the parameters given (the resistors and capacitors).

$$\frac{V_{in}(s)}{R} = -\frac{V_{out}(s)}{R_a \| 1/Cs} \qquad R_a \| \frac{1}{Cs} = \frac{R_a/Cs}{R_a + 1/Cs} = \frac{R_a}{R_aCs+1}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{-R_a/R}{R_aCs+1} = \frac{-\frac{R_a}{R}\frac{1}{R_aC}}{s+1/R_aC}$$

$$\boxed{K_{low} = \frac{R_a}{R} \quad \omega_{Low} = \frac{1}{R_aC}}$$

System A



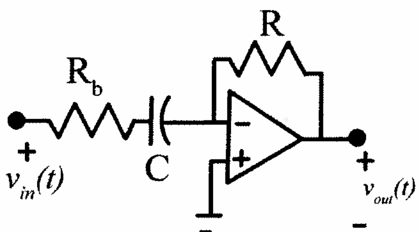
$$\frac{V_{in}(s)}{R_b + \frac{1}{Cs}} = \frac{-V_{out}(s)}{R} \qquad = \frac{V_{in}(s)\ Cs}{R_bCs+1}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \frac{-RCs}{R_bCs+1} = -\frac{RC\ s}{R_bC\,(s+1/R_bC)}$$

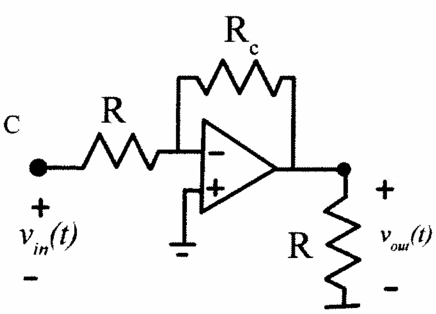
$$= \frac{-R/R_b\ s}{s+1/R_bC}$$

$$\boxed{K_{high} = -R/R_b \quad \omega_{high} = 1/R_bC}$$

System B



System C



$$\frac{V_{in}(s)}{R} = -\frac{V_{out}(s)}{R_c}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = -\frac{R_c}{R}$$

$$\boxed{K_{ap} = \frac{-R_c}{R}}$$

Midterm Exam 3

5) **(20 points)** For the following transfer functions, determine the **impulse response** of the system. Do not forget any necessary unit step functions.

a) $H(s) = \frac{e^{-s}}{s+2}$

$G(s) = \frac{1}{s+2} \quad g(t) = e^{-2t}u(t)$

$h(t) = g(t-2) = \boxed{e^{-2(t-2)}u(t-2) = h(t)}$

For the following transfer functions, determine the **unit step response** of the system. Do not forget any necessary unit step functions.

b) $H(s) = \frac{1}{(s+1)^2}$

$Y(s) = \frac{1}{s(s+1)^2} = \frac{A}{s} + \frac{B}{s+1} + \frac{C}{(s+1)^2}$

$A = 1 \quad C = -1, \quad \times s, \text{ let } s \to \infty, \quad 0 = A + B \quad B = -1$

$\boxed{y(t) = [1 - e^{-t} - te^{-t}]u(t)}$

c) $H(s) = \frac{1}{s^2+4s+20}$ $\quad Y(s) = \frac{1}{s^2+4s+20} \cdot \frac{1}{s} = \frac{1}{s[(s+2)^2+4^2]}$

$= \frac{A}{s} + B\left[\frac{4}{(s+2)^2+16}\right] + C\left[\frac{s+2}{(s+2)^2+16}\right] \quad A = \frac{1}{20} \quad \times s, \text{ let } s \to \infty \; 0 = A + C$

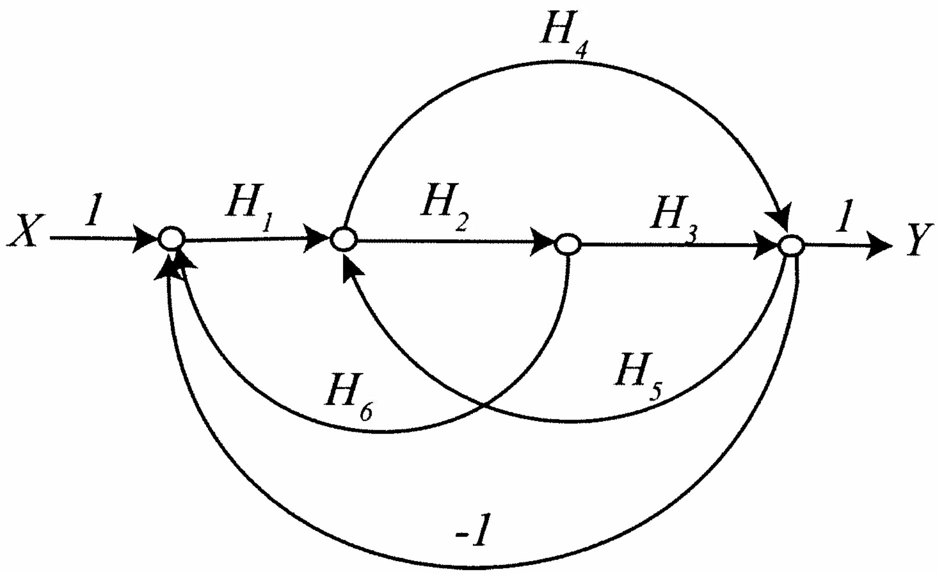
$C = -1/20$

let $s = -2 \quad \frac{-1}{32} = \frac{-1}{40} + \frac{B}{4} \quad B = 4\left[\frac{1}{40} - \frac{1}{32}\right] = \left[\frac{1}{10} - \frac{1}{8}\right] = \frac{8-10}{80} = \frac{-2}{80} = \frac{-1}{40}$

$y(t) = \left[\frac{1}{20} - \frac{1}{20}e^{-2t}\cos(4t) - \frac{1}{40}e^{-2t}\sin(4t)\right]u(t)$

Midterm Exam 3

6) **(10 points)** For the following signal flow graph, determine the transfer function between the input and output using Mason's gain formula. You do not need to simplify your final answer.



$P_1 = H_1 H_2 H_3$ $\qquad$ $P_2 = H_1 H_4$

$L_1 = H_1 H_2 H_6$ $\qquad$ $L_4 = -H_1 H_2 H_3$

$L_2 = H_2 H_3 H_5$ $\qquad$ $L_5 = -H_1 H_4$

$L_3 = H_4 H_5$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5)$

$\Delta_1 = 1$

$\Delta_2 = 1$

$$G_0 = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$