ECE-205 Quiz 2

1) A standard form for a first order system, with input x(t) and output y(t), is

a)
$$\frac{1}{\tau} \frac{dy(t)}{dt} + y(t) = Kx(t)$$
 b) $\tau \frac{dy(t)}{dt} + y(t) = Kx(t)$ c) $\frac{dy(t)}{dt} + \tau y(t) = Kx(t)$

d)
$$\frac{dy(t)}{dt} + \tau y(t) = \frac{1}{K}x(t)$$
 e) $\tau \frac{dy(t)}{dt} + y(t) = \frac{1}{K}x(t)$ f) $\frac{dy(t)}{dt} + \tau y(t) = Kx(t)$

2) The units of the time constant, τ , are a) 1/[time unit] b) [time unit] c) neither of these

Problems 3 -5 refer to a system described by the differential equation $2\dot{y}(t) + 2y(t) = 5x(t)$.

- 3) If the input is a step of amplitude 2, x(t) = 2u(t), then the steady state value of the output will be
- a) y(t) = 2.5 b) y(t) = 5 c) y(t) = 2 d) none of these
- **4)** The **time constant** of this system is
- a) $\tau = 5$ b) $\tau = 2.5$ c) $\tau = 1.0$ d) none of these
- 5) The static gain of this system is
- b) K = 2 c) K = 5a) K = 2.5d) none of these
- 6) Assume we have a first order system in standard form, and the input is a step. The usual form used to compute the response of the system is

a)
$$y(t) = [y(0) - y(\infty)]e^{-t/\tau} + y(0)$$
 b) $y(t) = [y(\infty) - y(0)]e^{-t/\tau} + y(0)$

c)
$$y(t) = [y(\infty) - y(0)]e^{-t/\tau} + y(\infty)$$
 d) $y(t) = [y(0) - y(\infty)]e^{-t/\tau} + y(\infty)$

7) A standard form for a second order system, with input x(t) and output y(t), is

a)
$$\ddot{y}(t) + \zeta \omega_n \dot{y}(t) + \omega_n^2 y(t) = K \omega_n^2 x(t)$$
 b) $\ddot{y}(t) + 2\zeta \omega_n \dot{y}(t) + \omega_n^2 y(t) = K x(t)$

b)
$$\ddot{y}(t) + 2\zeta\omega_{n}\dot{y}(t) + \omega_{n}^{2}y(t) = Kx(t)$$

c)
$$\ddot{y}(t) + 2\zeta \omega_n \dot{y}(t) + \omega_n^2 y(t) = K\omega_n^2 x(t)$$
 d) $\ddot{y}(t) + 2\zeta \omega_n \dot{y}(t) + y(t) = Kx(t)$

d)
$$\ddot{y}(t) + 2\zeta\omega_n \dot{y}(t) + y(t) = Kx(t)$$

Problems 8-11 refer to a system described by the differential equation $2\ddot{y}(t) + \dot{y}(t) + 4y(t) = 6x(t)$

8) If the input is a step of amplitude 2, x(t) = 2u(t), then the steady state value of the output will be

a)
$$v(t) = 3$$

a)
$$y(t) = 3$$
 b) $y(t) = 4$ c) $y(t) = 6$ d) $y(t) = 12$ e) none of these

d)
$$y(t) = 12$$

9) The natural frequency of this system is

a)
$$\omega_n = 1$$

a)
$$\omega_n = 1$$
 b) $\omega_n = \frac{1}{\sqrt{2}}$ c) $\omega_n = 2$ d) $\omega_n = \sqrt{2}$ e) none of these

c)
$$\omega_n = 2$$

d)
$$\omega_n = \sqrt{2}$$

10) The damping ratio of this system is

a)
$$\zeta = \frac{\sqrt{2}}{8}$$

$$b) \zeta = \frac{\sqrt{2}}{4}$$

c)
$$\zeta = \frac{1}{4}$$
 d)

$$d) \quad \zeta = \frac{1}{2\sqrt{2}}$$

a) $\zeta = \frac{\sqrt{2}}{9}$ b) $\zeta = \frac{\sqrt{2}}{4}$ c) $\zeta = \frac{1}{4}$ d) $\zeta = \frac{1}{2\sqrt{2}}$ e) none of these

11) The static gain of the system is

a)
$$K = 6$$

b)
$$K = 4$$

c)
$$K=1.5$$

c)
$$K=1.5$$
 d) none of these

12) For the differential equation $2\dot{y}(t) + y(t) = \cos(t)x(t)$ with intial time $t_0 = 2$ and initial value $y(t_0) = 2$, the output of the system at time t for an arbitrary input x(t) can be written as

a)
$$y(t) = 2e^{-\frac{t}{2}+1} + \int_{0}^{t} e^{-\frac{t}{2}+\frac{\lambda}{2}} \cos(\lambda) x(\lambda) d\lambda$$

a)
$$y(t) = 2e^{-\frac{t}{2}+1} + \int_{2}^{t} e^{-\frac{t}{2}+\frac{\lambda}{2}} \cos(\lambda) x(\lambda) d\lambda$$
 b) $y(t) = 2e^{-\frac{t}{2}+1} + \frac{1}{2} \int_{2}^{t} e^{-\frac{t}{2}+\frac{\lambda}{2}} \cos(\lambda) x(\lambda) d\lambda$

c)
$$y(t) = 2e^{-2t+4} + \int_{0}^{t} e^{-2t+2\lambda} \cos(\lambda) x(\lambda) d\lambda$$
 d) none of these

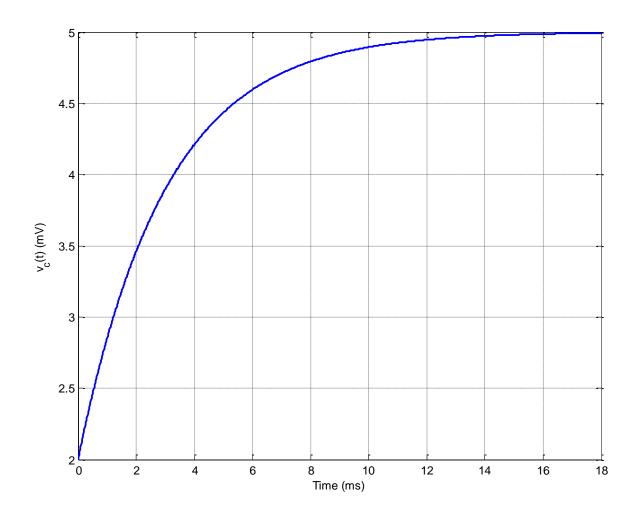
13) For the differential equation $\dot{y}(t) + 2ty(t) = x(t-1)$ with intial time $t_0 = 0$ and initial value $y(t_0) = 3$, the output of the system at time t for an arbitrary input x(t) can be written as

a)
$$y(t) = 3 + \int_{0}^{t} e^{-t^2 + \lambda^2} x(\lambda - 1) d\lambda$$

a)
$$y(t) = 3 + \int_{0}^{t} e^{-t^2 + \lambda^2} x(\lambda - 1) d\lambda$$
 b) $y(t) = 3e^{t^2} + \int_{0}^{t} e^{t^2 + \lambda^2} x(\lambda - 1) d\lambda$

c)
$$y(t) = 3e^{-t^2} + \int_0^t e^{-t^2 - \lambda^2} x(\lambda - 1) d\lambda$$
 d) none of these

14) The following figure shows a capacitor charging.

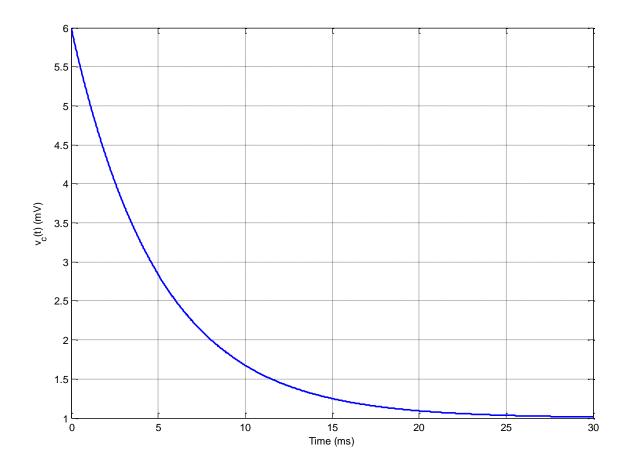


Based on this figure, the best estimate of the **time constant** for this system is

- a) 1.5 ms b) 3 ms
- c) 4.ms

- d) 12 me e) 16 ms f) 18 ms

15) The following figure shows a capacitor discharging.

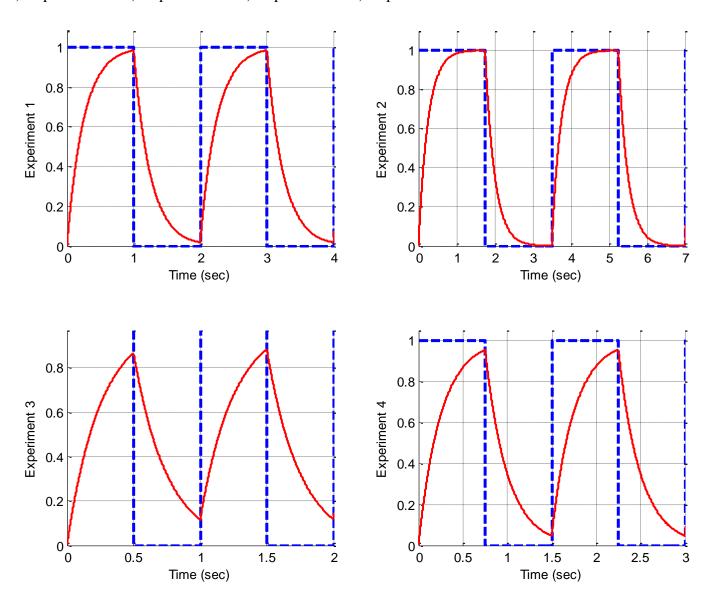


- Based on this figure, the best estimate of the **time constant** for this system is

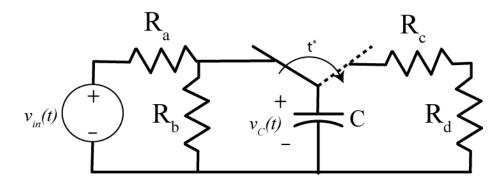
- a) 1 ms b) 3 ms c) 5 ms
 - d) 7 ms
- e) 15 ms
- f) 20 ms

16) Assume we are trying to use measure the time constant of a first order system experimentally using the rise time of the system. The input to the system is the rectangular pulse shown in the dotted line. Which of the experiments can we use? (Circle all that can be used)

a) Experiment 1 b) Experiment 2 c) Experiment 3 d) Experiment 4



Problems 17 and 18 refer to the following circuit:



Assume the system is initially at rest (initial conditions are zero), the input is a step of amplitude 4V, and the static gain is 0.5. Also assume the time constant during charging the capacitor is 1 ms, and the time constant during discharging the capacitor is 2 ms. The switch is connected to the source (charging the capacitor) until time $t^* = 10 \text{ ms}$ The following table may be helpful

Time (t)	t / τ	y(t)
0	0	$0 y_{ss}$
τ	1	$0.632 \ y_{ss}$
2τ	2	$0.865 \ y_{ss}$
3τ	3	$0.950 \ y_{ss}$
4τ	4	$0.982 \ y_{ss}$
5τ	5	$0.993 \ y_{ss}$

17) Which of the following is the best estimate of the voltage across the capacitor at 10 ms?

a)
$$0 V$$
 b) $2 V$ c) $3 V$ d) $4 V$ e) $6 V$

18) Which of the following is the best estimate of the voltage across the capacitor at 16 ms?