Name Solutions CM ____________

ECE-205

Exam 3

Fall 2015

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 ________/24

Problem 2 ________/17

Problem 3 ________/16

Problem 4 ________/19

Problems 5 ________/15

Problems 6-8 ________/9

Total ____________

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1) (24 points) For the following transfer functions, determine the <u>*unit step response*</u> of the system. *Do not forget any necessary unit step functions.*

a) $H(s)=\dfrac{e^{-2s}}{(s+1)^2}$

b) $H(s)=\dfrac{1}{(s+1)(s+2)}$

c) $H(s)=\dfrac{1}{s^2+2s+5}$

$A=1 \quad C=-1$

$\times s, \text{ let } s\to\infty \quad 0=A+B \quad B=-1$

a) $Y(s)=H(s)\frac{1}{s}=\dfrac{e^{-2s}}{s(s+1)^2}=e^{-2s}G(s) \qquad G(s)=\dfrac{1}{s(s+1)^2}=\dfrac{A}{s}+\dfrac{B}{s+1}+\dfrac{C}{(s+1)^2}$

$g(t)=[1-e^{-t}-te^{-t}]u(t) \qquad \boxed{y(t)=[1-e^{-(t-2)}-(t-2)e^{-(t-2)}]u(t-2)}$

b) $Y(s)=H(s)\frac{1}{s}=\dfrac{1}{s(s+1)(s+2)}=\dfrac{A}{s}+\dfrac{B}{s+1}+\dfrac{C}{s+2} \qquad A=\frac{1}{2} \quad B=-1 \quad C=\frac{1}{2}$

$\boxed{y(t)=\left[\frac{1}{2}-e^{-t}+\frac{1}{2}e^{-2t}\right]u(t)}$

c) $Y(s)=H(s)\frac{1}{s}=\dfrac{1}{s[s^2+2s+5]}=\dfrac{1}{s[(s+1)^2+2^2]}=\dfrac{A}{s}+B\left[\dfrac{2}{(s+1)^2+2^2}\right]+C\left[\dfrac{s+1}{(s+1)^2+2^2}\right]$

$A=\frac{1}{5} \qquad \times s \text{ let } s\to\infty \quad 0=A+C \quad C=-A=-\frac{1}{5}$

$s=-1 \qquad -\frac{1}{4}=\frac{-1}{5}+\frac{B}{2} \qquad \frac{1}{5}-\frac{1}{4}=\frac{4-5}{20}=\frac{-1}{20}=\frac{B}{2} \qquad B=-\frac{1}{10}$

$\boxed{y(t)=\left[\frac{1}{5}-\frac{1}{10}e^{-t}\sin(2t)-\frac{1}{5}e^{-t}\cos(2t)\right]u(t)}$

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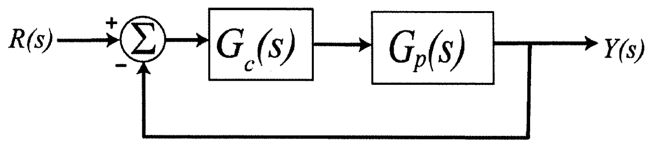
2) (17 points) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \frac{3}{s+8}$



a) Determine the settling time of the plant alone (assuming there is no feedback)

$$T_s = \frac{4}{8} = \boxed{\frac{1}{2} = T_s}$$

b) Determine the steady state error for plant alone assuming the input is a unit step (simplify your answer)

$$e_{ss} = 1 - G_p(0) = 1 - 3/8 = \boxed{5/8 = e_{ss}}$$

c) For a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function $G_0(s)$

$$G_0(s) = \frac{k_p \frac{3}{s+8}}{1 + k_p \frac{3}{s+8}} = \boxed{\frac{3k_p}{s+8+3k_p} = G_0(s)}$$

d) Determine the settling time of the closed loop system , in terms of k_p

$$\boxed{T_s = \frac{4}{8+3k_p}}$$

e) Determine the steady state error of the closed loop system for a unit step, in terms of k_p (simplify your answer)

$$e_{ss} = 1 - G_0(0) = 1 - \frac{3k_p}{8+3k_p} = \boxed{\frac{8}{8+3k_p} = e_{ss}}$$

f) For and integral controller, $G_c(s) = \frac{k_i}{s}$, determine the closed loop transfer function $G_0(s)$ and the steady state error for a unit step in terms of k_i

$$G_0(s) = \frac{\frac{k_i}{s} \frac{3}{s+8}}{1 + \frac{k_i}{s} \frac{3}{s+8}} = \boxed{\frac{3k_i}{s(s+8)+3k_i} = G_0(s)}$$

$$e_{ss} = 1 - G_0(0) = 1 - 1 = \boxed{0 = e_{ss}}$$

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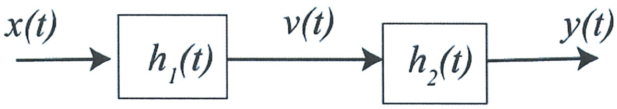
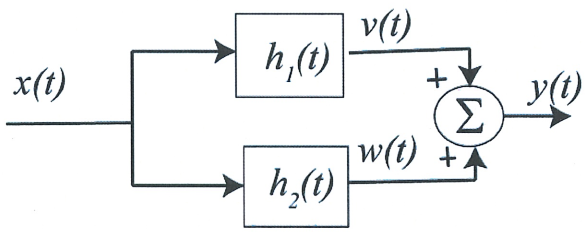
3) (16 points) For the following block diagram

For the following interconnected systems,

i) determine the overall impulse response (the impulse response between input $x(t)$ and output $y(t)$) and

ii) determine if the system is causal.





a) $h_1(t) = \delta(t+1)$, $h_2(t) = \delta(t+1)$

b) $h_1(t) = u(t+1)$, $h_2(t) = u(t-2) + \delta(t-2)$

Series (top) Connections:

a) $h(t) = h_1(t) * h_2(t) = \boxed{\delta(t+2) = h(t)}$ $\boxed{\text{not causal}}$

b) $h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} u(\lambda+1)\,u(t-\lambda-2)\,d\lambda + u(t-1)$

$= \int_{-1}^{t-2} d\lambda + u(t-1) = (t-1)u(t-1) + u(t-1) = \boxed{t\,u(t-1) = h(t)}$ $\boxed{\text{Causal}}$

Parallel (bottom) Connections:

a) $h(t) = h_1(t) + h_2(t) = \delta(t+1) + \delta(t+1) = \boxed{2\,\delta(t+1) = h(t)}$ $\boxed{\text{not causal}}$

b) $h(t) = h_1(t) + h_2(t) = \boxed{u(t+1) + u(t-2) + \delta(t-2) = h(t)}$ $\boxed{\text{not causal}}$

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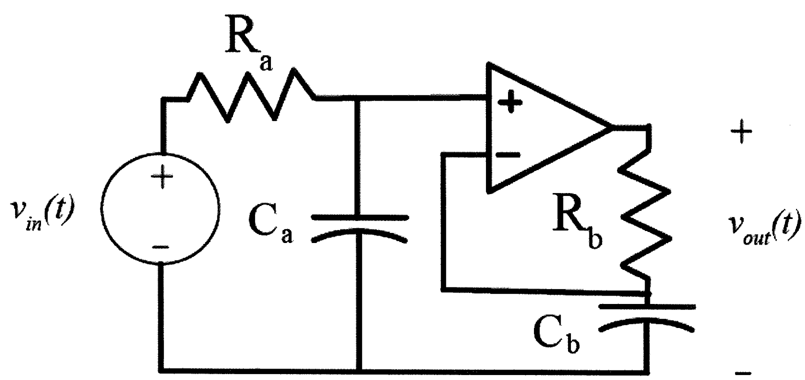
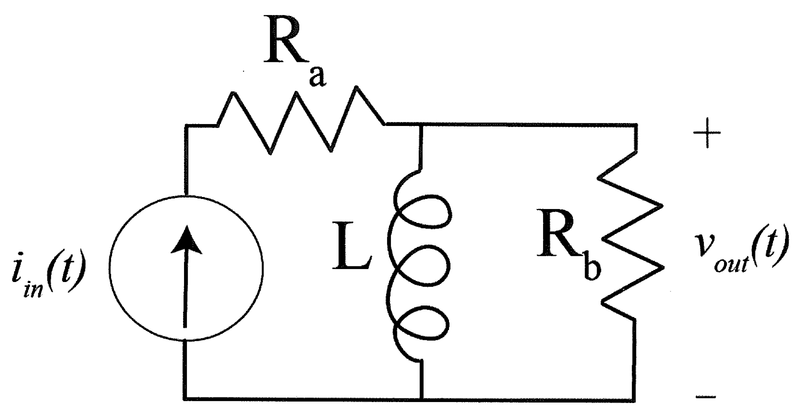
4) (19 points) Determine the transfer function for the following circuits:



$$V^+ = \frac{\frac{1}{C_a s}}{R_a + \frac{1}{C_a s}} V_{in}(s) = \frac{V_{in}(s)}{R_a C_a s + 1}$$

$$V^- = \frac{\frac{1}{C_b s}}{R_b + \frac{1}{C_b s}} V_{out}(s) = \frac{V_{out}(s)}{R_b C_b s + 1}$$

$$V^+ = V^- \quad \boxed{H(s) = \frac{V_{out}(s)}{V_{in}(s)} = \frac{R_b C_b s + 1}{R_a C_a s + 1}}$$



$$\frac{V_{out}(s)}{R_b} + \frac{V_{out}(s)}{Ls} = I_{in}(s)$$

$$V_{out}(s)\left[\frac{1}{R_b} + \frac{1}{Ls}\right] = I_{in}(s)$$

$$V_{out}(s)\left[\frac{Ls + R_b}{R_b L s}\right] = I_{in}(s)$$

$$\boxed{H(s) = \frac{V_{out}(s)}{I_{in}(s)} = \frac{R_b L s}{Ls + R_b}}$$

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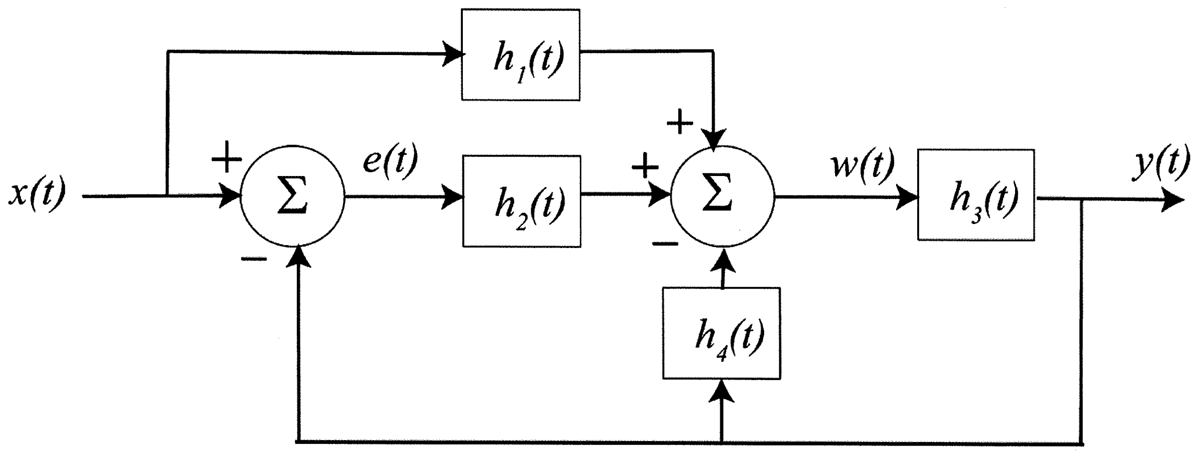
5) (15 points) For the following block diagram



Draw the corresponding signal flow graph, labeling each branch and direction. *Feel free to insert as many branches with a gain of 1 as you think you may need.*

Determine the system transfer function using Mason's gain rule. *You must clearly indicate all of the paths, the loops, the determinant and the cofactors.* **You need to simplify your final answer!**

$P_1 = H_2(s)\, H_3(s)$ $\quad P_2 = H_1(s)\, H_3(s)$

$L_1 = -H_2(s) H_3(s)$ $\quad L_2 = -H_3(s) H_4(s)$

$\Delta = 1-(L_1+L_2) = 1 + H_2(s) H_3(s) + H_3(s) H_4(s)$

$\Delta_1 = 1 \quad \Delta_2 = 1$

$$G_0(s) = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta} = \boxed{\frac{H_2(s)\, H_3(s) + H_1(s)\, H_3(s)}{1 + H_2(s) H_3(s) + H_3(s) H_4(s)}} = G_0(s)$$

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Problems 6 and 7 refer to the impulse responses of six different systems given below:

$$h_1(t) = [\sin(\mathrm{t}) + e^{-t}]u(t) \quad m$$
$$h_2(t) = e^{-2t}u(t) \quad S$$
$$h_3(t) = \mathrm{t}^2\, u(t) \quad U$$
$$h_4(t) = \delta(t-1) \quad S$$
$$h_5(t) = [t\sin(t) + e^{-t}]u(t) \quad U$$
$$h_6(t) = [te^{-t}\cos(5t) + e^{-2t}\sin(3t)]u(t) \quad S$$

6) The number of (asymptotically) **maginally stable systems** is a) 0 b) 1 c) 2 d) 3

7) The number of (asymptotically) **unstable systems** is a) 0 b) 1 c) 2 d) 3

8) Which of the following transfer functions represents a (asymptotically) **stable** system?

$$G_a(s) = \frac{s-1}{s+1} \qquad G_b(s) = \frac{1}{(\mathrm{s}+2)(s+1)} \qquad \cancel{G_c(s) = \frac{s}{s^2-1}}$$

$$G_d(s) = \frac{s+1}{(s+1+j)(s+1-j)} \qquad G_e(s) = \frac{(s-1-j)(s-1+j)}{s+1} \qquad G_f(s) = \frac{(s-1-j)(s-1+j)}{(s+1-j)(s+1+j)}$$

a) all but G_c b) only G_a, G_b, and G_d c) only G_a, G_d, and G_f

d) only G_d and G_f e) only G_a and G_d