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# ECE-205

# Exam 2

# Fall 2015

**Calculators and computers are not allowed. You must show your work to receive credit.**

**Problem 1 ________/22**

**Problem 2 ________/15**

**Problem 3 ________/18**

**Problem 4 ________/25**

**Problem 5 ________/25**

**Total ____________**

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**1) (22 points)** Fill in the non-shaded part of the following table. You should assume $0^- < t < \infty$ ($t$ starts just before time zero, so we include all of any delta functions at the origin.)

| | Linear? (Y/N) | Time Invariant? (Y/N) | BIBO Stable? (Y/N) |
|---|---|---|---|
| $y(t) = tx(t) + 2$ | No | No | |
| $\dot{y}(t) + ty(t) = \cos(t)x(t)$ | Yes | No | |
| $y(t) = x(1-t)$ | Yes | No | |
| $y(t) = \int_{-\infty}^{t} e^{\lambda} x(\lambda)d\lambda$ | | | No |
| $y(t) = \int_{0}^{t} e^{-\lambda} x(\lambda)d\lambda$ | | | Yes |
| $y(t) = \cos\left(\frac{1}{x(t)}\right)$ | | | Yes |
| $h(t) = \delta(t)$ | | | Yes |
| $h(t) = e^{t}u(t)$ | | | No |

$y(t) \le \int_{-\infty}^{t} e^{\lambda} N d\lambda = Ne^{t}$ not BIBO stable

$y(t) \le \int_{0}^{t} e^{-\lambda} N \, d\lambda = (1-e^{-t})N$ BIBO stable

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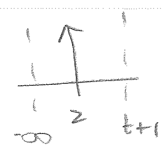
**2) (15 points)** Simplify the following as much as posible. Be sure to include any necessary unit step functions

$$y(t) = \delta(t-2) * \delta(t-1) \;=\; \int_{-\infty}^{\infty} \delta(\lambda-2)\,\delta(t-\lambda-1)\,d\lambda = \boxed{\delta(t-3)}$$

$$y(t) = \int_{-\infty}^{t+1} \delta(\lambda-2)\,d\lambda$$




need $t+1 > 2$

$t-1 > 0$

$$\boxed{u(t-1)}$$

$$y(t) = e^{t}\delta(t-2) \qquad \boxed{e^{2}\,\delta(t-2)}$$

$$y(t) = h(t) \star \delta(t) \;=\; \int_{-\infty}^{\infty} h(\lambda)\,\delta(t-\lambda)\,d\lambda = \boxed{h(t)}$$

$$y(t) = \int_{-\infty}^{\infty} \delta(\lambda-2)\,\delta(t-\lambda)\,d\lambda \;=\; \boxed{\delta(t-2)}$$

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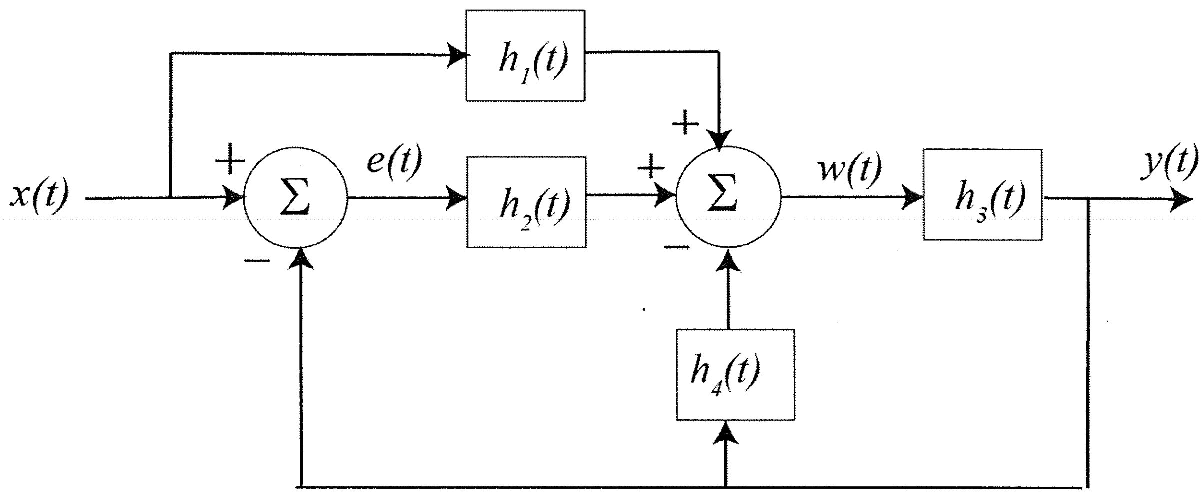
**3) (18 points)** The input-output relationship for the following system can be written as

$$y(t) * A(t) = x(t) * B(t)$$

Determine $A(t)$ and $B(t)$.

***Hint:*** Determine an expression for $e(t)$, then $w(t)$, then $y(t)$




$e(t) = x(t) - y(t) \qquad y(t) = w(t) * h_3(t)$

$w(t) = x(t) * h_1(t) + e(t) * h_2(t) - y(t) * h_4(t)$

$$y(t) = \left\{x(t) * h_1(t) + [x(t) - y(t)] * h_2(t) - y(t) * h_4(t)\right\} * h_3(t)$$

$$y(t) + y(t) * h_2(t) * h_3(t) + y(t) * h_4(t) * h_3(t) = x(t) * h_1(t) * h_3(t) + x(t) * h_2(t) * h_3(t)$$

$$y(t) * \underbrace{\left[\delta(t) + h_3(t) * h_4(t) + h_3(t) * h_2(t)\right]}_{A(t)} = x(t) * \underbrace{\left[h_1(t) * h_3(t) + h_2(t) * h_3(t)\right]}_{B(t)}$$

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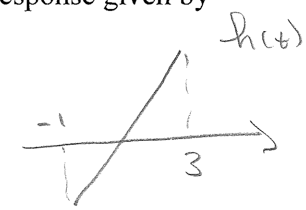
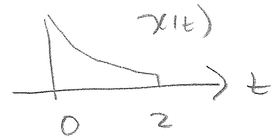
**4) (20 points)** Consider a linear time invariant system with impulse response given by

$$h(t) = t[u(t+1) - u(t-3)]$$



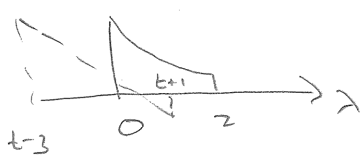

The input to the system is

$$x(t) = e^{-t}[u(t) - u(t-2)]$$




Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of $t$ for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals **cannot contain any unit step functions**
- ***DO NOT EVALUATE THE INTEGRALS!!***

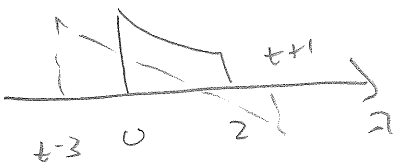



$0 \le t+1 \le 2$ $\quad$ $t-3 \le 0$

$-1 \le t \le 1$ $\quad$ $t < 3$

$\boxed{-1 \le t \le 1}$

$$\boxed{y(t) = \int_0^{t+1} (t-\lambda) e^{-\lambda} \, d\lambda}$$

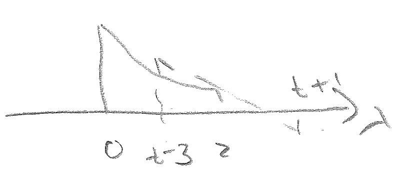



$t+1 \ge 2$ $\quad$ $t-3 \le 0$

$t \ge 1$ $\quad$ $t \le 3$

$\boxed{1 \le t \le 3}$

$$\boxed{y(t) = \int_0^{2} (t-\lambda) e^{-\lambda} \, d\lambda}$$




$0 \le t-3 \le 2$

$\boxed{3 \le t \le 5}$

$$\boxed{y(t) = \int_{t-3}^{2} (t-\lambda) e^{-\lambda} \, d\lambda}$$

$$\boxed{y(t) = 0 \text{ for } t \le -1 \text{ and } t \ge 5}$$

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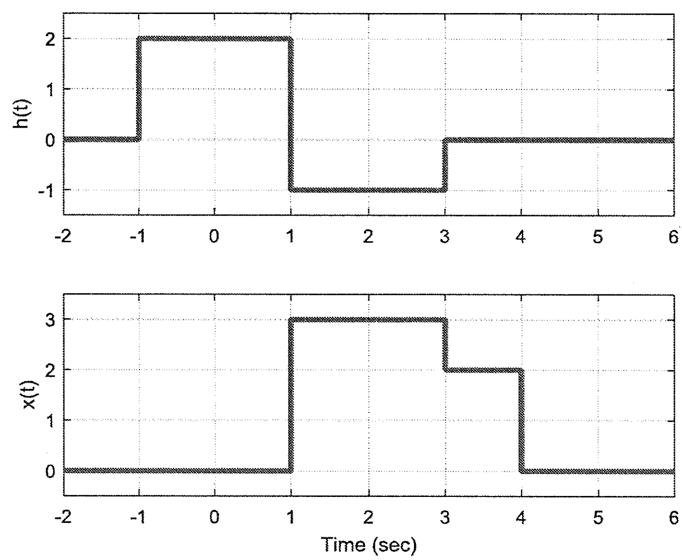
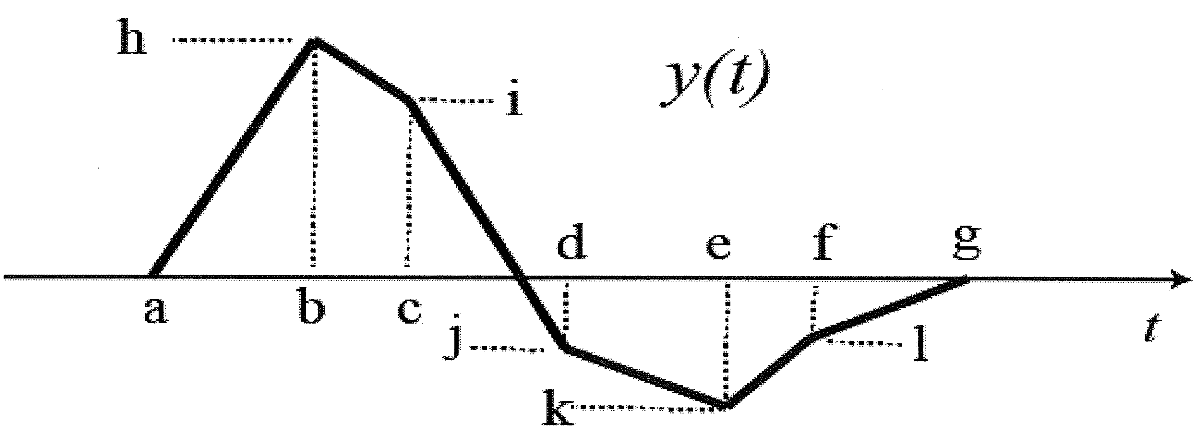
**5) (26 Points)** An LTI system has input, impulse response, and output as shown below. Determine numerical values for the parameters *a-l.* Note that parameters *a-g* correspond to *times*, and *h-l* correspond to *amplitudes.*

*Hints:*

- *Note that the output is not drawn to scale, it just represents the general shape of the output.*
- *A good way to check your answer is to flip and slide one of them, then flip and slide the other one.*
- *It is probably much easier to get the times correct than the amplitudes.*



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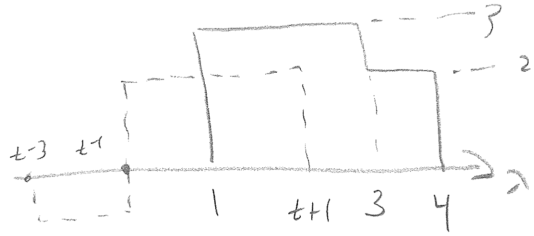
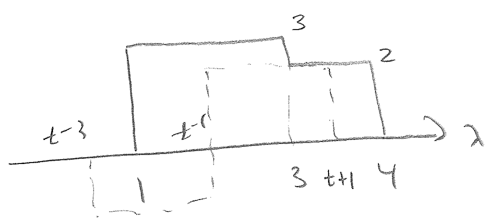
$y(t) = 0$ for $t+1 \le 1$ or $t \le 0$

$\boxed{a = 0}$



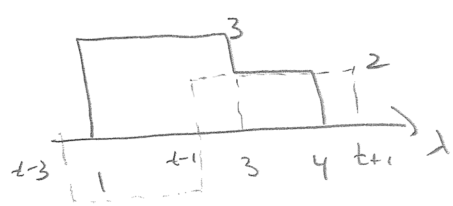

$1 \le t+1 \le 3$

$0 \le t \le 2$ $\boxed{b = 2}$ $y_b = 2 \cdot 3 \cdot 2 = \boxed{12 = h}$




$3 \le t+1 \le 4$

$2 \le t \le 3$ $\boxed{c = 3}$ $y_c = 2 \cdot 2 \cdot 1 + 3 \cdot 2 \cdot 1 + 3 \cdot 1 \cdot (-1) = 4 + 6 - 3 = \boxed{7 = i}$

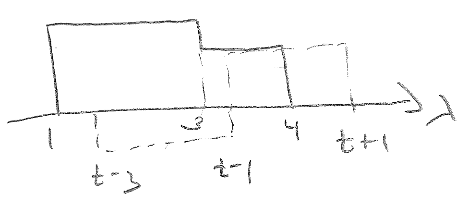



$t-1 \le 3$ $t \le 4$

$t-3 \le 1$ $t \le 4$

$d = 4$

$y_d = 2 \cdot 2 \cdot 1 + 3 \cdot 2(-1) = 4 - 6 = \boxed{-2 = j}$

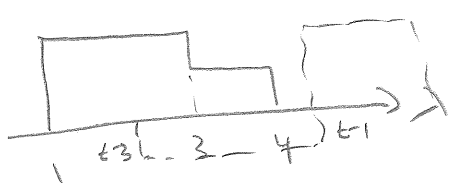



$1 \le t-3 \le 3$ $3 \le t-1 \le 4$

$4 \le t \le 6$ $4 \le t \le 5$

$e = 5$

$y_e = (-1)(3)(1) + (-1)(2)(1) = \boxed{-5 = k}$



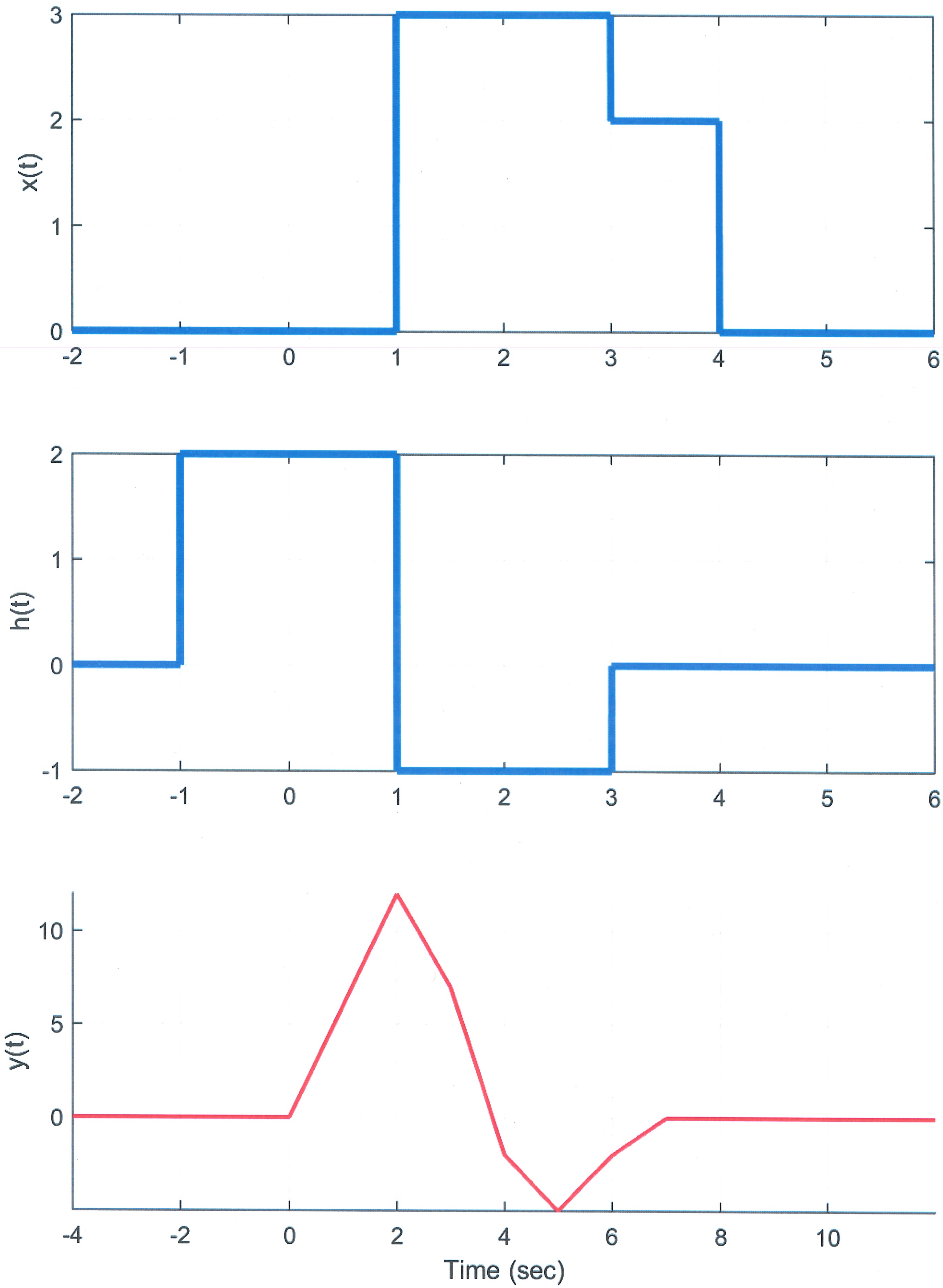

$t-3 \le 3$ $t \le 6$

$f = 6$

$y_f = (-1)(1)(2) = \boxed{-2 = l}$

$y = 0$ for $t-3 \ge 4$

$t = 7 = g$


x(t)
h(t)
y(t)
Time (sec)