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ECE-205

Exam 1

Fall 2015

Calculators can only be used for simple calculations. Solving integrals, differential equations, systems of equations, etc. does not count as a simple calculation.

You must show your work to receive credit.

Problem 1	________/16
Problem 2	________/20
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Problem 5	________/10
Problem 6	________/10
Problem 7	________/12
Problem 8	________/5

Total ____________

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1) (16 points) Assume we have a first order system with the governing differential equation

$$0.5\dot{y}(t) + y(t) = 2x(t)$$

The system has the initial value of 0, so $y(0) = 0$. The input to this system is

$$x(t) = \begin{cases} 0 & t < 0 \\ 2 & 0 \le t < 1 \\ 3 & 1 \le t \end{cases}$$

Determine the output of the system in each of the above time intervals. *Simplify your final answer as much as possible and box it. Be sure to include the correct initial value in each interval.*

$\tau = 0.5 \quad K = 2 \qquad y(t) = [y(t_0) - y(\infty)]e^{-(t-t_0)/\tau} + y(\infty)$

$KA = y(\infty)$

$0 \le t < 1 \qquad A = 2 \quad KA = 4 \quad t_0 = 0 \quad y(t_0) = 0$

$y(t) = [0 - 4]e^{-t/0.5} + 4 = 4(1 - e^{-2t})$

$1 \le t \qquad A = 3 \quad KA = 6 \quad t_0 = 1 \quad y(t_0) = y(1) = 4(1 - e^{-2}) = 3.459$

$y(t) = [3.459 - 6]e^{-(t-1)/0.5} + 6 = 6 - 2.541e^{-2(t-1)}$

$$\boxed{y(t) = \begin{cases} 0 & t < 0 \\ 4(1 - e^{-2t}) & 0 \le t < 1 \\ 6 - 2.541e^{-2(t-1)} & t \ge 1 \end{cases}}$$

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2) **(20 points)** For the following differential equations, assume the input is $x(t) = 4u(t)$ (the input is equal to four for time greater than zero), and the initial conditions are $y(0) = \dot{y}(0) = 0$

Determine the solution to each of the following differential equations and put your final answer in a box. Be sure to use the initial conditions to solve for all unknowns. You must show all your work to receive credit.

a) $\ddot{y}(t) + 5\dot{y}(t) + 6y(t) = 3x(t)$

$6y_f = 3 \cdot 4 = 12 \quad y_f = 2$

$r^2 + 5r + 6 = (r+2)(r+3) = 0$

$y(t) = 2 + c_1 e^{-2t} + c_2 e^{-3t}$

$\dot{y}(t) = 0 - 2c_1 e^{-2t} - 3c_2 e^{-3t}$

$y(0) = 2 + c_1 + c_2 = 0$

$\dot{y}(0) = -2c_1 - 3c_2 = 0$

$2c_1 + 2c_2 + 4 = 0$

$+ \quad -2c_1 - 3c_2 = 0$

$-c_2 + 4 = 0$

$c_2 = 4 \qquad c_1 = -c_2 - 2 = -6$

$$\boxed{y(t) = 2 - 6e^{-2t} + 4e^{-3t}}$$

b) $\ddot{y}(t) + 4\dot{y}(t) + 4y(t) = 8x(t)$

$4y_f = 8 \cdot 4 \qquad y_f = 8$

$r^2 + 4r + 4 = (r+2)^2 = 0$

$y(t) = 8 + c_1 e^{-2t} + c_2 t e^{-2t}$

$\dot{y}(t) = -2c_1 e^{-2t} + c_2 e^{-2t} - 2c_2 t e^{-2t}$

$y(0) = 0 = 8 + c_1 \qquad c_1 = -8$

$\dot{y}(0) = 0 = -2c_1 + c_2 \qquad c_2 = 2c_1 = -16$

$$\boxed{y(t) = 8 - 8e^{-2t} - 16t\,e^{-2t}}$$

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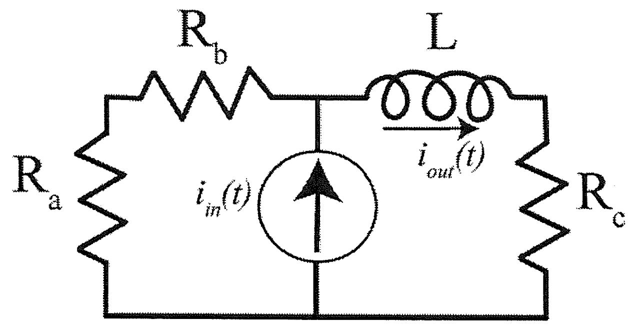
3) (12 points) For the following circuit:



- Determine the time constant
- Determine the static gain
- Determine the governing differential equation

You may solve this problem using the "short-cut methods", or you can derive the governing differential equation to find the time constant and static gain. Be sure you answer all three parts.

$$L\frac{di_{out}(t)}{dt} + i_{out}(t)R_c = [i_{in}(t) - i_{out}(t)](R_a + R_b)$$

$$L\frac{di_{out}(t)}{dt} + (R_a + R_b + R_c)i_{out}(t) = (R_a + R_b)\, i_{in}(t)$$

$$\boxed{\left(\frac{L}{R_a+R_b+R_c}\right)\frac{di_{out}(t)}{dt} + i_{out}(t) = \left(\frac{R_a+R_b}{R_a+R_b+R_c}\right) i_{in}(t)}$$

$$\boxed{\tau = \frac{L}{R_a+R_b+R_c}} \qquad \boxed{K = \frac{R_a+R_b}{R_a+R_b+R_c}}$$

$R_{th} = R_a + R_b + R_c$ use current divider to get K

$\tau = L/R_{th}$

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4) (15 points) Determine the response ($y(t)$) for the following underdamped second order system,

$$\ddot{y}(t) + 3\dot{y}(t) + 225y(t) = 450x(t)$$

with intitial conditions $y(0) = 1$, $\dot{y}(0) = -2$ and input $x(t) = 3u(t)$ (a step input of amplitude 3 at time zero).

$225 y_f = 450 \cdot 3$ $\boxed{y_f = 6}$

$r^2 + 3r + 225 = 0$ $r = \frac{-3 \pm \sqrt{9 - 4(225)}}{2} = -1.5 \pm j14.925$

$y(t) = 6 + ce^{-1.5t} \sin(14.925t + \theta)$

$y(0) = 1 = 6 + c\sin(\theta)$ $\boxed{c\sin(\theta) = -5}$

$\dot{y}(t) = 0 - 1.5ce^{-1.5t} \sin(14.925t + \theta) + 14.925ce^{-1.5t} \cos(14.925t + \theta)$

$\dot{y}(0) = -1.5c\sin(\theta) + 14.925c\cos(\theta) = -2$

$(-1.5)(-5) + 14.925c\cos(\theta) = -2$

$c\cos(\theta) = \frac{-2 - 7.5}{14.925} = \boxed{-0.637 = c\cos(\theta)}$

$\tan(\theta) = \frac{c\sin(\theta)}{c\cos(\theta)} = \frac{-5}{-0.637} = 1.444 \pm \pi = 4.586$

$c = \frac{-5}{\sin(\theta)} = 5.040$

$$\boxed{y(t) = 6 + 5.040e^{-1.5t} \sin(14.925t + 4.586)}$$

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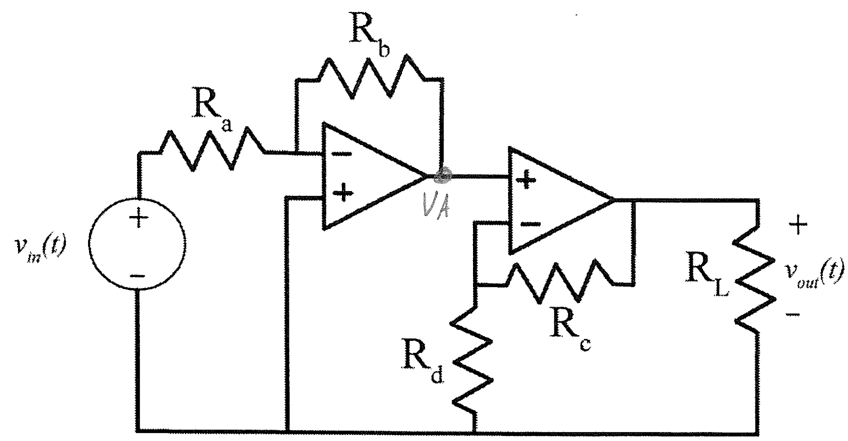
5) (10 points) For the following circuit, we can write $v_{out}(t) = G\, v_{in}(t)$. Determine an expression for the constant G in terms of the resisters given below.



Note that the polarity of the two op-amps are different!

$$\frac{V_{in}}{R_a} + \frac{V_A}{R_b} = 0 \qquad V_A = \frac{-R_b}{R_a} V_{in}$$

$$V_A = \frac{V_{out}}{R_c + R_d} \cdot R_d$$

$$V_A = \frac{-R_b}{R_a} V_{in} = \frac{R_d}{R_c + R_d} V_{out}$$

$$V_{out} = \left[\frac{-R_b}{R_a} \frac{R_c + R_d}{R_d} \right] V_{in}$$

$$\boxed{G = \frac{-R_b}{R_a} \frac{R_c + R_d}{R_d}}$$

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6) (10 points) For a first order system described by the differential equation

$$\dot{y}(t) + \frac{1}{t} y(t) = \cos(t) x(t)$$

with $t_0 = 1$ and $y(t_0) = 1$, use integrating factors to solve the differential equation. Include the initial conditions in your solution.

$$\frac{d}{dt}\left[y(t) e^{\ln(t)}\right] = \frac{d}{dt}\left[t\, y(t)\right] = t\cos(t)\, x(t)$$

$$t y(t) - 1 = \int_1^t \lambda \cos(\lambda)\, x(\lambda)\, d\lambda$$

$$t y(t) = 1 + \int_1^t \lambda \cos(\lambda)\, x(\lambda)\, d\lambda$$

$$\boxed{y(t) = \frac{1}{t} + \int_1^t \frac{\lambda}{t} \cos(\lambda)\, x(\lambda)\, d\lambda}$$

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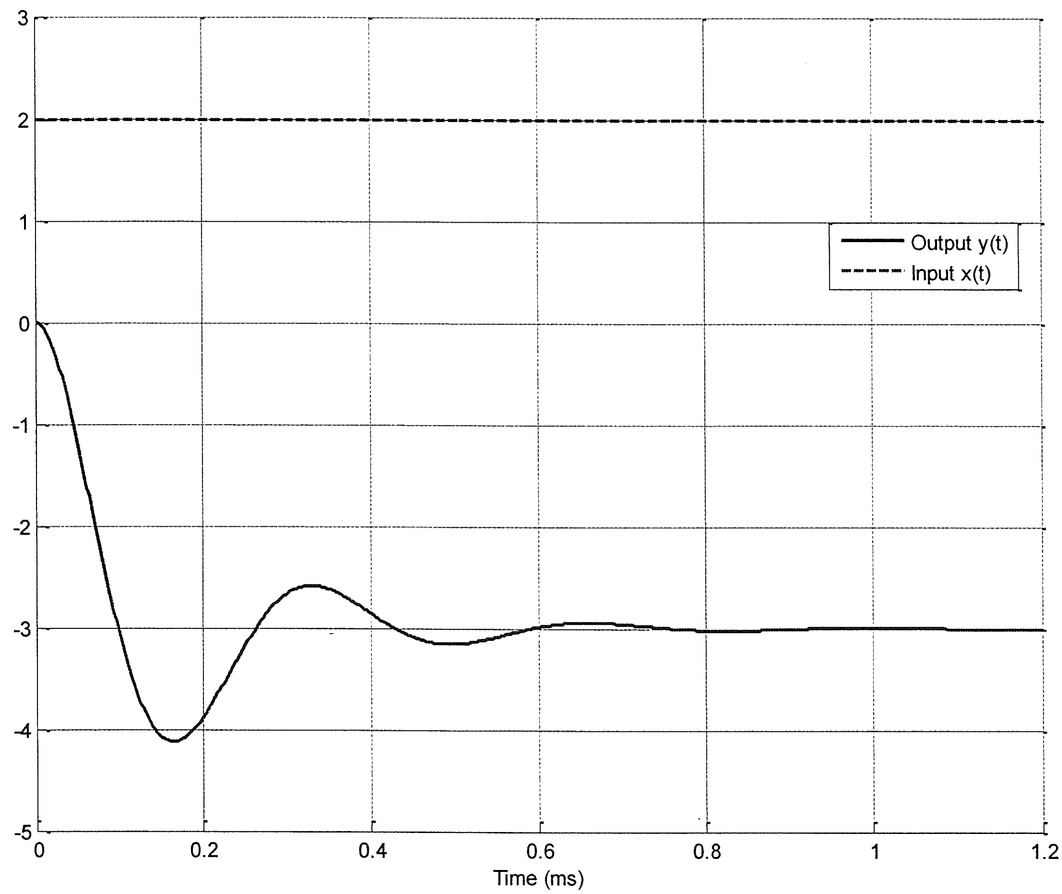
7) (12 points) The following graph showing the response of a second order system to a step input of amplitude 2.



a) Estimate the percent overshoot

$\frac{-4.1-(-3)}{-3} \times 100\% = \boxed{36.7\% = PO}$

b) Estimate the (2%) settling time

$\boxed{T_s \approx 0.6 ms}$

c) Estimate the static gain

$K(2) = -3$ $\boxed{K = -1.5}$

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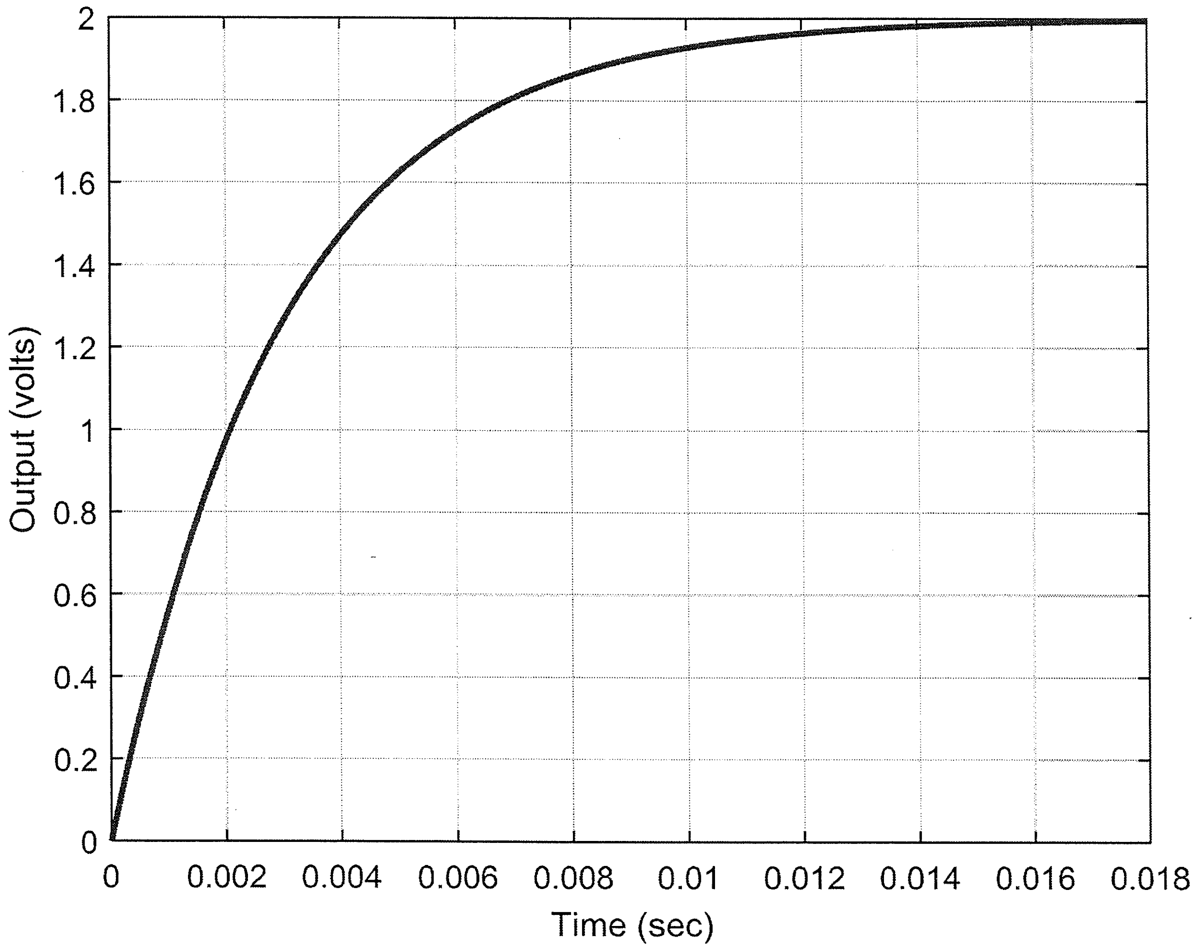
8) (6 points) The following graph showing the response of a first order system to a step input. Estimate the ***time constant*** of the system.



at 3τ at 95% of final value = 1.9

$3\tau \approx 0.009$ $\boxed{\tau \approx 0.003 \text{ sec}}$