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ECE-205

Exam 3

Fall 2013

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 _________/20

Problem 2 _________/15

Problem 3 _________/15

Problem 4 _________/15

Problems 5 ________/14

Problems 6-12 _________/21

Total _____________

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1) (20 points) For the following transfer functions, determine **both**

- the *impulse response*
- the *unit step response*

Do not forget any necessary unit step functions.

a) $H(s) = \dfrac{4}{s^2 + 4s + 8}$

b) $H(s) = \dfrac{se^{-s}}{(s+1)^2}$

a) $H(s) = 2 \dfrac{2}{(s+2)^2 + 2^2}$ $\boxed{h(t) = 2e^{-2t}\sin(t)\,u(t)}$

$Y(s) = \dfrac{4}{s(s^2+4s+8)} = \dfrac{A}{s} + B\left[\dfrac{2}{(s+2)^2+4}\right] + C\left[\dfrac{s+2}{(s+2)^2+4}\right]$

$A = \frac{1}{2}$ $\quad \times s$, let $s \to \infty$ $\quad 0 = A + C$ $\quad C = -\frac{1}{2}$

$s = -2$ $\quad -\frac{1}{2} = -\frac{1}{4} + \frac{B}{2}$ $\quad -\frac{1}{4} = \frac{B}{2}$ $\quad B = -\frac{1}{2}$

$\boxed{y(t) = \left[\frac{1}{2} - \frac{1}{2}e^{-2t}\sin(2t) - \frac{1}{2}e^{-2t}\cos(2t)\right]u(t)}$

b) $G(s) = \dfrac{s}{(s+1)^2} = \dfrac{A}{s+1} + \dfrac{B}{(s+1)^2}$ $\quad B = -1$ $\quad \times s$ let $s \to \infty$ $\quad 1 = A$

$g(t) = \left[e^{-t} - te^{-t}\right]u(t)$ $\quad \boxed{h(t) = g(t-1) = \left[e^{-(t-1)} - (t-1)e^{-(t-1)}\right]u(t-1)}$

$G(s) = \dfrac{1}{(s+1)^2}$ $\quad g(t) = te^{-t}u(t)$ $\quad \boxed{y(t) = g(t-1) = (t-1)e^{-(t-1)}u(t-1)}$

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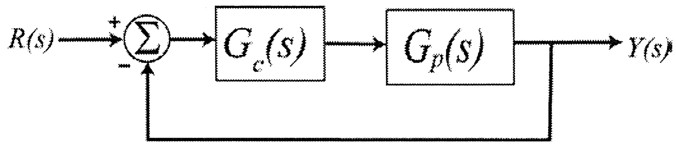
2) (15 points) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \dfrac{3}{s+5}$



a) Determine the settling time of the plant alone (assuming there is no feedback)

$$\boxed{T_s = \frac{4}{5}}$$

b) Determine the steady state error for plant alone assuming the input is a unit step (simplify your answer as much as possible)

$$e_{ss} = 1 - 3/5 = \boxed{\frac{2}{5} = e_{ss}}$$

c) For a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function $G_0(s)$

$$\boxed{G_0(s) = \frac{3k_p}{s+5+3k_p}}$$

d) Determine the settling time of the closed loop system, in terms of k_p

$$\boxed{T_s = \frac{4}{5+3k_p}}$$

e) Determine the steady state error of the closed loop system for a unit step, in terms of k_p (simplify your answer as much as possible)

$$e_{ss} = 1 - \frac{3k_p}{5+3k_p} = \boxed{\frac{5}{5+3k_p} = e_{ss}}$$

f) For and integral controller, $G_c(s) = \dfrac{k_i}{s}$, determine the closed loop transfer function $G_0(s)$ and the steady state error for a unit step in terms of k_i

$$\boxed{G_0(s) = \frac{3k_i}{s^2+5s+3k_i}} \qquad \boxed{e_{ss} = 0}$$

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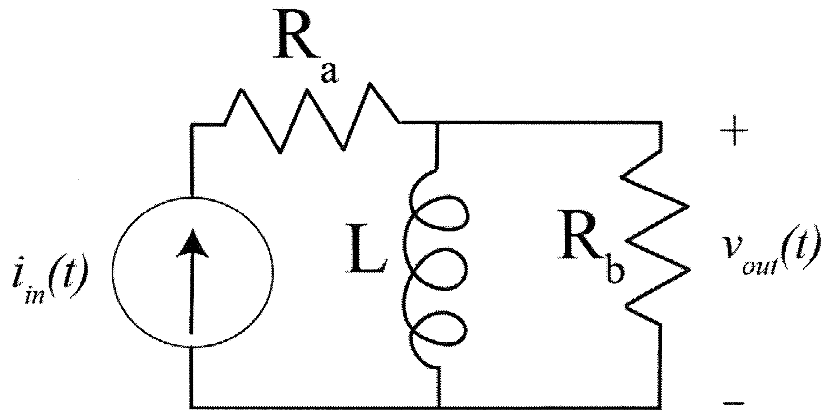
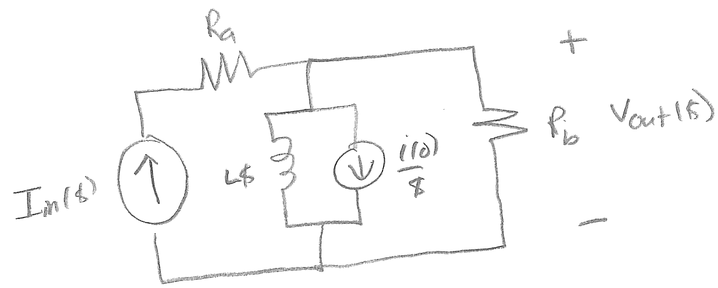
3) (15 points) For the following circuit



- Determine the ZIR
- Determine the ZSR
- Determine the transfer function



$$I_{in}(s) = \frac{V_{out}(s)}{Ls} + \frac{i(0)}{s} + \frac{V_{out}(s)}{R_b} = \frac{i(0)}{s} + V_{out}(s)\left[\frac{1}{Ls} + \frac{1}{R_b}\right]$$

$$= \frac{i(0)}{s} + V_{out}(s)\left[\frac{R_b + Ls}{R_b L s}\right]$$

$$\boxed{V_{out}(s) = \underbrace{\left[\frac{R_b L s}{Ls + R_b} I_{in}(s)\right]}_{ZSR} + \underbrace{\left[\frac{-i(0) R_b L}{Ls + R_b}\right]}_{ZIR}}$$

$$H(s) = \frac{V_{out}(s)}{I_{in}(s)} = \boxed{\frac{R_b L s}{Ls + R_b} = H(s)}$$

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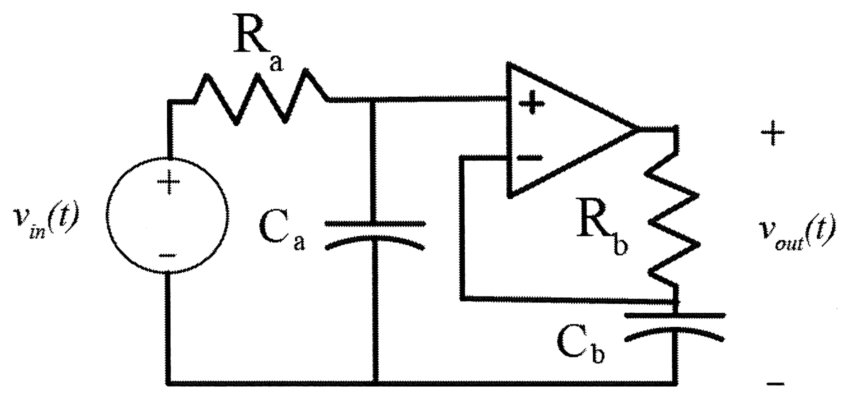
4) (15 Points) Determine the transfer function for the following circuit.



$$V^+ = \frac{V_{in}(s)\frac{1}{C_a s}}{R_a + \frac{1}{C_a s}} = \frac{V_{in}(s)}{R_a C_a s + 1} = V^- = \frac{V_{out}(s)\frac{1}{C_b s}}{R_b + \frac{1}{C_b s}} = \frac{V_{out}(s)}{R_b C_b s + 1}$$

$$\frac{V_{out}(s)}{V_{in}(s)} = \boxed{H(s) = \frac{R_b C_b s + 1}{R_a C_a s + 1}}$$

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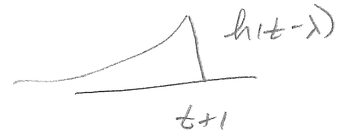
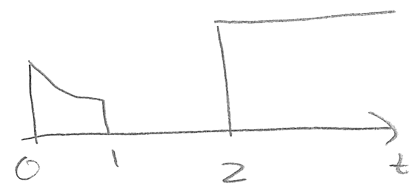
5) (15 points) Consider a linear time invariant system with impulse response given by

$$h(t) = e^{-(t+1)}u(t+1)$$



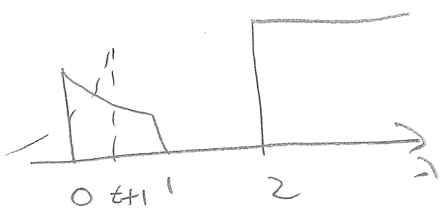
The input to the system is given by

$$x(t) = e^{-t}[u(t) - u(t-1)] + 2u(t-2)$$

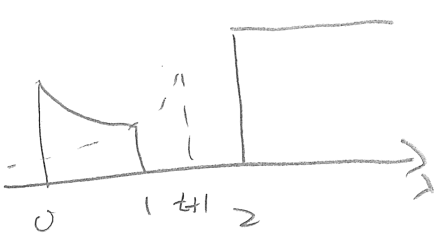


Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, **NOT** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

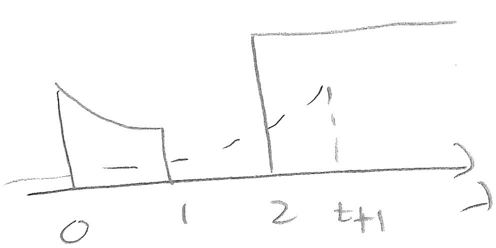


$-1 \le t \le 0 \quad y(t) = \int_0^{t+1} e^{-(t-\lambda+1)} e^{-\lambda}\, d\lambda$



$0 \le t \le 1 \quad y(t) = \int_0^{1} e^{-(t-\lambda+1)} e^{-\lambda}\, d\lambda$

$t \ge 1 \quad y(t) = \int_0^{1} e^{-(t-\lambda+1)} e^{-\lambda}\, d\lambda + \int_2^{t+1} e^{-(t-\lambda+1)}\, 2\, d\lambda$



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Problems 6 and 7 refer to the impulse responses of six different systems given below:

$$h_1(t) = [1 + e^{-t}]u(t) \quad m$$
$$h_2(t) = e^{-2t}u(t) \quad S$$
$$h_3(t) = [2 + \sin(t)]u(t) \quad m$$
$$h_4(t) = [1 - t^3 e^{-0.1t}]u(t) \quad m$$
$$h_5(t) = [t\sin(t) + e^{-t}]u(t) \quad u$$
$$h_6(t) = [te^{-t}\cos(5t) + e^{-2t}\sin(3t)]u(t) \quad S$$

6) The number of (asymptotically) **maginally stable systems** is a) 0 b) 1 c) 2 d) 3

7) The number of (asymptotically) **unstable systems** is a) 0 b) 1 c) 2 d) 3

8) Which of the following transfer functions represents a (asymptotically) **stable** system?

$$G_a(s) = \frac{s-1}{s+1} \quad S \qquad G_b(s) = \frac{1}{s(s+1)} \quad m \qquad G_c(s) = \frac{s}{s^2-1} \quad u$$

$$G_d(s) = \frac{s+1}{(s+1+j)(s+1-j)} \quad S \qquad G_e(s) = \frac{(s-1-j)(s-1+j)}{s} \quad m \qquad G_f(s) = \frac{(s-1-j)(s-1+j)}{(s+1-j)(s+1+j)} \quad S$$

a) all but G_c b) only G_a, G_b, and G_d c) only G_a, G_d, and G_f

d) only G_d and G_f e) only G_a and G_d

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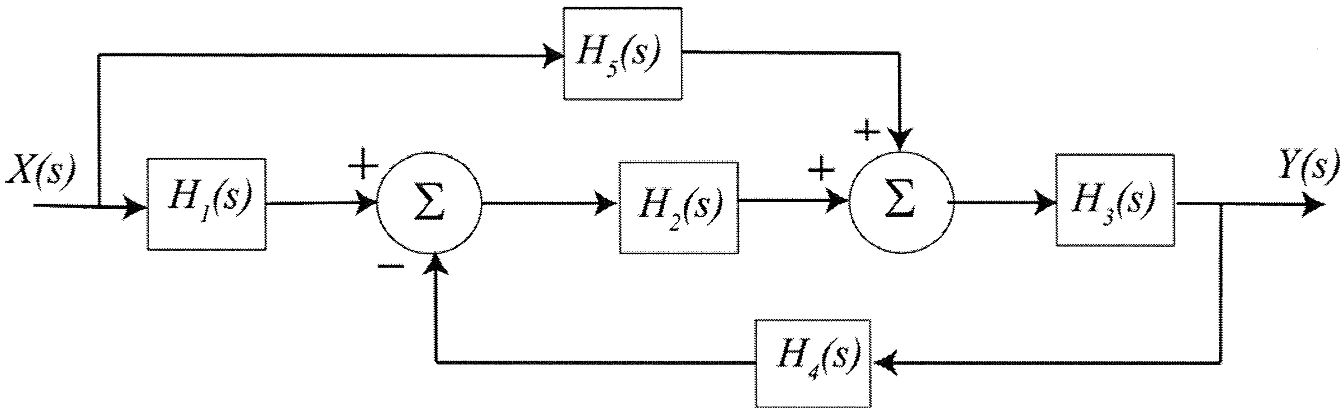
For problems 9-12, consider the signal flow graph representation of the following block diagram.



9) How many **paths** are there? a) 0 b) 1 c) 2 d) 3 e) 4

10) How many **loops** are there? a) 0 b) 1 c) 2 d) 3 e) 4

11) The **determinant** (Δ) is a) 1 b) $1-H_2H_3H_4$ c) $1+H_2H_3H_4$ d) none of these

12) The **transfer function** is a) 1 b) $\dfrac{H_3H_5+H_1H_2H_3}{1+H_2H_3H_4}$ c) $\dfrac{H_3H_5+H_1H_2H_3}{1-H_2H_3H_4}$