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ECE-205

Exam 1

Fall 2013

Calculators can only be used for simple calculations. Solving integrals, differential equations, systems of equations, etc. does not count as a simple calculation.

You must show your work to receive credit.

Problem 1 ________/10

Problem 2 ________/20

Problem 3 ________/30

Problem 4 ________/22

Problems 5-10 ________/18

Total ____________

Name ______________________________ Mailbox ______________

1) (10 points) For a first order system described by the differential equation

$$\dot{y}(t) + 2ty(t) = e^{t}x(t-1)$$

with $t_0 = 0$ and $y(t_0) = 1$, use integrating factors to solve the differential equation. Include the initial conditions in your solution.

$$\frac{d}{dt}\left[y(t)e^{t^2}\right] = e^{t^2}e^{t}x(t-1)$$

$$y(t)e^{t^2} - y(t_0)e^{t_0^2} = \int_{t_0}^{t} e^{\lambda^2}e^{\lambda}x(\lambda-1)\,d\lambda$$

$$y(t)e^{t^2} - 1 = \int_0^t e^{\lambda^2}e^{\lambda}x(\lambda-1)\,d\lambda$$

$$\boxed{y(t) = e^{-t^2} + e^{-t^2}\int_0^t e^{\lambda^2}e^{\lambda}x(\lambda-1)\,d\lambda}$$

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2) (20 points) Assume we have a first order system with the governing differential equation

$$0.4\dot{y}(t) + y(t) = 3x(t)$$

$K = 3 \quad \tau = 0.4$

The system has the initial value of 1, so $y(0) = 1$. The input to this system is

$y(t) = [y(t_0) - y(\infty)]e^{-(t-t_0)/\tau} + y(\infty)$

$y(\infty) = KA$

$$x(t) = \begin{cases} 0 & t < 0 \\ -1 & 0 \le t < 1 \\ 3 & 1 \le t < 2 \\ -2 & 2 < t \end{cases}$$

Determine the output of the system in each of the above time intervals. *Simplify your final answer as much as possible and box it. Be sure to include the correct initial value in the first interval!*

$0 \le t \le 1 \quad y(0) = 1 \quad y(\infty) = KA = -3$

$y(t) = [1 - (-3)]e^{-t/0.4} - 3 = \boxed{4e^{-t/0.4} - 3 = y(t)}$

$1 \le t \le 2 \quad y(1) = 4e^{-1/0.4} - 3 = -2.672 \quad y(\infty) = KA = 9$

$y(t) = [-2.672 - 9]\, e^{-(t-1)/0.4} + 9 = \boxed{-11.672e^{-(t-1)/0.4} + 9 = y(t)}$

$2 \le t \quad y(2) = -11.672e^{-1/0.4} + 9 = 8.042 \quad y(\infty) = KA = -6$

$y(t) = [8.042 - (-6)]\, e^{-(t-2)/0.4} - 6 = \boxed{14.042\, e^{-(t-2)/0.4} - 6 = y(t)}$

Name ______________________________ Mailbox ____________

3) (30 points) For the following three differential equations, assume the input is $x(t) = 4u(t)$ (the input is equal to four for time greater than zero), and the initial conditions are $y(0) = \dot{y}(0) = 0$

Determine the solution to each of the following differential equations and put your final answer in a box. Be sure to use the initial conditions to solve for all unknowns. You must show all your work to receive credit.

a) $\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = x(t)$ $\quad r^2 + 3r + 2 = (r+2)(r+1) = 0 \quad 2y_f = 4 \quad y_f = 2$

$y(t) = 2 + c_1 e^{-t} + c_2 e^{-2t} \quad \dot{y}(t) = -c_1 e^{-t} - 2c_2 e^{-2t}$

$y(0) = 2 + c_1 + c_2 = 0$
$\dot{y}(0) = -c_1 - 2c_2 = 0$ $\quad$ adding $\quad 2 - c_2 = 0 \quad c_2 = 2 \quad c_1 = -4$

$$\boxed{y(t) = 2 - 4e^{-t} + 2e^{-2t}}$$

b) $\ddot{y}(t) + 4\dot{y}(t) + 4y(t) = 8x(t)$ $\quad r^2 + 4r + 4 = (r+2)^2 = 0 \quad 4y_p = 32 \quad y_f = 8$

$y(t) = 8 + c_1 e^{-2t} + c_2 t e^{-2t} \quad \dot{y}(t) = -2c_1 e^{-2t} + c_2 e^{-2t} - 2c_2 t e^{-2t}$

$y(0) = 8 + c_1 = 0 \quad c_1 = -8 \quad \dot{y}(0) = -2c_1 + c_2 = 0 \quad c_2 = 2c_1 = -16$

$$\boxed{y(t) = 8 - 8e^{-2t} - 16te^{-2t}}$$

c) $\ddot{y}(t) + 4\dot{y}(t) + 16y(t) = 4x(t)$ $\quad r^2 + 4r + 16 = 0 \quad 16y_f = 16 \quad y_f = 1$

$r = \dfrac{-4 \pm \sqrt{16 - 64}}{2} = -2 \pm j2\sqrt{3}$

$y(t) = 1 + ce^{-2t}\sin(2\sqrt{3}t + \theta) \quad y(0) = 1 + c\sin(\theta) = 0 \quad c = \dfrac{-1}{\sin(\theta)}$

$\dot{y}(t) = -2ce^{-2t}\sin(2\sqrt{3}t + \theta) + 2\sqrt{3}ce^{-2t}\cos(2\sqrt{3}t + \theta)$

$\dot{y}(0) = -2\sin(\theta) + 2\sqrt{3}\cos(\theta) = 0 \quad \tan(\theta) = \sqrt{3} \quad \theta = 60^\circ \quad c = -1.155$

$$\boxed{y(t) = 1 - 1.155e^{-2t}\sin(2\sqrt{3}t + 60^\circ)}$$

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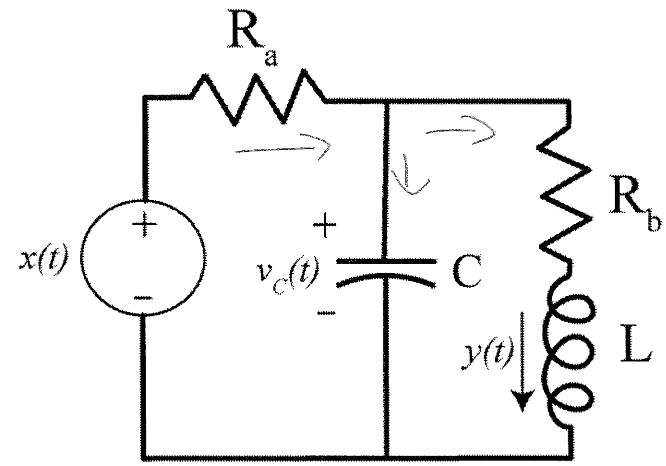
4) (22 points) Derive the governing differential equation for the following circuit. You do not need to put the differential equation into standard form.



$$\frac{x - V_c}{R_a} = C\frac{dV_c}{dt} + y \qquad V_c - yR_b - L\dot{y} = 0$$

$$V_c = yR_b + L\dot{y}$$

$$\frac{x - [yR_b + L\dot{y}]}{R_a} = C\frac{d}{dt}[yR_b + L\dot{y}] + y$$

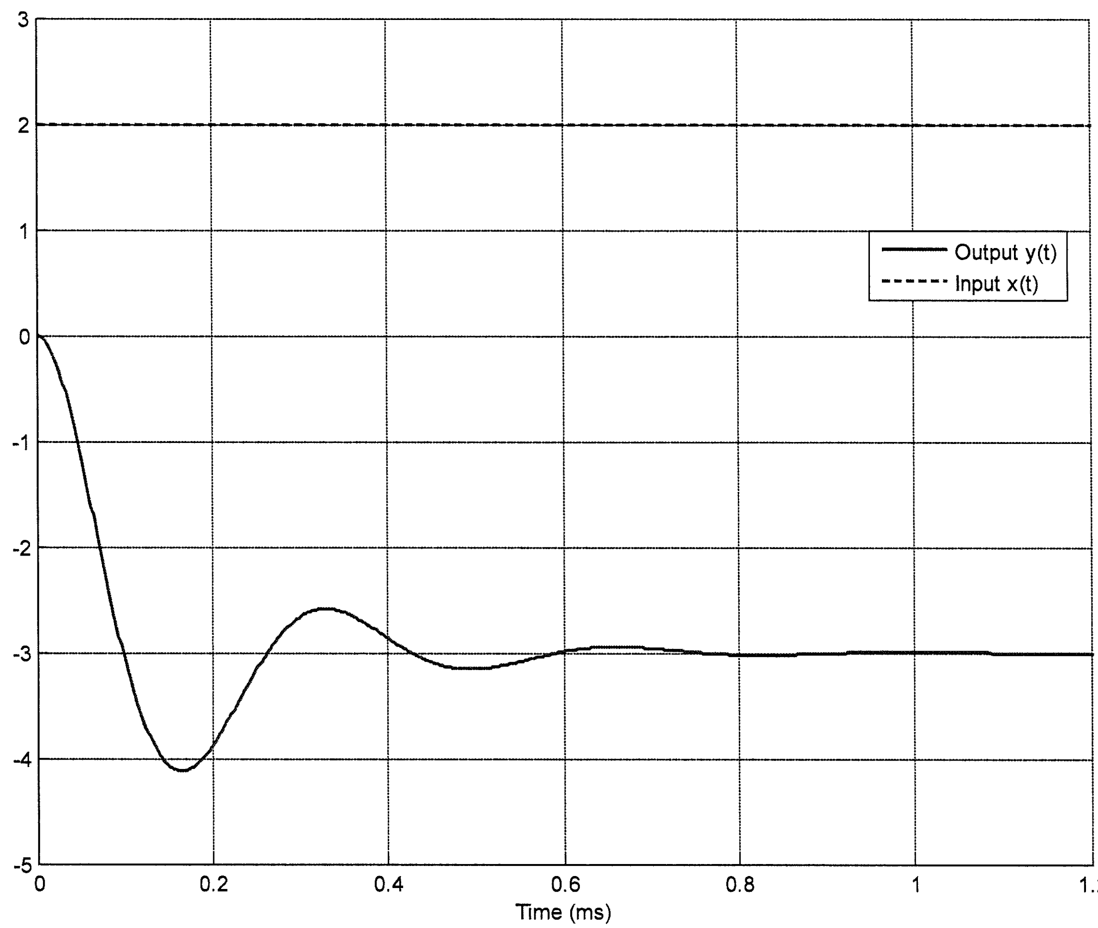
$$x - yR_b - L\dot{y} = R_a C[\dot{y}R_b + L\ddot{y}] + R_a y$$

$$\boxed{x = R_a C L\ddot{y} + (R_a R_b C + L)\dot{y} + (R_a + R_b)y}$$

Name ____ Mailbox __________________

Problems 5-10, 3 points each, no partial credit (18 points)

Problems 5-7 refer the following graph showing the response of a second order system to a step input.



5) The percent overshoot for this system is best estimated as

a) 400%   b) -400 %   c) 300%   d) -300 %   e) -33%   f) 33%

$\frac{-4-(-3)}{-3} \times 100\% \simeq 33\%$

6) The (2%) settling time for this system is best estimated as

a) 0.3 ms   b) 0.6 ms   c) 1.0 ms   d) 1.2 ms

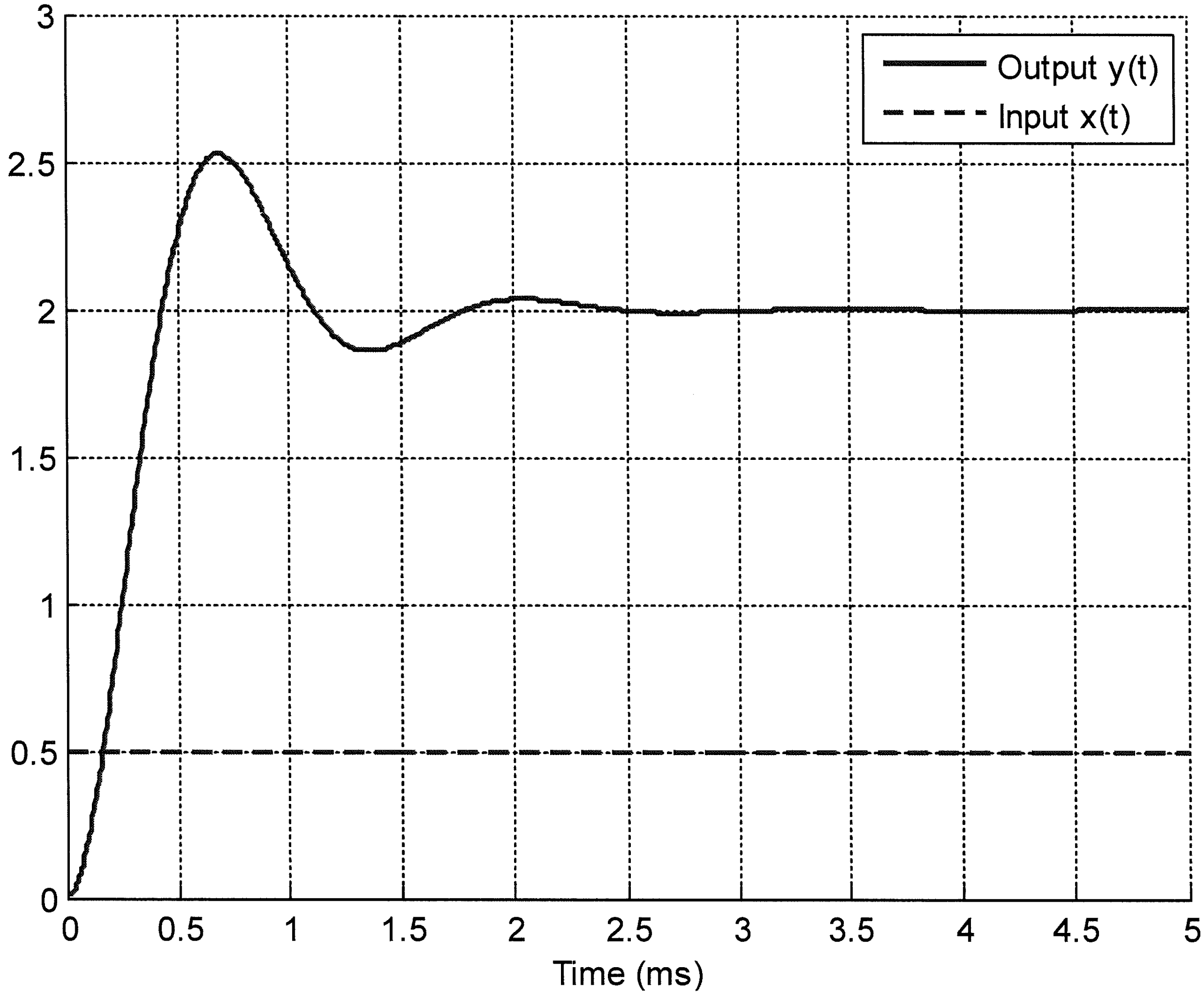
7) The static gain for this system is best estimated as

$K(2) = -3 \quad K = -3/2$

a) 1.5   b) 3   c) -1.5   d) -3

Name ____ Mailbox __________________

Problems 8-10 refer the following graph showing the response of a second order system to a step input.



8) The percent overshoot for this system is best estimated as $\frac{2.5-2}{2} \times 100\% \approx \frac{1}{4} \times 100\%$

a) 400% b) 250 % c) 200% d) 150 % e) 100 % f) 25%

9) The (2%) settling time for this system is best estimated as

a) 1.5 ms b) 2.5 ms c) 4 ms d) 5 ms

10) The static gain for this system is best estimated as $K(0.5) = 2$ $K = 4$

a) 1 b) 2 c) 3 d) 4