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Name Solutions CM ______________

ECE-205

Exam 3

Fall 2012

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 _________/20

Problem 2 _________/15

Problem 3 _________/15

Problem 4 _________/20

Problems 5 ________/30

Total ______________

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1) (20 points) For the following transfer functions, determine the <u>*unit step response*</u> of the system. *Do not forget any necessary unit step functions.*

a) $H(s) = \dfrac{e^{-2s}}{s}$

b) $H(s) = \dfrac{1}{(s+1)^2}$

c) $H(s) = \dfrac{1}{s^2+2s+5}$

a) $Y(s) = H(s)\frac{1}{s} = \frac{e^{-2s}}{s^2}$ $\boxed{y(t) = (t-2)u(t-2)}$

b) $Y(s) = H(s)\frac{1}{s} = \frac{1}{s(s+1)^2} = \frac{a_1}{s} + \frac{a_2}{s+1} + \frac{a_3}{(s+1)^2}$

$a_1 = 1 \quad a_3 = -1$

$\times s$, let $s \to \infty$ $\quad 0 = a_1 + a_2$

$a_2 = -a_1 = -1$

$\boxed{y(t) = [1 - e^{-t} - te^{-t}]u(t)}$

c) $Y(s) = H(s)\frac{1}{s} = \frac{1}{s[(s+1)^2+4]} = \frac{A}{s} + B\left[\frac{2}{[(s+1)^2+2^2]}\right] + C\left[\frac{s+1}{(s+1)^2+4}\right]$

$A = \frac{1}{5}$ $\quad \times s$, let $s \to \infty$ $\quad 0 = A + C \quad C = -\frac{1}{5}$

let $s = -1$ $\quad -\frac{1}{4} = \frac{-1}{5} + \frac{B}{2}$

$-5 = -4 + 10B$

$-1 = 10B \quad B = -\frac{1}{10}$

$\boxed{y(t) = \left[\frac{1}{5} - \frac{1}{10}e^{-t}\sin(2t) - \frac{1}{5}e^{-t}\cos(2t)\right]u(t)}$

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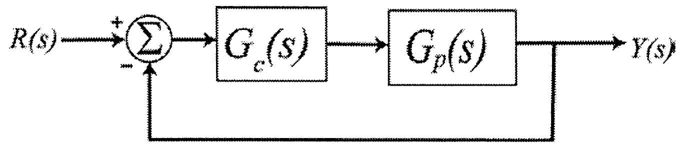
2) (15 points) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \dfrac{5}{s+3}$



a) Determine the settling time of the plant alone (assuming there is no feedback)

$$\boxed{T_s = \frac{4}{3}}$$

b) Determine the steady state error for plant alone assuming the input is a unit step (simplify your answer)

$$e_{ss} = 1 - G_p(0) = 1 - 5/3 = \boxed{-2/3 = e_{ss}}$$

c) For a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function $G_0(s)$

$$\boxed{G_0(s) = \frac{5k_p}{s+3+5k_p}}$$

d) Determine the settling time of the closed loop system , in terms of k_p

$$\boxed{T_s = \frac{4}{3+5k_p}}$$

e) Determine the steady state error of the closed loop system for a unit step, in terms of k_p (simplify your answer)

$$e_{ss} = 1 - G_0(0) = 1 - \frac{5k_p}{3+5k_p} = \boxed{\frac{3}{3+5k_p} = e_{ss}}$$

f) For and integral controller, $G_c(s) = \dfrac{k_i}{s}$, determine the closed loop transfer function $G_0(s)$ and the steady state error for a unit step in terms of k_i

$$G_0(s) = \frac{\frac{k_i}{s}\,\frac{5}{s+3}}{1+\frac{k_i}{s}\,\frac{5}{s+3}} = \boxed{\frac{5k_i}{s^2+3s+5k_i} = G_0(s)}$$

$$e_{ss} = 1 - G_c(0) = 1 - 1 = \boxed{0 = e_{ss}}$$

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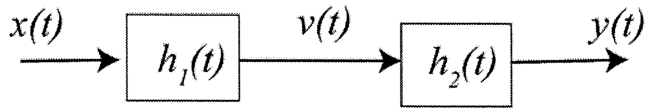
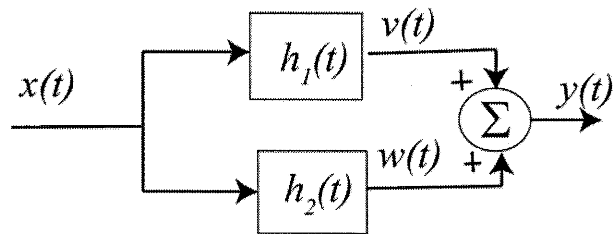
3) (15 points) For the following block diagram

For the following interconnected systems,

i) determine the overall impulse response (the impulse response between input $x(t)$ and output $y(t)$) and

ii) determine if the system is causal.





a) $h_1(t) = \delta(t-2)$, $h_2(t) = \delta(t+1)$

b) $h_1(t) = u(t+1)$, $h_2(t) = u(t-2) + \delta(t-2)$

Series Connections:

a) $h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(t-\lambda)h_2(\lambda)\,d\lambda = \int_{-\infty}^{\infty} \delta(t-\lambda-2)\,\delta(\lambda+1)\,d\lambda = \boxed{\delta(t-1)}$ $\boxed{\text{causal}}$

b) $h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(t-\lambda)h_2(\lambda)\,d\lambda = \int_{-\infty}^{\infty} u(t-\lambda+1)u(\lambda-2)\,d\lambda + \int_{-\infty}^{\infty} u(t-\lambda+1)\,\delta(\lambda-2)\,d\lambda$

$= \int_{2}^{t+1} d\lambda + u(t-1) = \boxed{(t-1)u(t-1) + u(t-1) = t\,u(t-1)}$ $\boxed{\text{causal}}$

Parallel Connections:

a) $h(t) = h_1(t) + h_2(t) = \boxed{\delta(t-2) + \delta(t+1)}$ $\boxed{\text{not causal}}$

b) $h(t) = h_1(t) + h_2(t) = \boxed{u(t+1) + u(t-2) + \delta(t-2)}$ $\boxed{\text{not causal}}$

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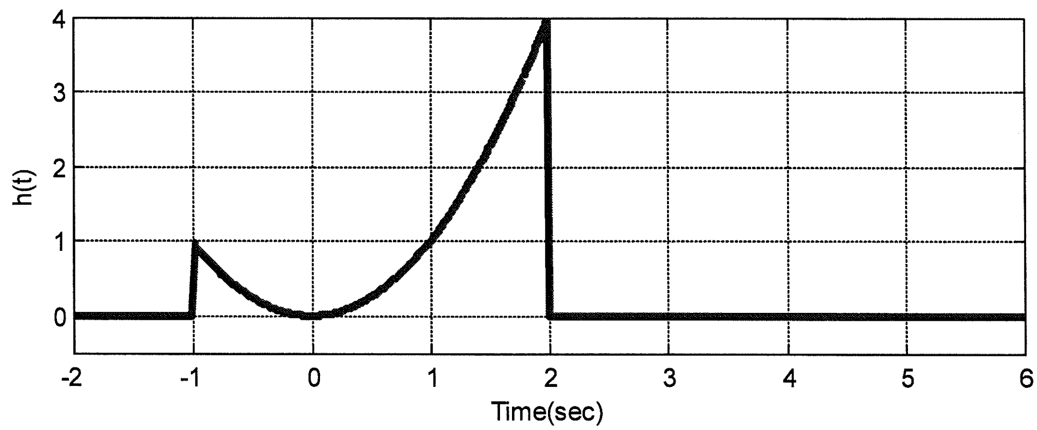
4) (20 points) Consider a linear time invariant system with impulse response given by

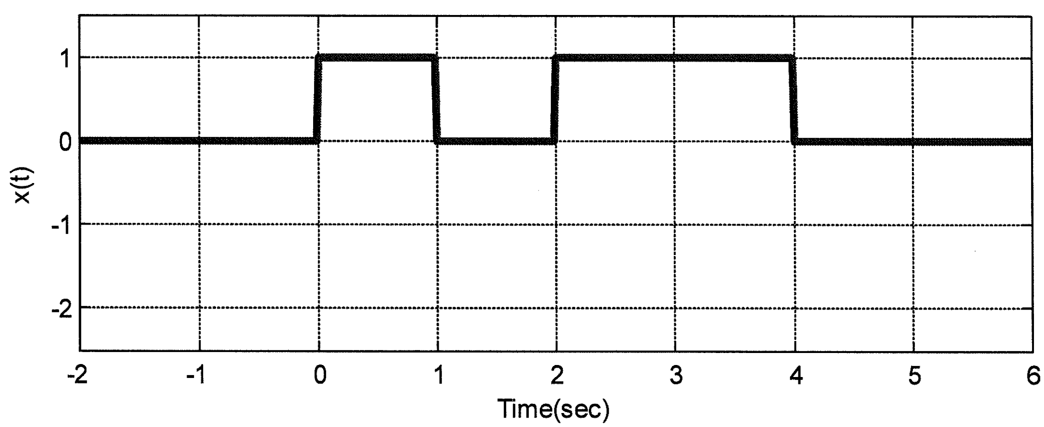
$$h(t) = t^2[u(t+1) - u(t-2)]$$

The input to the system is given by

$$x(t) = [u(t) - u(t-1)] + [u(t-2) - u(t-4)]$$

The impulse response and input are shown below:





Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

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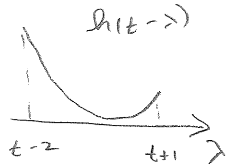
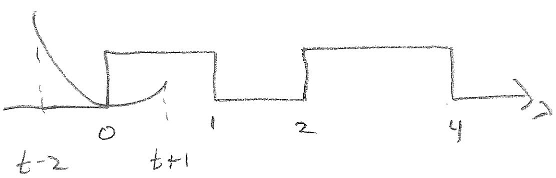
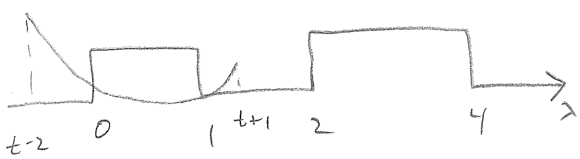


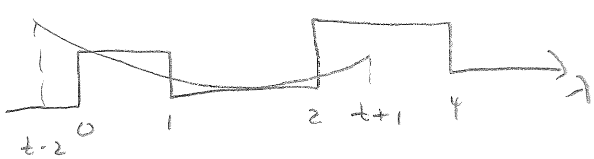
$y(t) = 0 \quad t < -1$



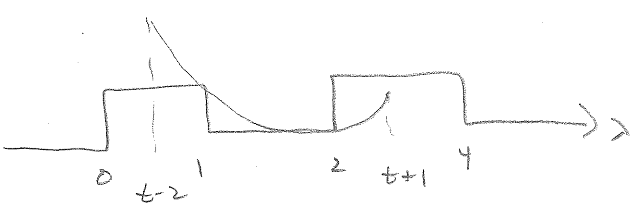
$-1 \leq t \leq 0 \quad y(t) = \int_0^{t+1} (t-\lambda)^2 \, d\lambda$



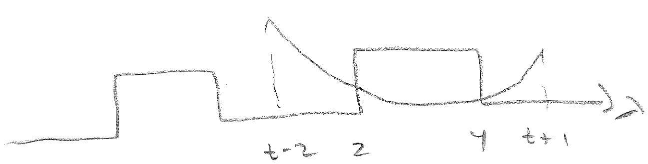
$0 \leq t \leq 1 \quad y(t) = \int_0^1 (t-\lambda)^2 \, d\lambda$



$1 \leq t \leq 2 \quad y(t) = \int_0^1 (t-\lambda)^2 \, d\lambda + \int_2^{t+1} (t-\lambda)^2 \, d\lambda$

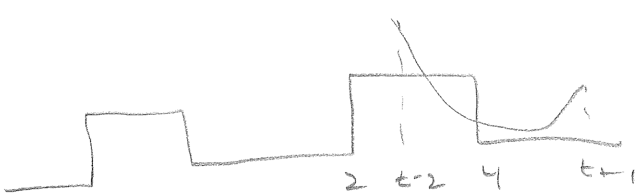


$2 \leq t \leq 3 \quad y(t) = \int_{t-2}^1 (t-\lambda)^2 \, d\lambda + \int_2^{t+1} (t-\lambda)^2 \, d\lambda$



$3 \leq t \leq 4 \quad y(t) = \int_2^4 (t-\lambda)^2 \, d\lambda$

$4 \leq t \leq 6 \quad y(t) = \int_{t-2}^4 (t-\lambda)^2 \, d\lambda$



$t \geq 6 \quad y(t) = 0$

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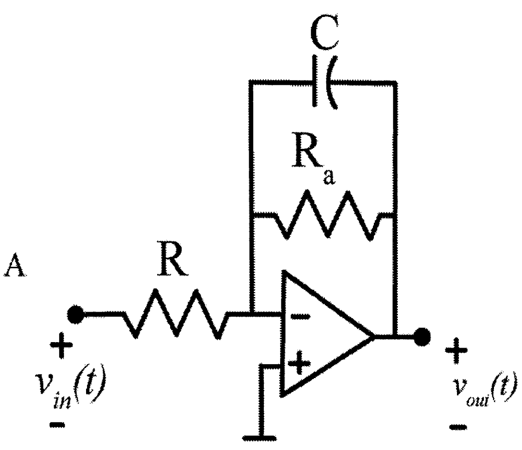
5) (30 points) The following figure shows three different circuits, which are subsystems for a larger system. We can write the transfer functions for these systems as

$$G_a(s) = \frac{-K_{low}\omega_{low}}{s+\omega_{low}} \qquad G_b(s) = \frac{-K_{high}s}{s+\omega_{high}} \qquad G_c(s) = -K_{ap}$$

Determine the parameters $K_{low}, \omega_{low}, K_{high}, \omega_{high}$, and K_{ap} in terms of the parameters given (the resistors and capacitors).

System A



$$\frac{V_{in}}{R} + \frac{V_{out}}{R_a} + \frac{V_{out}}{1/Cs} = 0$$

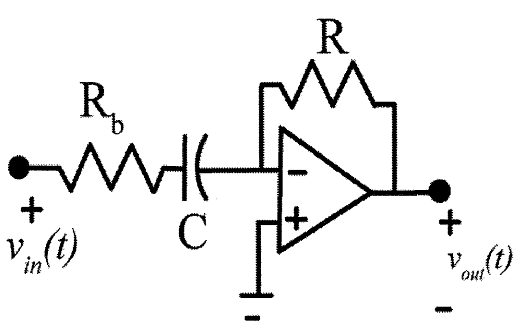
$$-\frac{V_{in}}{R} = V_{out}\left[\frac{1}{R_a} + Cs\right] = V_{out}\left[\frac{R_aCs+1}{R_a}\right]$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_a}{R}\,\frac{1}{(R_aCs+1)} = \frac{-R_a/R}{R_aC(s+1/R_aC)}$$

$$= \frac{-R_a/R \;\; 1/R_aC}{s+1/R_aC}$$

$\boxed{K_{low} = \frac{R_a}{R} \quad \omega_{low} = 1/R_aC}$

System B



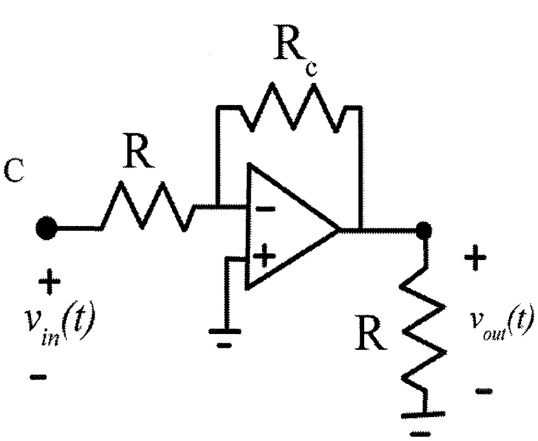
$$\frac{V_{in}}{R_b + \frac{1}{Cs}} + \frac{V_{out}}{R} = 0$$

$$\frac{V_{in}Cs}{R_bCs+1} = -\frac{V_{out}}{R}$$

$$\frac{V_{out}}{V_{in}} = \frac{-RCs}{R_bCs+1} = -\frac{RCs}{R_bC(s+1/R_bC)} = \frac{-\frac{R}{R_b}s}{s+1/R_bC}$$

$\boxed{K_{high} = R/R_b}$ $\boxed{\omega_{high} = 1/R_bC}$

System C



$$\frac{V_{in}}{R} + \frac{V_{out}}{R_c} = 0$$

$$\frac{V_{out}}{V_{in}} = -\frac{R_c}{R}$$

$\boxed{K_{ap} = \frac{R_c}{R}}$