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# ECE-205

# Exam 2

# Fall 2012

**Calculators and computers are not allowed. You must show your work to receive credit.**

**Problem 1 _________/18**

**Problem 2 _________/15**

**Problem 3 _________/18**

**Problems 4 ________/10**

**Problem 5 _________/15**

**Problem 6 _________/24**

**Total ______________**

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**1) (18 points)** Fill in the non-shaded part of the following table.

| | Linear? (Y/N) | Time Invariant? (Y/N) | BIBO Stable? (Y/N) |
|---|---|---|---|
| $y(t) = e^{t-1}x(t)$ | Yes | No | |
| $\dot{y}(t) + \cos(t)y(t) = \sin(t)x(t)$ | Yes | No | |
| $y(t) = x\left(\frac{t}{2}\right)$ | Yes | No | |
| $y(t) = \int_{-\infty}^{t} e^{\lambda}x(\lambda)d\lambda$ | | | No |
| $y(t) = \int_{-\infty}^{t} e^{-\lambda}x(\lambda)d\lambda$ | | | No |
| $y(t) = \cos\left(\frac{1}{x(t)}\right)$ | | | Yes |

$|y(t)| \le \int_{-\infty}^{t} e^{\lambda} N d\lambda = N e^{\lambda}\Big|_{-\infty}^{t} = N e^{t}$ Not BIBO

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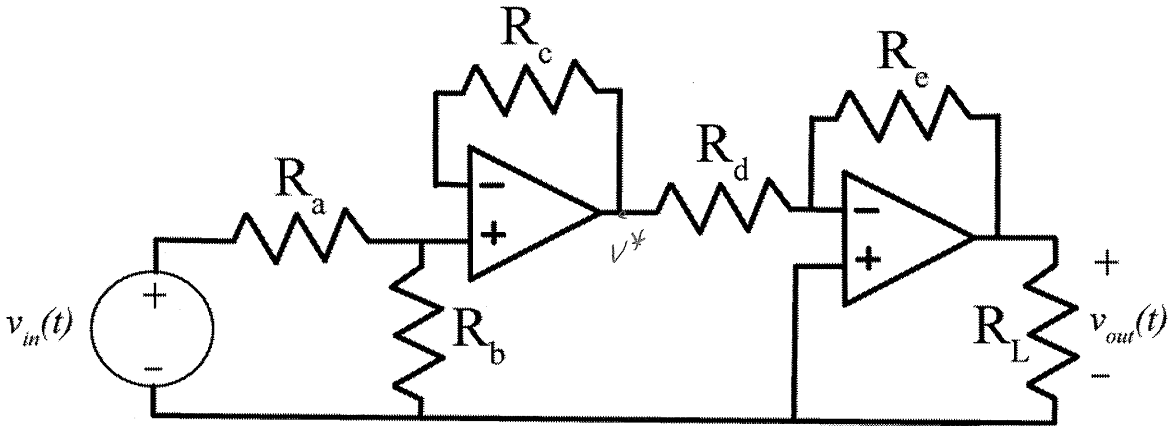
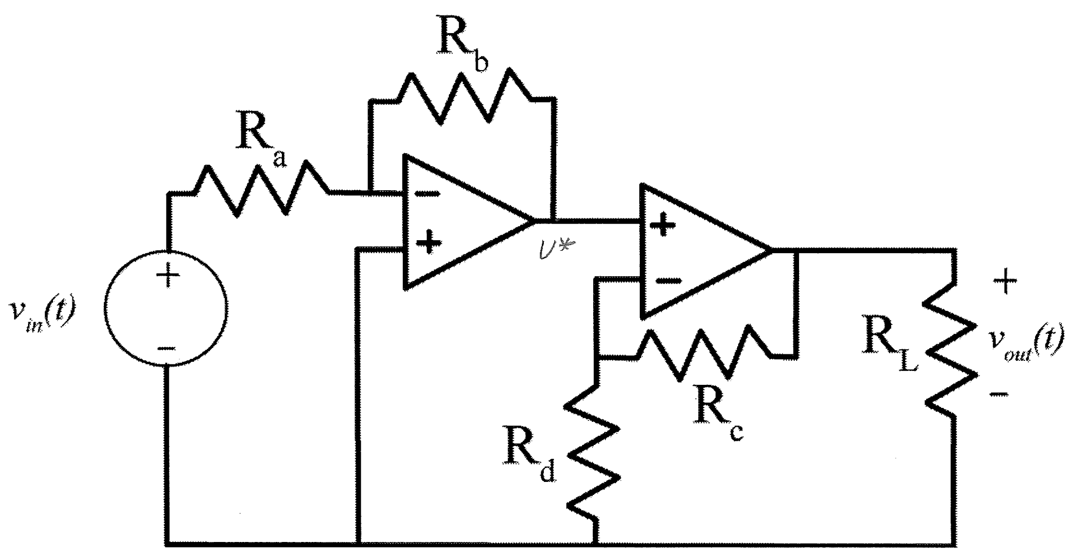
**2) (15 points)** For the following two op-amps circuits, we can write $v_{out}(t) = G\, v_{in}(t)$. Determine the value of $G$ for each circuit.




$V^* = \frac{V_{in} R_b}{R_a + R_b}$ $\quad \frac{V^*}{R_d} + \frac{V_{out}}{R_e} = 0$ $\quad V_{out} = -\frac{R_e}{R_d} V^* = \left[-\frac{R_e}{R_d}\frac{R_b}{R_a+R_b}\right] V_{in}$

$$\boxed{G = -\frac{R_e}{R_d}\frac{R_b}{R_a+R_b}}$$




$\frac{V_{in}}{R_a} + \frac{V^*}{R_b} = 0$ $\quad V^* = \frac{V_{out}}{R_d + R_c} R_d$

$V^* = -\frac{R_b}{R_a} V_{in}$ $\quad V_{out} = \frac{R_c + R_d}{R_d} V^* = \left[-\frac{R_b}{R_a}\frac{R_c+R_d}{R_d}\right] V_{in}$

$$\boxed{G = -\frac{R_b}{R_a}\frac{R_c+R_d}{R_d}}$$

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**3) (18 Points)** Determine the ***impulse response*** for the following systems. Don't forget any necessary unit step functions

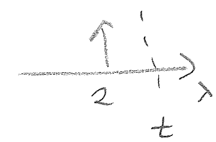
a) $y(t) = x(t-1) + x(t+1)$

b) $y(t) = \int_{-\infty}^{t} e^{-(t+\lambda)} x(\lambda - 2) d\lambda$

c) $\frac{1}{3}\dot{y}(t) + y(t) = 2x(t)$

a) $\boxed{h(t) = \delta(t-1) + \delta(t+1)}$

b) $h(t) = \int_{-\infty}^{t} e^{-(t+\lambda)} \delta(\lambda - 2) d\lambda = \boxed{e^{-(t+2)} u(t-2) = h(t)}$




c) $\dot{h}(t) + 3h(t) = 6\delta(t)$

$\frac{d}{dt}\left[h(t) e^{3t}\right] = 6e^{3t}\delta(t) = 6\delta(t)$

$h(t) e^{3t} = \int_{0}^{t} 6\delta(\lambda) d\lambda = 6u(t)$

$\boxed{h(t) = 6e^{-3t} u(t)}$

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**4) (10 points)** For an LTI system with impulse response $h(t) = e^{-(t-1)}u(t-1) + \delta(t-2)$ determine the system output using **analytical evaluation** (directly evaluating the integrals) for input $x(t) = e^{-t}u(t)$

$$y(t) = \int_{-\infty}^{\infty} h(\lambda)\, x(t-\lambda)\, d\lambda$$

$$= \int_{-\infty}^{\infty} \left[e^{-(\lambda-1)}u(\lambda-1) + \delta(\lambda-2)\right] e^{-(t-\lambda)} u(t-\lambda)\, d\lambda$$

$$= \int_{-\infty}^{\infty} e^{-(\lambda-1)} e^{-(t-\lambda)} u(\lambda-1)\, u(t-\lambda)\, d\lambda + \int_{-\infty}^{\infty} \delta(\lambda-2)\, e^{-(t-\lambda)} u(t-\lambda)\, d\lambda$$

$u(\lambda-1) = 1$ for $\lambda - 1 > 0$, $\lambda > 1$

$u(t-\lambda) = 1$ for $t - \lambda > 0$, $t > \lambda$

$$= e^{-(t-1)} \int_{1}^{t} d\lambda + e^{-(t-2)} u(t-2)$$

$$\boxed{y(t) = (t-1)\, e^{-(t-1)} u(t-1) + e^{-(t-2)} u(t-2)}$$

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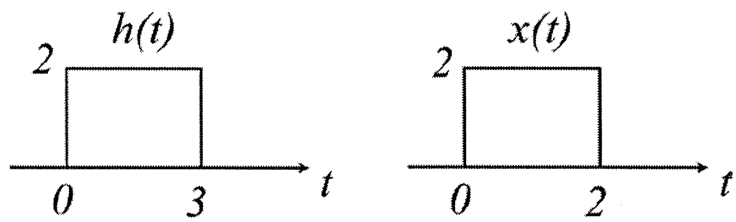
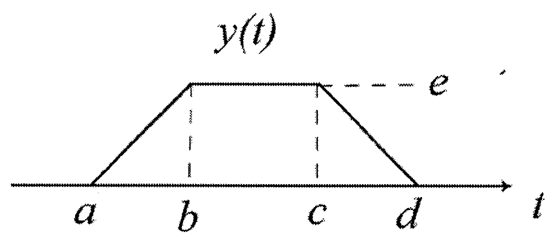
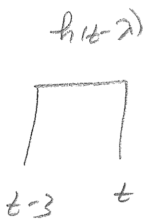
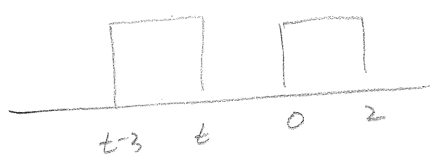
**5) (15 Points)** An LTI system has impulse response, input, and output as shown below. Determine numerical values for the parameters *a, b, c, d*, and *e*.




$a = 0$

$b = 2$

$c = 3$

$d = 5$

$e = \underbrace{2 \cdot 2}_{\text{height}} \cdot \underbrace{2}_{\text{width}} = 8$

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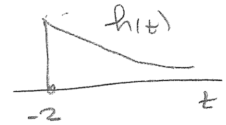
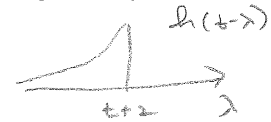
**6) (24 points)** Consider a linear time invariant system with impulse response given by

$$h(t) = e^{-(t+2)}u(t+2)$$




The input to the system is given by

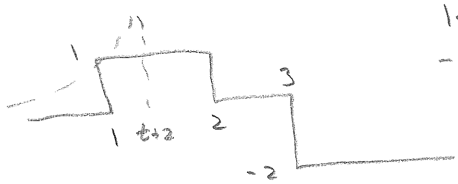
$h(-2) = h(t-\lambda)$

$-2 = t-\lambda \quad \lambda = t+2$

$$x(t) = [u(t-1) - u(t-2)] - 2u(t-3)$$

Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

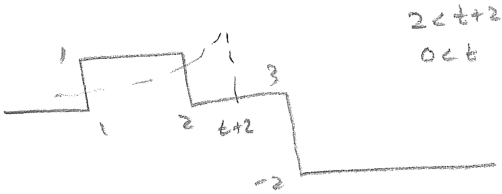
- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of $t$ for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***




$1 < t+2 \quad t+2 < 2$

$-1 < t \quad t < 0$

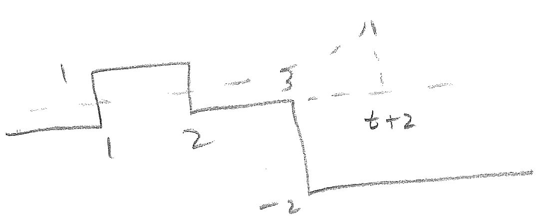
$\boxed{-1 < t < 0}$ $\quad y(t) = \int_1^{t+2} e^{-(t-\lambda+2)}(1)\,d\lambda$




$2 < t+2 \quad t+2 < 3$

$0 < t \quad t < 1$

$\boxed{0 < t < 1}$ $\quad y(t) = \int_1^{2} e^{-(t-\lambda+2)}(1)\,d\lambda$




$\boxed{t > 1}$ $\quad y(t) = \int_1^{2} e^{-(t-\lambda+2)}(1)\,d\lambda + \int_3^{t+2} e^{-(t-\lambda+2)}(-2)\,d\lambda$

$t < -1 \quad y(t) = 0$