Name Solutions ______________________ Mailbox ______________

ECE-205

Exam 1

Fall 2012

Calculators can only be used for simple calculations. Solving integrals, differential equations, systems of equations, etc. does not count as a simple calculation.

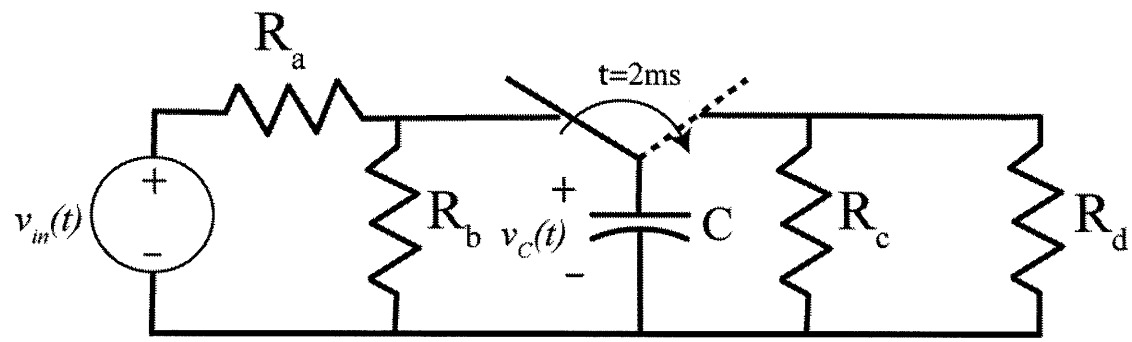
You must show your work to receive credit.

Problem 1	**________/15**
Problem 2	**________/15**
Problem 3	**________/10**
Problem 4	**________/15**
Problem 5	**________/15**
Problem 6-15	**________/30**

Total ____________

Name ______________________________ Mailbox ______________

1) (15 points) Consider the circuit shown in the figure below which is initially at rest:



a) Assume during charging ($0 \le t < 0.002$ seconds) that the time constant $\tau = 0.001$ seconds, the static gain is $K = 2$ and the input is a step with amplitude $A = 2$ volts. Determine the voltage on the capacitor at time $t = 0.002$ seconds

b) Assume during discharging ($t \ge 0.002$ seconds) that the time constant $\tau = 0.002$ seconds. Determine the voltage on the capacitor at time $t = 0.006$ seconds

c) Sketch the voltage on the capacitor from $t = 0$ seconds to $t = 0.010$ seconds. The most important part of this graph is the shape of the curves (i.e., what does the voltage on the capacitor look like when it is charging compared to when it is discharging)

$$V_c(t) = (V_c(t_0) - V_c(\infty))e^{-(t-t_0)/\tau} + V_c(\infty)$$

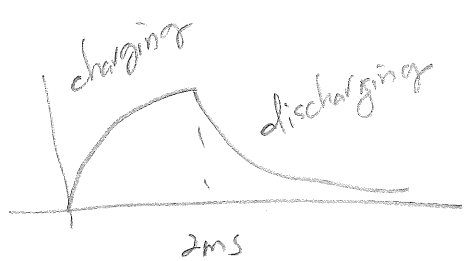
a) $K = 2 \quad A = 2 \quad t_0 = 0 \quad V_c(0) = 0 \quad V_c(\infty) = KA = 4 \quad \tau = 0.001 \quad t = 0.002$

$$V_c(0.002) = (0-4)e^{-2} + 4 = 4(1-e^{-2}) = \boxed{3.458V}$$

b) $K = 0 \quad t_0 = 0.002 \quad V_c(t_0) = 3.458 \quad V_c(\infty) = 0 \quad \tau = 0.002 \quad t = 0.006$

$$V_c(0.006) = (3.458 - 0)e^{-2} + 0 = 3.458e^{-2} = \boxed{0.467V}$$

c)



Name ________________________________ Mailbox ______________

2) (15 points) For the following differential equation, assume the input is $x(t) = 4u(t)$ (the input is equal to four for time greater than zero), and the initial conditions are $y(0) = \dot{y}(0) = 0$

$$\ddot{y}(t) + 2\dot{y}(t) + 3y(t) = 3x(t)$$

Determine the solution and put your final answer in a box. Be sure to use the initial conditions to solve for all unknowns. You must show all your work to receive credit. *You cannot just use the answer to problem 5, though you can check your answer if you want.*

$r^2 + 2r + 3 = 0 \qquad r = \frac{-2 \pm \sqrt{4-12}}{2} = \frac{-2 \pm j\sqrt{8}}{2} = \boxed{-1 \pm j\sqrt{2} = r}$

$3y_F = 3 \cdot 4 \qquad \boxed{y_F = 4}$

$y(t) = 4 + ce^{-t}\sin(\sqrt{2}t + \theta)$

$y(0) = 0 = 4 + c\sin(\theta) \qquad c = \frac{-4}{\sin(\theta)}$

$\dot{y}(t) = -ce^{-t}\sin(\sqrt{2}t + \theta) + c\sqrt{2}e^{-t}\cos(\sqrt{2}t + \theta)$

$\dot{y}(0) = 0 = -\sin(\theta) + \sqrt{2}\cos(\theta) = 0$

$\frac{\sin(\theta)}{\cos(\theta)} = \tan(\theta) = \sqrt{2} \qquad \boxed{\theta = 54.7^\circ}$

$c = \frac{-4}{\sin(54.7^\circ)} = -4.90$

$$\boxed{y(t) = 4 - 4.90e^{-t}\sin(\sqrt{2}t + 54.7^\circ)}$$

Name ______________________________ Mailbox ______________

3) (10 points) For the following first order differential equation,

$$\frac{1}{t}\dot{y}(t) + y(t) = \cos(t)x(t)$$

determine an expression for the output assuming $t_0 = 0$ and $y(t_0) = y(0) = 0$.

$$\dot{y}(t) + t y(t) = t\cos(t)x(t)$$

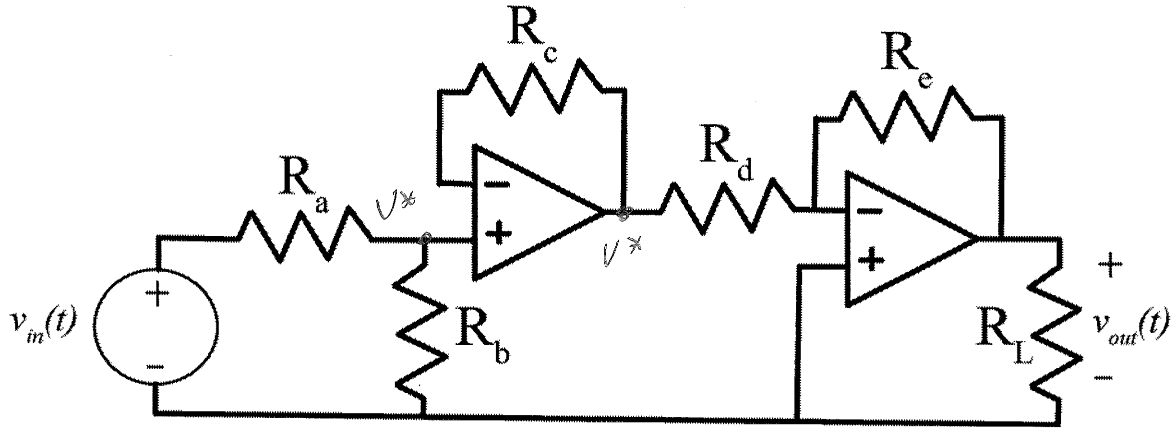
$$\frac{d}{dt}\left(y(t)e^{t^2/2}\right) = e^{t^2/2}\, t\cos(t)\, x(t)$$

$$y(t)e^{t^2/2} = \int_0^t e^{\frac{\lambda^2}{2}} \lambda\cos(\lambda)x(\lambda)\,d\lambda$$

$$\boxed{y(t) = \int_0^t e^{-\frac{t^2}{2}} e^{\frac{\lambda^2}{2}} \lambda\cos(\lambda)\,x(\lambda)\,d\lambda}$$

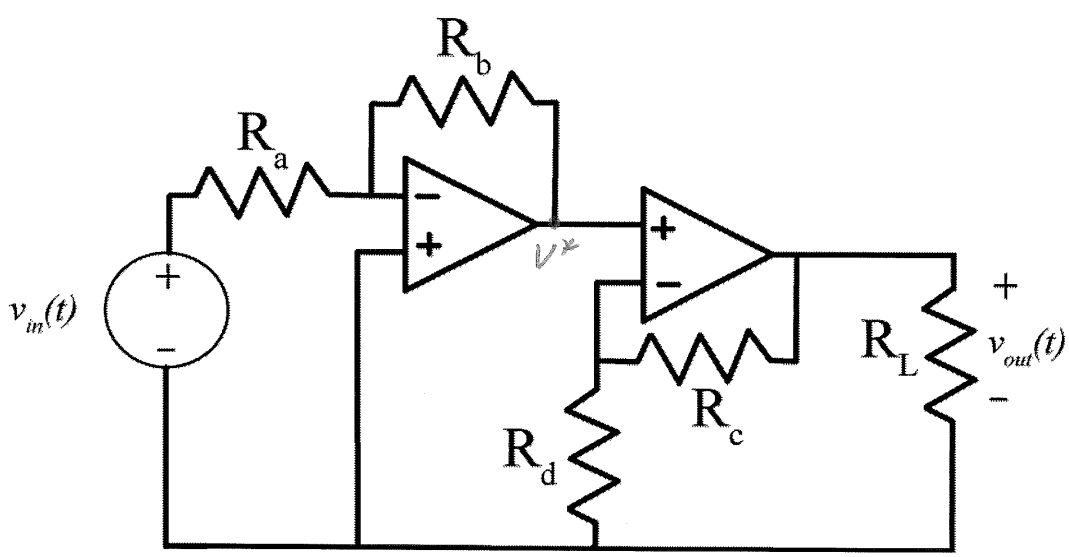
Name ______________________________ Mailbox ______________

4) (15 points) For the following two op-amps circuits, we can write $v_{out}(t) = G\, v_{in}(t)$. Determine the value of G for each circuit.



$V^* = \frac{V_{in} R_b}{R_a + R_b}$ $\quad \frac{V^*}{R_d} + \frac{V_{out}}{R_e} = 0$ $\quad V_{out} = \boxed{\frac{-R_e}{R_d}\,\frac{R_b}{R_a + R_b}}\, V_{in}$

G



$\frac{V_{in}}{R_a} + \frac{V^*}{R_b} = 0$ $\quad V^* = \frac{V_{out} R_d}{R_c + R_d}$

$V^* = \frac{-R_b}{R_a} V_{in}$ $\quad V_{out} = \boxed{\left(\frac{R_c + R_d}{R_d}\right)\left(\frac{-R_b}{R_a}\right)}\, V_{in}$

G

Name ____________________ Mailbox ____________

5) (15 points) The form of the under damped ($0 < \zeta < 1$) solution to the second order differential equation

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2 y(t) = \mathrm{K}\ \omega_n^2 x(t)$$

for a step input $x(t) = Au(t)$ is

$$y(t) = KA\left[1 - \frac{e^{-\zeta\omega_n t}}{\sqrt{1-\zeta^2}}\sin(\omega_d t + \phi)\right]$$

where the damped frequency is $\omega_d = \omega_n\sqrt{1-\zeta^2}$, $\sin(\phi) = \sqrt{1-\zeta^2}$, and $\tan(\phi) = \dfrac{\sqrt{1-\zeta^2}}{\zeta}$

a) Determine an expression for the maximum value of the output, which occurs at the time to peak, $T_p = \dfrac{\pi}{\omega_d}$. Hence you are to determine $y(T_p)$. Note that $\sin(\pi + \phi) = -\sin(\phi)$. ***To receive credit you must simplify your answer as much as possible.***

b) Determine an expression for $y(\infty)$, the steady state value of the output.

c) Determine an expression for the Percent Overshoot (PO), defined as

$$\text{a)}\quad PO = \left[\frac{y(T_p) - y(\infty)}{y(\infty)}\right]\times 100\%$$

To receive full credit you must simplify your answer as much as possible.

a) $y(T_p) = KA\left[1 - \dfrac{e^{-\zeta\omega_n \pi/\omega_d}}{\sqrt{1-\zeta^2}}\sin\left(\omega_d \dfrac{\pi}{\omega_d} + \phi\right)\right]$ $\quad \dfrac{-\zeta\omega_n\pi}{\omega_d} = \dfrac{-\zeta\omega_n\pi}{\omega_n\sqrt{1-\zeta^2}} = \dfrac{-\zeta\pi}{\sqrt{1-\zeta^2}}$

$= KA\left[1 - \dfrac{e^{-\zeta\pi/\sqrt{1-\zeta^2}}}{\sqrt{1-\zeta^2}}\sin(\pi+\phi)\right] = KA\left[1 + \dfrac{e^{-\zeta\pi/\sqrt{1-\zeta^2}}}{\sqrt{1-\zeta^2}}\sin(\phi)\right]$

$= \boxed{KA\left[1 + e^{-\zeta\pi/\sqrt{1-\zeta^2}}\right] = y(T_p)}$

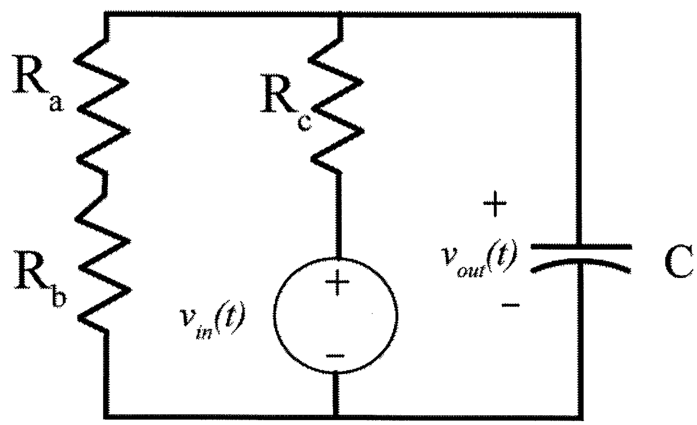
b) $\boxed{y(\infty) = KA}$

c) $PO = \left[\dfrac{y(T_p) - y(\infty)}{y(\infty)}\right]\times 100\% = \boxed{e^{-\zeta\pi/\sqrt{1-\zeta^2}} \times 100\% = PO}$

Name ______________________________ Mailbox ______________

Problems 6-15, 3 points each, no partial credit (30 points)

Problems 6 and 7 refer to the following circuit:



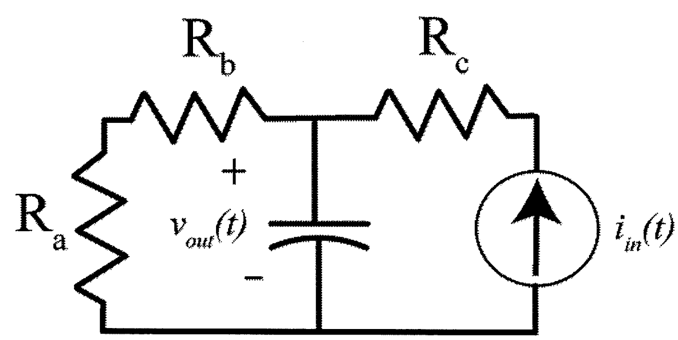
6) The Thevenin resistance seen from the ports of the capacitor is

a) $R_{th} = R_a + R_b$ b) $R_{th} = R_c$ c) $R_{th} = R_c \,||\, (R_a + R_b)$ d) $R_{th} = R_a + R_b + R_c$ e) none of these

7) The static gain for the system is

a) $K = 1$ b) $K = \dfrac{R_c}{R_a + R_b + R_c}$ c) $K = \dfrac{R_a + R_b}{R_a + R_b + R_c}$ d) $K = \dfrac{R_c}{R_a + R_b}$ e) none of these

Problems 8 and 9 refer to the following circuit



8) The Thevenin resistance seen from the ports of the capacitor is

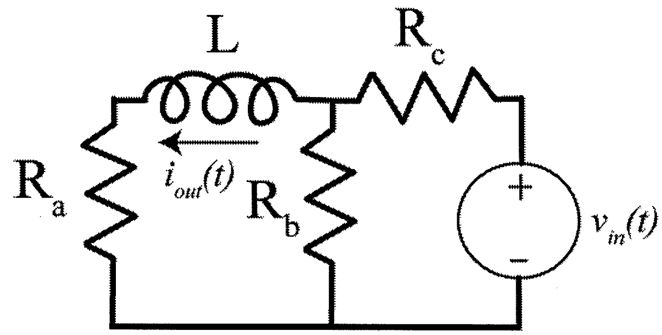
a) $R_{th} = R_a + R_b$ b) $R_{th} = R_c$ c) $R_{th} = R_c \,||\, (R_a + R_b)$ d) $R_{th} = R_a + R_b + R_c$ e) none of these

9) The static gain for the system is

a) $K = 1$ b) $K = R_c$ c) $K = R_a + R_b$ d) $K = R_c \,||\, (R_a + R_b)$ e) none of these

Name ____ Mailbox ____________________

Problems 10 and 11 refer to the following circuit



10) The Thevenin resistance seen from the ports of the inductor is

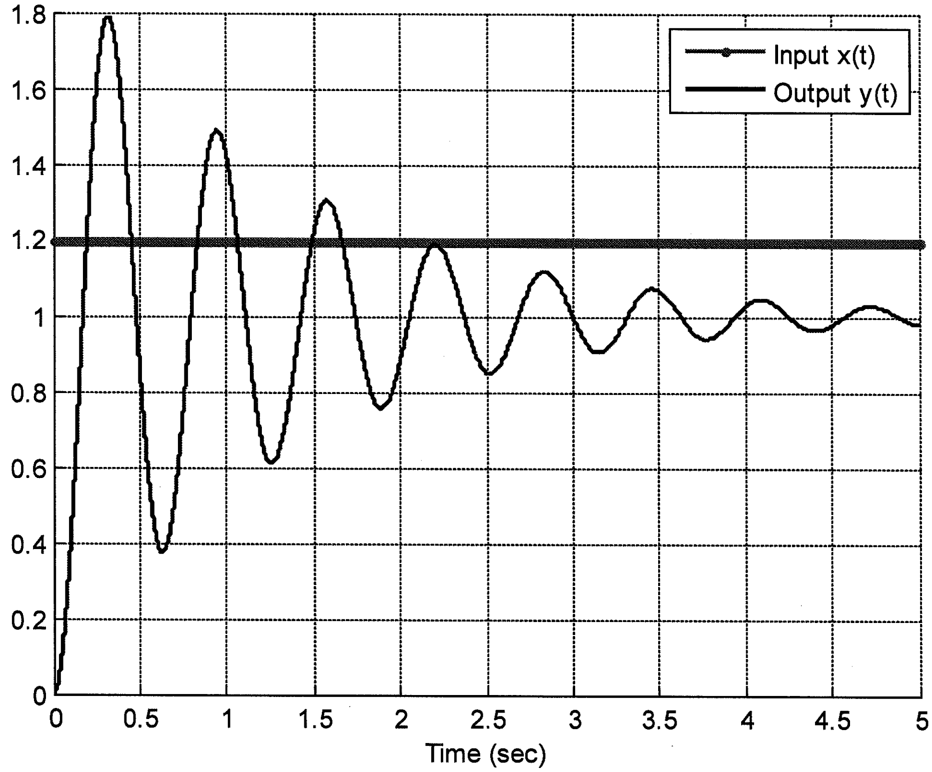
a) $R_{th} = R_a + R_b \| R_c$ b) $R_{th} = R_c + R_a \| R_b$ c) $R_{th} = R_a + R_b$ d) $R_{th} = R_a + R_c$ e) none of these

11) The static gain for the system is

a) $K = 1$ b) $K = \dfrac{R_b}{R_a + R_b}$ c) $K = \dfrac{R_a}{R_a + R_b}$ d) $K = \dfrac{R_b}{R_c + R_b}$ e) none of these

Name ________________________________ Mailbox ______________

Problems 12 and 13 refer the following graph showing the response of a second order system to a step input.



12) The percent overshoot for this system is best estimated as

$\frac{1.8-1}{1} = 0.8$

a) 180% b) 150 % c) 100% d) 80 % e) 60% f) 50%

13) The static gain for this system is best estimated as

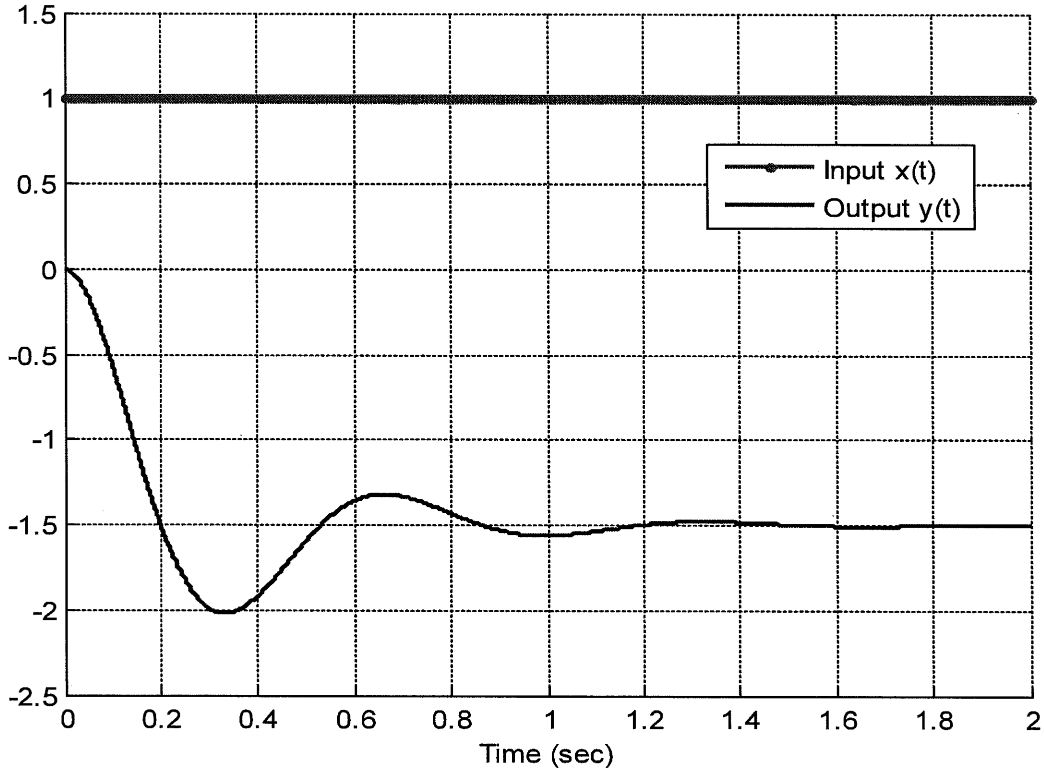
$K(1.2) = 1$

a) 1.8 b) 1.2 c) 1.00 d) 0.83 e) 0.5

$K = 0.83$

Name ____________________ Mailbox ____________

Problems 14 and 15 refer the following graph showing the response of a second order system to a step input.



14) The percent overshoot for this system is best estimated as

a) 300% b) -300 % c) 200% d) -200 % e) 33% f) -33%

$$\frac{-2-(-1.5)}{-1.5} = \frac{-0.5}{-1.5} = \frac{1}{3}$$

15) The static gain for this system is best estimated as

a) -3 b) 3 c) 2.5 d) -2.5 e) 1.5 f) -1.5

$K(1) = -1.5$