Midterm Exam 2

ECE205 Dynamical Systems

Midterm Exam 2
4/14/11

NAME: Solutions CM: ________

- You must **show work** to receive partial and full credit.
- Put a box around your final answer and it must include units, if necessary.
- Time allowed : 50 minutes.

Question #	Possible Points	Awarded Points
1	10	
2	30	
3	15	
4	15	
5	30	
Total	100	

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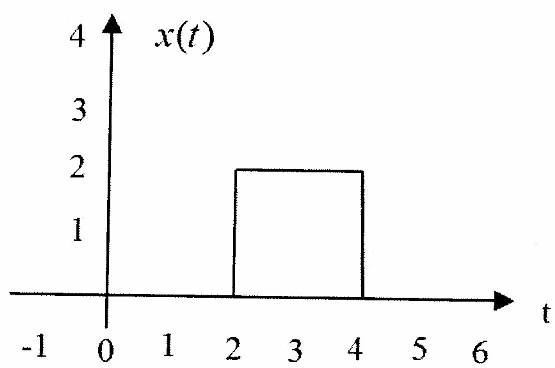
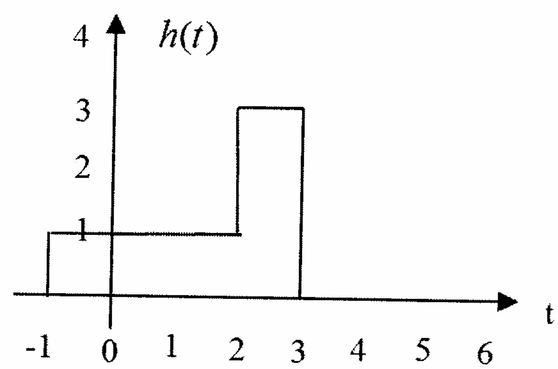
1) (10 points)

A linear time invariant (LTI) system has the following input, x(t) and the impulse response, h(t). The output of the system, $y(t)$, is the convolution of the impulse response with the input, $y(t) = h(t) * x(t)$.





a) Is the system causal, why or why not? no, $h(t) \neq 0$ for $t < 0$

b) Is the system BIBO stable, why or why not? yes, $\int_{-\infty}^{\infty} |h(\lambda)| d\lambda$ is clearly finite

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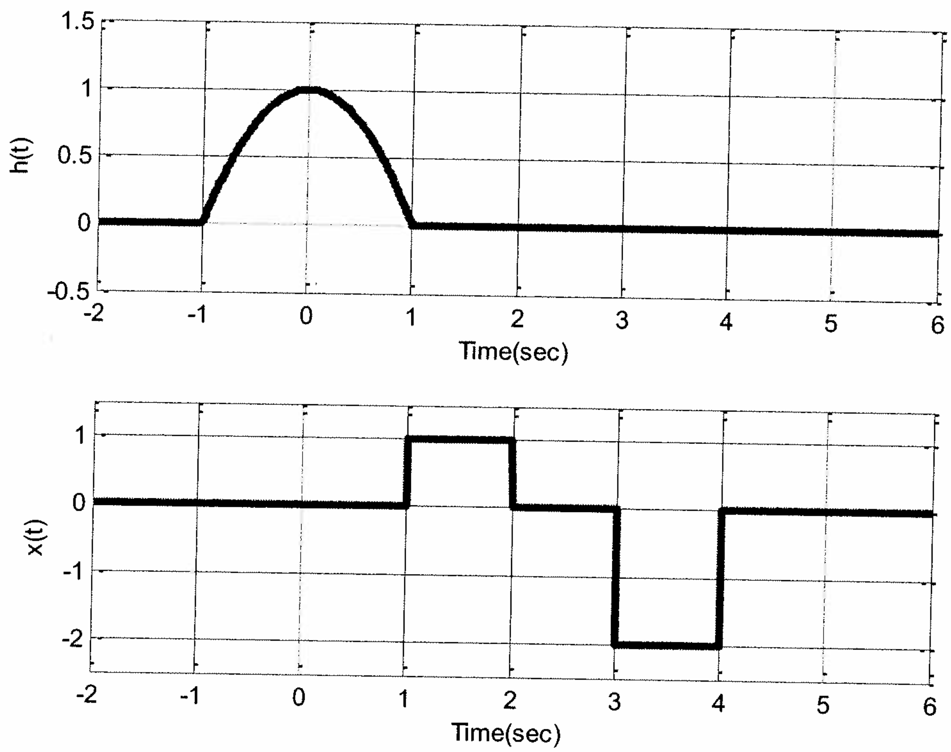
2) (30 points) Consider a linear time invariant system with impulse response given by

$$h(t) = (1-t^2)[u(t+1) - u(t-1)]$$

The input to the system is given by $x(t) = [u(t-1) - u(t-2)] - 2[u(t-3) - u(t-4)]$



Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, **NOT** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

Problem 2 continued on the next page.

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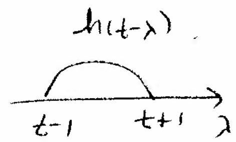
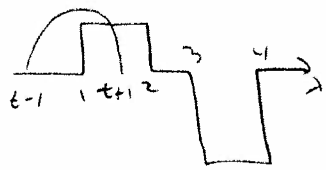
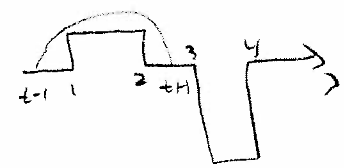
2) continued

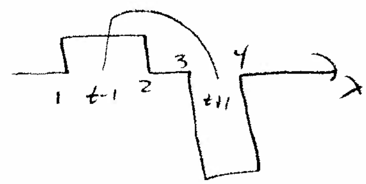




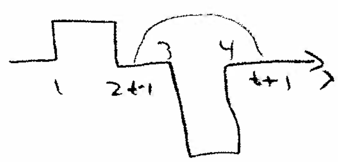
$0 < t < 1$ $\quad y(t) = \int_1^{t+1} [1-(t-\lambda)^2](1)\,d\lambda$



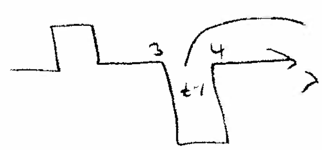
$1 \le t \le 2$ $\quad y(t) = \int_1^{2} [1-(t-\lambda)^2](1)\,d\lambda$



$2 \le t \le 3$ $\quad y(t) = \int_{t-1}^{2} [1-(t-\lambda)^2](1)\,d\lambda + \int_3^{t+1} [1-(t-\lambda)^2](-2)\,d\lambda$



$3 \le t \le 4$ $\quad y(t) = \int_3^{4} [1-(t-\lambda)^2](-2)\,d\lambda$



$4 \le t \le 5$ $\quad y(t) = \int_{t-1}^{4} [1-(t-\lambda)^2](-2)\,d\lambda$

$y(t) = 0 \quad t < 0$ or $t > 5$

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3) (15 points) The LTI system in Figure 1 has the following input, $x(t)$, impulse response, $h(t)$, and output, $y(t)$. Use the convolution to determine the time parameters (*a, b, c, d*) and the amplitude, e. Note that these figures are not drawn to scale and you must show detailed work to justify your answer.

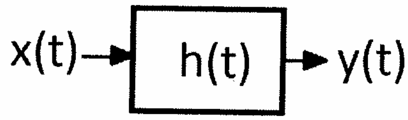
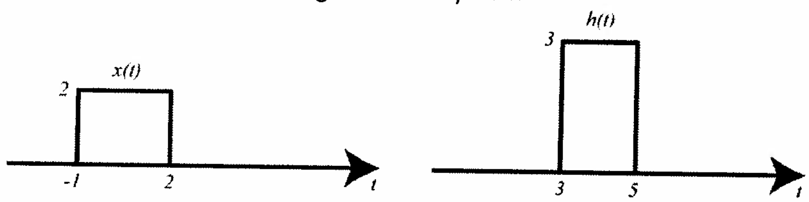
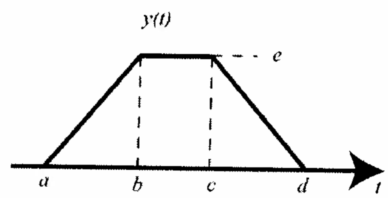
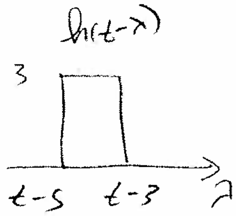


Figure 1. LTI system







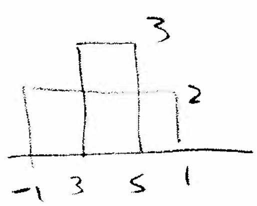
$t-3=-1 \quad t=2$

$t-5=-1 \quad t=4$

$t-3=2 \quad t=5$

$t-5=2 \quad t=7$

$\boxed{a=2,\ b=4,\ c=5,\ d=7}$



$max = 3 \cdot 2 \cdot 2 = \boxed{12 = e}$

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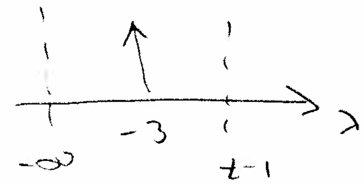
4) (15 points)

For the following LTI systems, detrmine the **impulse response**.

a) $y(t) = 6x(t) + \int_{-\infty}^{t-1} e^{-(t+\lambda)} x(\lambda+3)d\lambda$



$t-1 > -3$

$t+2 > 0$

$$\boxed{h(t) = 6\delta(t) + e^{-(t-3)} u(t+2)}$$

b) $3\dot{y}(t) - y(t) = 2x(t+1)$

$3\dot{h} - h = 2\,\delta(t+1)$

$\dot{h} - \frac{1}{3}h = \frac{2}{3}\delta(t+1)$

$\frac{d}{dt}\left[h e^{-t/3}\right] = \frac{2}{3} e^{-t/3}\,\delta(t+1) = \frac{2}{3} e^{1/3}\delta(t+1)$

$h(t)\, e^{-t/3} = \frac{2}{3} e^{1/3} u(t+1)$

$$\boxed{h(t) = \frac{2}{3} e^{(t+1)/3} u(t+1)}$$

For the following LTI systems, detrmine the **step response**.

c) $y(t) = 6x(t) + \int_{-\infty}^{t-1} e^{-\lambda} x(\lambda)d\lambda$

$s(t) = 6u(t) + \int_{-\infty}^{t-1} e^{-\lambda} u(\lambda)d\lambda = 6u(t) + \int_{0}^{t-1} e^{-\lambda}d\lambda$

$= 6u(t) + \left[-e^{-\lambda}\Big|_{0}^{t-1}\right] u(t-1)$

$$\boxed{s(t) = 6u(t) + \left(1 - e^{-(t-1)}\right)u(t-1)}$$

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5) (30 points) Fill in the following table with a Y (yes) or N (no) for each of the system models given. Assume $-\infty < t < \infty$ for all of the systems and all initial conditions are zero. *You do not need to show any work!*

System	System Model	Causal	Memoryless	Linear	Time-Invariant	BIBO Stable	Invertible
1	$y(t) = x(1-t)$	N	N	Y	N	Y	Y
2	$y(t) = \cos\left(\frac{1}{1+x(t)}\right)$	Y	Y	N	Y	Y	N
3	$y(t) = x(t-1) + x(t-2)$	Y	N	Y	Y	Y	N
4	$y(t) = tx(t)$	Y	Y	Y	N	N	N
5	$y(t) = \sqrt{x(t)}$	Y	Y	N	Y	Y	Y