ECE-205

Exam 3

Fall 2011

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 ________/30

Problem 2 ________/15

Problem 3 ________/20

Problem 4 ________/15

Problems 5 ________/20

Solutions

Total ____________

1) (30 points) For the following transfer functions, determine ***both*** the *impulse response* and the *unit step response* of the system. *Do not forget any necessary unit step functions.*

a) $H(s) = \dfrac{e^{-s}}{(s+2)}$

b) $H(s) = \dfrac{1}{(s+1)^2}$

c) $H(s) = \dfrac{s}{s^2+4s+5}$

(a) $\boxed{h(t) = e^{-2(t-1)} u(t-1)}$

$Y(s) = \dfrac{e^{-s}}{s(s+2)}$ $\quad \dfrac{1}{s(s+2)} = \dfrac{A}{s} + \dfrac{B}{s+2}$ $\quad A = \frac{1}{2}$ $\quad B = -\frac{1}{2}$

$\boxed{y(t) = \left[\frac{1}{2} - \frac{1}{2} e^{-2(t-1)}\right] u(t-1)}$

(b) $\boxed{h(t) = t e^{-t} u(t)}$

$Y(s) = \dfrac{1}{s(s+1)^2} = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{C}{(s+1)^2}$ $\quad A = 1$ $\quad C = -1$ $\quad 0 = A + B$ $\quad B = -1$

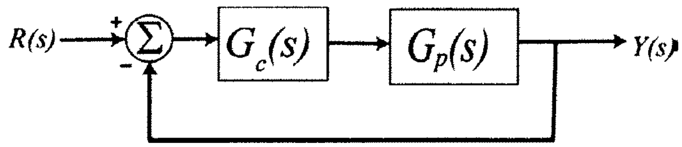
$\boxed{y(t) = \left[1 - e^{-t} - t e^{-t}\right] u(t)}$

(c) $H(s) = \dfrac{s}{s^2+4s+5} = \dfrac{s}{(s+2)^2+1} = \dfrac{A}{(s+2)^2+1} + \dfrac{B(s+2)}{(s+2)^2+1}$ $\quad 1 = 0 + B$ $\quad B = 1$ $\quad -2 = A$

$\boxed{h(t) = \left[-2 e^{-2t} \sin(t) + e^{-2t} \cos(t)\right] u(t)}$

$Y(s) = \dfrac{1}{(s+2)^2+1}$ $\quad \boxed{y(t) = e^{-2t} \sin(t)\, u(t)}$

2) (15 points) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \dfrac{3}{s+5}$



a) Determine the settling time of the plant alone (assuming there is no feedback)

$$\boxed{T_s = \frac{4}{5}}$$

b) Determine the steady state error for plant alone assuming the input is a unit step (simplify your answer)

$$e_{ss} = 1 - G_p(0) = 1 - \frac{3}{5} = \boxed{\frac{2}{5} = e_{ss}}$$

c) For a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function $G_0(s)$

$$G_0(s) = \frac{\left(\frac{3k_p}{s+5}\right)}{1+\left(\frac{3k_p}{s+5}\right)} = \boxed{\frac{3k_p}{s+5+3k_p} = G_0(s)}$$

d) Determine the settling time of the closed loop system , in terms of k_p

$$\boxed{T_s = \frac{4}{5+3k_p}}$$

e) Determine the steady state error of the closed loop system for a unit step, in terms of k_p (simplify your answer)

$$e_{ss} = 1 - G_0(0) = 1 - \frac{3k_p}{5+3k_p} = \frac{5+3k_p-3k_p}{5+3k_p} = \boxed{\frac{5}{5+3k_p} = e_{ss}}$$

f) For and integral controller, $G_c(s) = \dfrac{k_i}{s}$, determine the closed loop transfer function $G_0(s)$ and the steady state error for a unit step in terms of k_i

$$G_0(s) = \frac{\frac{k_i}{s}\frac{3}{s+5}}{1+\frac{k_i}{s}\frac{3}{s+5}} = \boxed{\frac{3k_i}{s^2+5s+3k_i} = G_0(s)}$$

$$e_{ss} = 1 - G_0(0) = \boxed{0 = e_{ss}}$$

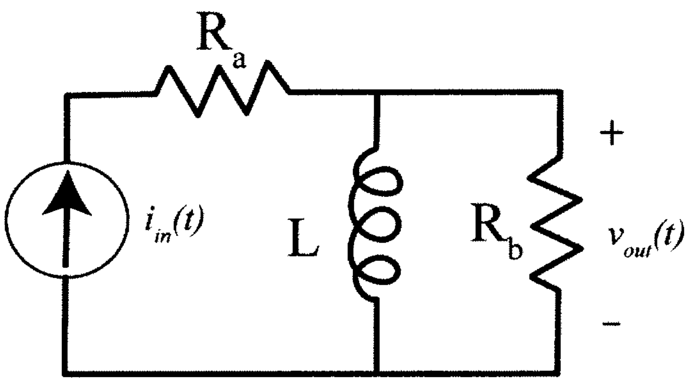
3) (20 points) For the following circuit determine

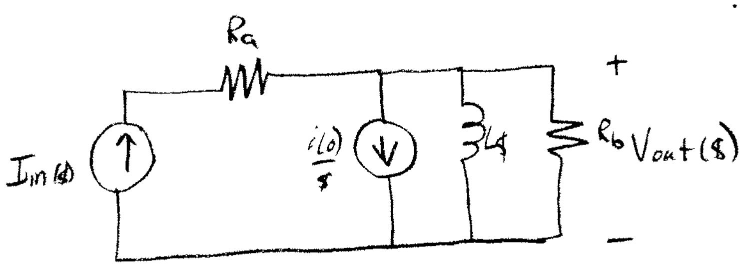
a) the zero input response (ZIR)

b) the zero state response (ZSR)

c) the <u>*transfer function*</u> $H(s)$

Note; You will need to include initial conditions for some of this problem.





$$I_{in}(s) = \frac{i(0)}{s} + \frac{V_{out}(s)}{Ls} + \frac{V_{out}(s)}{R_b}$$

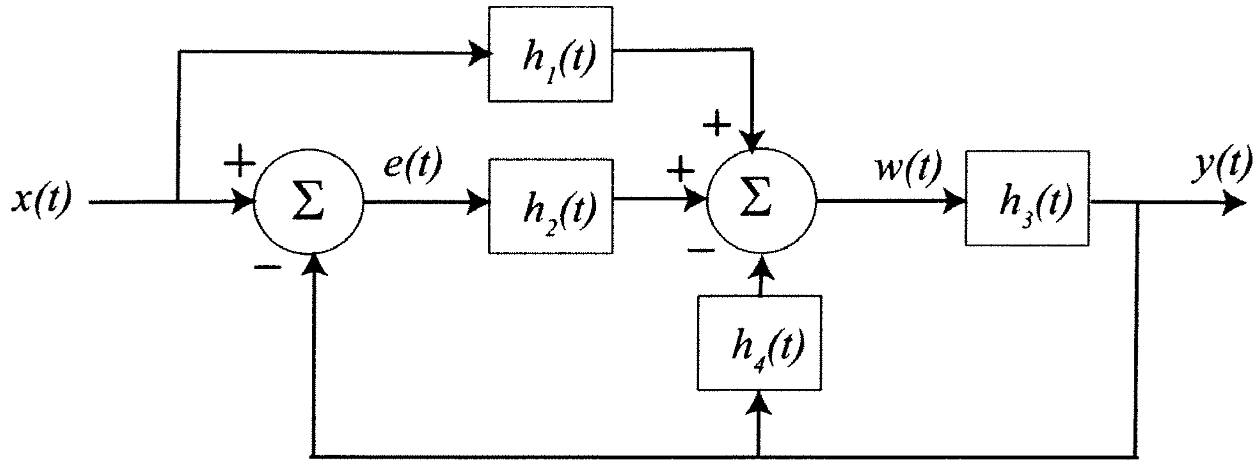
$$I_{in}(s) - \frac{i(0)}{s} = V_{out}(s)\left[\frac{1}{Ls} + \frac{1}{R_b}\right]$$

$$\left[I_{in}(s) - \frac{i(0)}{s}\right] = V_{out}(s)\left[\frac{R_b + Ls}{R_b L s}\right]$$

$$\boxed{V_{out}(s) = \underbrace{\left[\frac{R_b L s}{Ls + R_b} I_{in}(s)\right]}_{ZSR} + \underbrace{\left[\frac{-i(0) R_b L}{Ls + R_b}\right]}_{ZIR}}$$

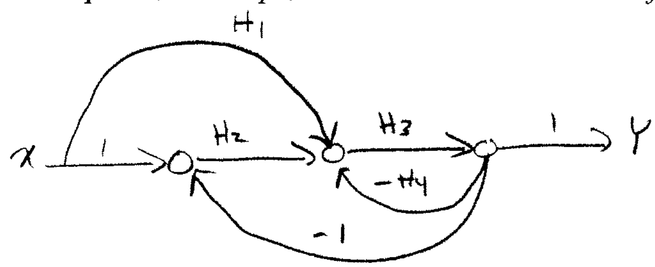
$$H(s) = \frac{V_{out}(s)}{I_n(s)} = \boxed{\frac{R_b L s}{Ls + R_b} = H(s)}$$

4) (15 points) For the following block diagram



Draw the corresponding signal flow graph, labeling each branch and direction. *Feel free to insert as many branches with a gain of 1 as you think you may need.*

Determine the system transfer function using Mason's gain rule. *You must clearly indicate all of the paths, the loops, the determinant and the cofactors.* **You need to simplify your final answer!**



$P_1 = H_1 H_3 \quad P_2 = H_2 H_3 \qquad L_1 = -H_3 H_4 \qquad L_2 = -H_2 H_3$

$\Delta = 1 - (L_1 + L_2) = 1 + H_3 H_4 + H_2 H_3$

$\Delta_1 = 1 \qquad \Delta_2 = 1$

$$G_0 = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta} = \boxed{\frac{H_1 H_3 + H_2 H_3}{1 + H_3 H_4 + H_2 H_3} = G_0}$$

Note The transfer function needs to be written in terms of other transfer functions, not impulse responses!

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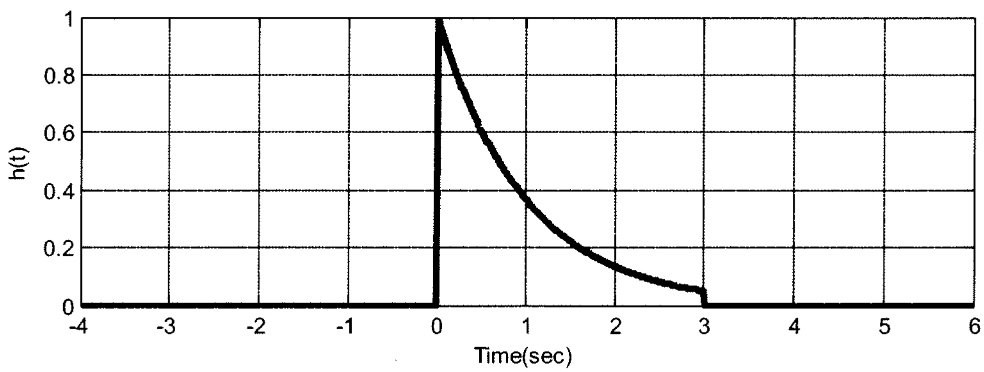
5) (20 points) Consider a linear time invariant system with impulse response given by

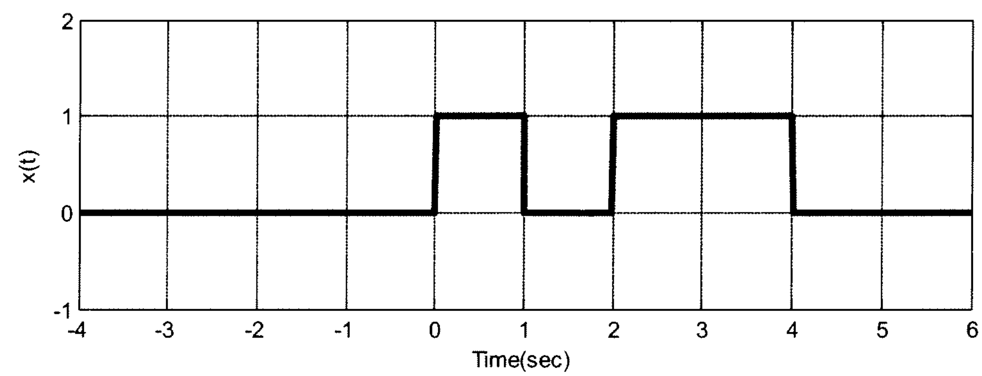
$$h(t) = e^{-t}[u(t) - u(t-3)]$$

The input to the system is given by

$$x(t) = [u(t) - u(t-1)] + [u(t-2) - u(t-4)]$$

The impulse response and input are shown below:





Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

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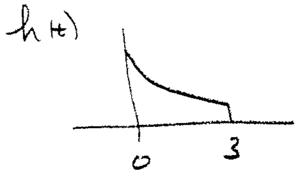
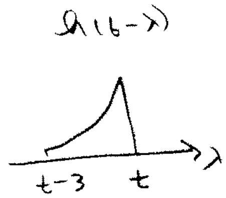
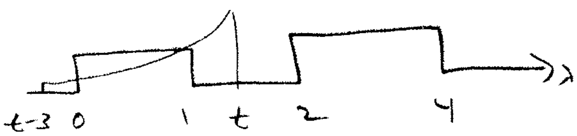
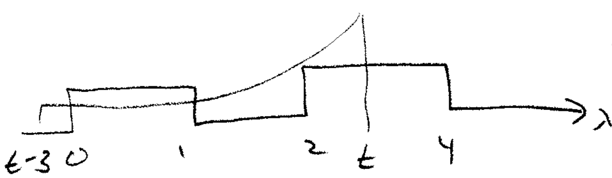




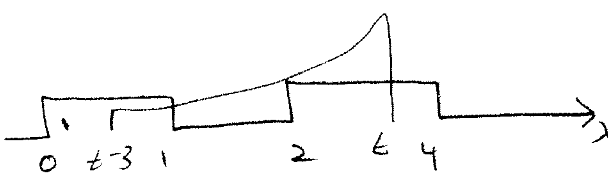
$0 \le t \le 1 \quad y(t) = \int_0^t e^{-(t-\lambda)}(1)\,d\lambda$



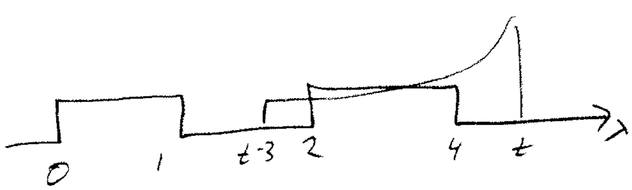
$1 \le t \le 2 \quad y(t) = \int_0^1 e^{-(t-\lambda)}(1)\,d\lambda$



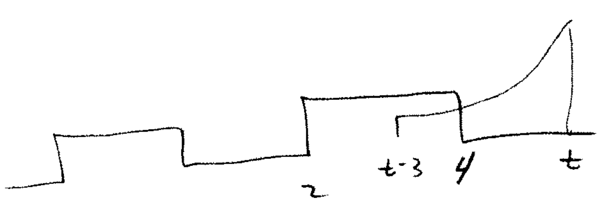
$2 \le t \le 3 \quad y(t) = \int_0^1 e^{-(t-\lambda)}(1)\,d\lambda + \int_2^t e^{-(t-\lambda)}(1)\,d\lambda$



$3 \le t \le 4 \quad y(t) = \int_{t-3}^1 e^{-(t-\lambda)}(1)\,d\lambda + \int_2^t e^{-(t-\lambda)}(1)\,d\lambda$



$4 \le t \le 5 \quad y(t) = \int_2^4 e^{-(t-\lambda)}(1)\,d\lambda$



$5 \le t \le 7 \quad y(t) = \int_{t-3}^4 e^{-(t-\lambda)}(1)\,d\lambda$

$y(t) = 0$ for $t \le 0$ and $t \ge 7$