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ECE-205

Exam 2

Fall 2011

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 ________/22

Problem 2 ________/20

Problem 3 ________/15

Problems 4 ________/18

Problem 5 ________/25

Total ____________

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1) (22 points) Fill in the non-shaded part of the following table.

	Linear? (Y/N)	Time Invariant? (Y/N)	BIBO Stable? (Y/N)
$y(t)=\cos(t)x(t)$	Y	N	
$\dot{y}(t)+y(t)=e^{-t}x(t)$	Y	N	
$y(t)=x\left(\frac{t}{2}\right)$	Y	N	
$y(t)=\int_{-\infty}^{t} e^{\lambda}x(\lambda)d\lambda$			N
$y(t)=\int_{-\infty}^{t} e^{-\lambda}x(\lambda)d\lambda$			N
$y(t)=\cos\left(\frac{1}{x(t)}\right)$			Y
$h(t)=\delta(t)$			Y
$h(t)=u(t)$			N

$y(t) = \int_{-\infty}^{t} e^{\lambda} x(\lambda) d\lambda \le \int_{-\infty}^{t} e^{\lambda} N d\lambda = e^{t} N$ not BIBO

$y(t) = \int_{-\infty}^{t} e^{-\lambda} x(\lambda) d\lambda \le \int_{-\infty}^{t} e^{-\lambda} N d\lambda = -e^{-\lambda}\Big|_{-\infty}^{t} N$ not BIBO

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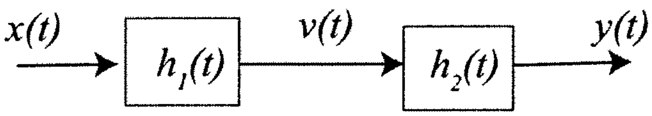
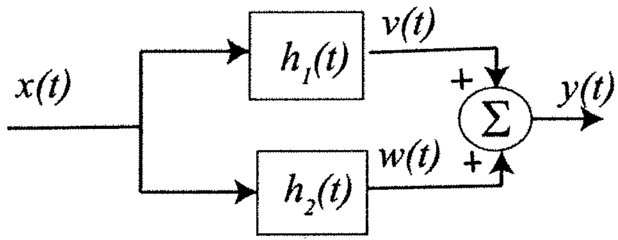
2) (20 points) For the following interconnected systems,

i) determine the overall impulse response (the impulse response between input *x(t)* and output *y(t)*) and

ii) determine if the system is causal.





a) $h_1(t) = \delta(t+2)$, $h_2(t) = \delta(t-1)$

b) $h_1(t) = u(t+1)$, $h_2(t) = u(t-2)$

Parallel Connections:

a) $h(t) = h_1(t) + h_2(t) = \boxed{\delta(t+2) + \delta(t-1) = h(t)}$ $\boxed{\text{not causal}}$

b) $h(t) = h_1(t) + h_2(t) = \boxed{u(t+1) + u(t-2) = h(t)}$ $\boxed{\text{not causal}}$

Series Connections:

a) $h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(\lambda)\, h_2(t-\lambda)\, d\lambda = \int_{-\infty}^{\infty} \delta(\lambda+2)\, \delta(t-\lambda-1)\, d\lambda = \delta(t+1)$

$\boxed{h(t) = \delta(t+1)}$ $\boxed{\text{not causal}}$

b) $h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(\lambda)\, h_2(t-\lambda)\, d\lambda = \int_{-\infty}^{\infty} u(\lambda+1)\, u(t-\lambda-2)\, d\lambda = \int_{-1}^{t-2} d\lambda = (t-1)\,u(t-1)$

$\boxed{h(t) = (t-1)\,u(t-1)}$ $\boxed{\text{causal}}$

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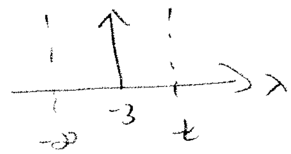
3) (18 Points) Determine the impulse response for the following systems. Don't forget any necessary unit step functions

a) $y(t) = x(t-1) + x(t+1)$

b) $y(t) = \int_{-\infty}^{t} e^{-(t-\lambda)} x(\lambda+3) d\lambda$

c) $2\dot{y}(t) + y(t) = 3x(t)$

a) $\boxed{h(t) = \delta(t-1) + \delta(t+1)}$



b) $h(t) = \int_{-\infty}^{t} e^{-(t-\lambda)} \delta(\lambda+3) d\lambda$

$= \boxed{e^{-(t+3)} u(t+3) = h(t)}$

c) $2\dot{h}(t) + h(t) = 3\,\delta(t)$

$\dot{h}(t) + \frac{1}{2} h(t) = \frac{3}{2}\,\delta(t)$

$\frac{d}{dt}\left[h(t)\, e^{+\frac{t}{2}}\right] = e^{t/2}\, \frac{3}{2}\,\delta(t) = \frac{3}{2}\,\delta(t)$

$h(t)\, e^{t/2} = \int_{-\infty}^{t} \frac{3}{2}\,\delta(\lambda) d\lambda = \frac{3}{2} u(t)$

$\boxed{h(t) = \frac{3}{2} e^{-t/2} u(t)}$

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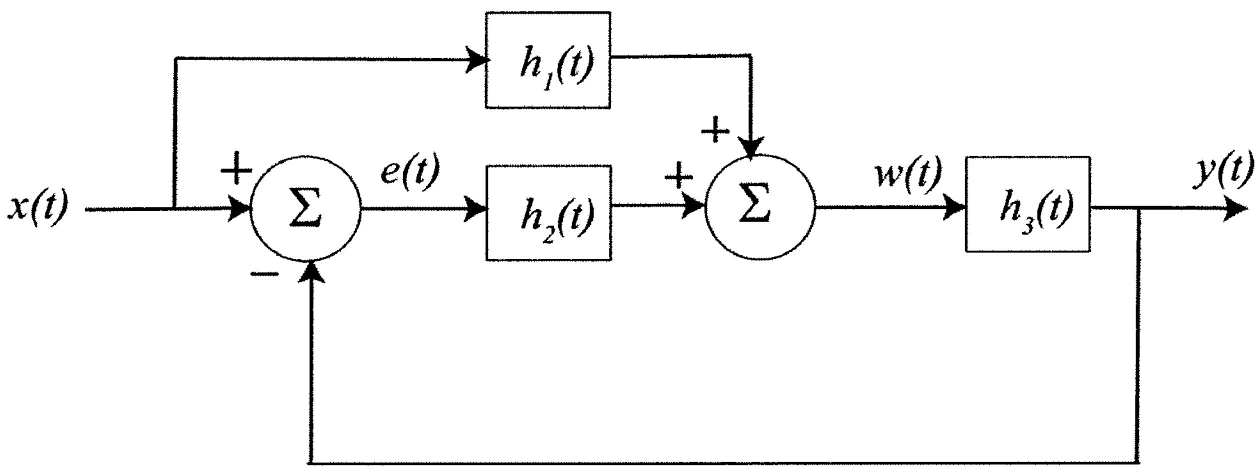
4) (15 points) The input-output relationship for the following system can be written as

$$y(t) * A(t) = x(t) * B(t)$$

Determine $A(t)$ and $B(t)$



$e(t) = x(t) - y(t)$ $\quad w(t) = e(t) * h_2(t) + h_1(t) * x(t)$ $\quad y(t) = w(t) * h_3(t)$

$y(t) = \left[e(t) * h_2(t) + h_1(t) * x(t)\right] * h_3(t)$

$= e(t) * h_2(t) * h_3(t) + x(t) * h_1(t) * h_3(t)$

$= (x(t) - y(t)) * h_2(t) * h_3(t) + x(t) * h_1(t) * h_3(t)$

$= x(t) * h_2(t) * h_3(t) - y(t) * h_2(t) * h_3(t) + x(t) * h_1(t) * h_3(t)$

$y(t) + y(t) * h_2(t) * h_3(t) = x(t) * \left[h_2(t) * h_3(t) + h_1(t) * h_3(t)\right]$

$y(t) * \underbrace{\left[\delta(t) + h_2(t) * h_3(t)\right]}_{A(t)} = x(t) * \underbrace{\left[h_2(t) * h_3(t) + h_1(t) * h_3(t)\right]}_{B(t)}$

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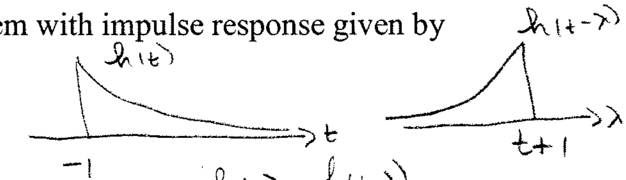
5) (25 points) Consider a linear time invariant system with impulse response given by

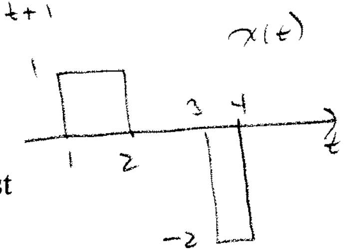
$$h(t) = e^{-(t+1)}u(t+1)$$



$h(-1) = h(t-\lambda)$
$-1 = t - \lambda$
$\lambda = t+1$

The input to the system is given by

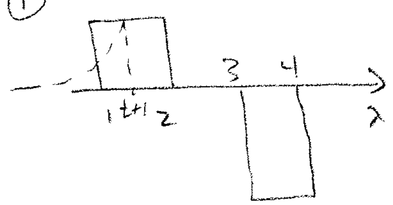
$$x(t) = [u(t-1) - u(t-2)] - 2[u(t-3) - u(t-4)]$$



Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

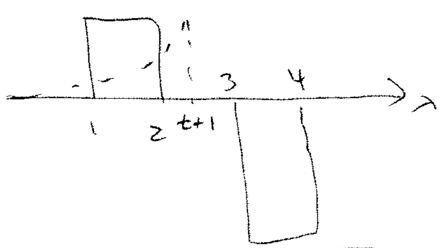
①



$1 < t+1 < 2$ $\boxed{0 < t < 1}$

$$\boxed{y(t) = \int_1^{t+1} e^{-(t-\lambda+1)}(1)\,d\lambda}$$

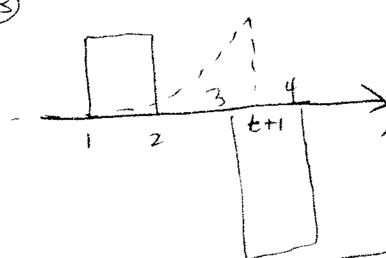
②



$2 < t+1 < 3$ $\boxed{1 < t < 2}$

$$\boxed{y(t) = \int_1^{2} e^{-(t-\lambda+1)}(1)\,d\lambda}$$

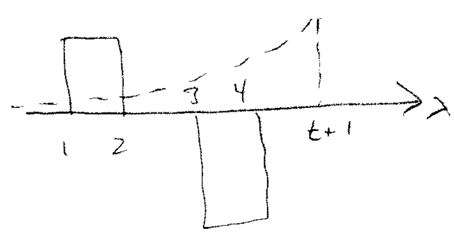
③



$3 < t+1 < 4$ $\boxed{2 < t < 3}$

$$\boxed{y(t) = \int_1^{2} e^{-(t-\lambda+1)}(1)\,d\lambda + \int_3^{t+1} e^{-(t-\lambda+1)}(-2)\,d\lambda}$$

④



$4 < t+1$ $\boxed{t > 3}$

$$\boxed{y(t) = \int_1^{2} e^{-(t-\lambda+1)}(1)\,d\lambda + \int_3^{4} e^{-(t-\lambda+1)}(-2)\,d\lambda}$$