

# **ECE-205**

## **Exam 1**

### **Fall 2011**

**Calculators can only be used for simple calculations. Solving integrals, differential equations, systems of equations, etc. does not count as a simple calculation.**

**You must show your work to receive credit.**

**Problem 1** \_\_\_\_\_/20

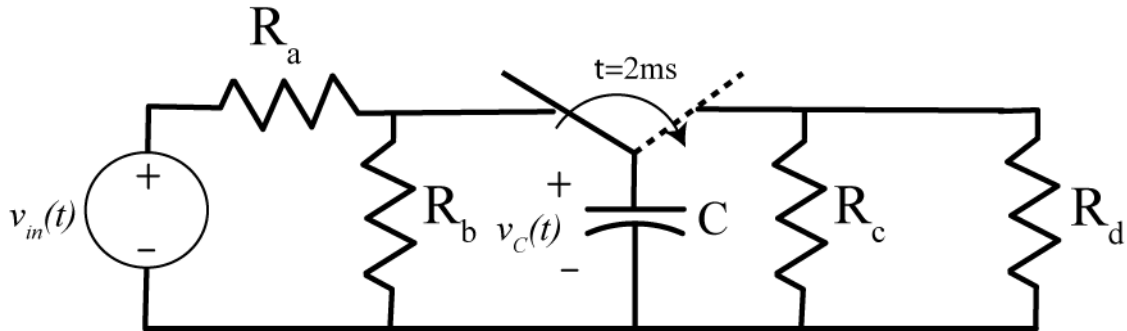
**Problem 2** \_\_\_\_\_/30

**Problem 3** \_\_\_\_\_/18

**Problem 4-11** \_\_\_\_\_/32

**Total** \_\_\_\_\_

1) (20 points) Consider the circuit shown in the figure below:



- Determine an expression for the time constant of the circuit for the time when the capacitor is charging ( $t < 0.002$  seconds) and discharging ( $t > 0.002$  seconds) in terms of the parameters  $C, R_a, R_b, R_c$  and  $R_d$ . (Do not use numbers).
- Determine an expression for the static gain of the circuit for  $t < 0.002$  seconds in terms of the parameters  $C, R_a, R_b, R_c$  and  $R_d$ . (Do not use numbers).
- For  $R_a = 1k\Omega, R_b = 1k\Omega, R_c = 2k\Omega, R_d = 10k\Omega, C = 2\mu F, V_{in} = 6V$  accurately *sketch* the voltage across the capacitor from 0 to 12 ms. You need to specifically label the voltages at  $t = 0.002$  seconds and  $t = 0.012$  seconds. You need to primarily determine the appropriate time constants and steady state values, and use the following table as a guide.

| Time ( $t$ ) | $t / \tau$ | $y(t)$         |
|--------------|------------|----------------|
| 0            | 0          | $0 y_{ss}$     |
| $\tau$       | 1          | $0.632 y_{ss}$ |
| $2\tau$      | 2          | $0.865 y_{ss}$ |
| $3\tau$      | 3          | $0.950 y_{ss}$ |
| $4\tau$      | 4          | $0.982 y_{ss}$ |
| $5\tau$      | 5          | $0.993 y_{ss}$ |

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**2) (30 points)** For the following three differential equations, assume the input is  $x(t) = 4u(t)$  (the input is equal to four for time greater than zero), and the initial conditions are  $y(0) = \dot{y}(0) = 0$

Determine the solution to each of the following differential equations and put your final answer in a box. Be sure to use the initial conditions to solve for all unknowns. You must show all your work to receive credit.

**a)**  $\ddot{y}(t) + 2\dot{y}(t) + y(t) = 3x(t)$

**b)**  $\ddot{y}(t) + 5\dot{y}(t) + 4y(t) = 2x(t)$

**c)**  $\ddot{y}(t) + 4\dot{y}(t) + 16y(t) = 4x(t)$

**3) (18 points)** The form of the under damped ( $0 < \zeta < 1$ ) solution to the second order differential equation

$$\ddot{y}(t) + 2\zeta\omega_n\dot{y}(t) + \omega_n^2y(t) = K\omega_n^2x(t)$$

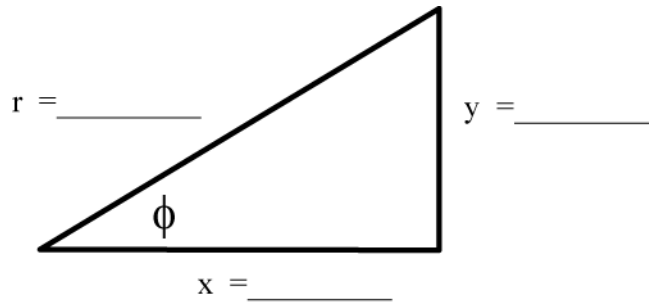
for a step input  $x(t) = Au(t)$  is

$$y(t) = KA + ce^{-\zeta\omega_n t} \sin(\omega_d t + \phi)$$

where  $c$  and  $\phi$  are constants to be determined and the damped frequency  $\omega_d = \omega_n\sqrt{1-\zeta^2}$

a) Using the initial condition  $\dot{y}(0) = 0$  show that  $\tan(\phi) = \frac{\sqrt{1-\zeta^2}}{\zeta}$

b) We can express the relationship in part a using the following triangle. Fill in the blanks and then use this triangle determine an expression for  $\sin(\phi)$ .

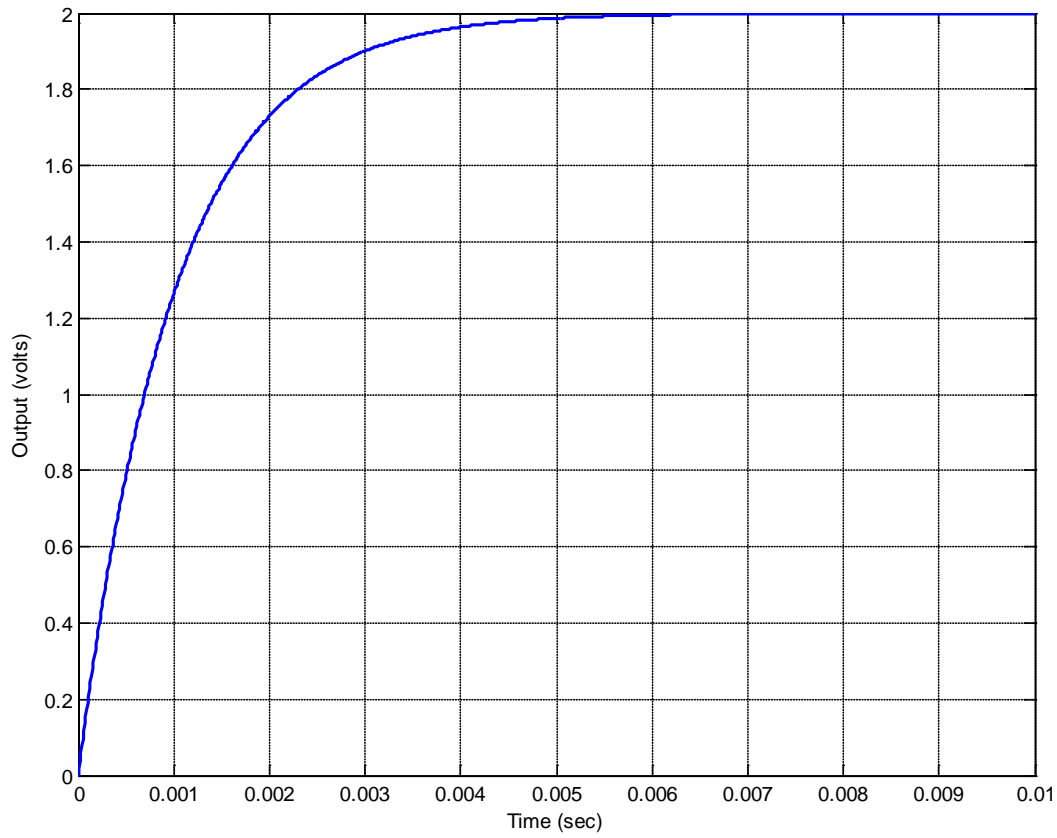


c) Use your answer to part b, and the initial condition  $y(0) = 0$  to determine the remaining unknown constant, and write out the complete solution for  $y(t)$ .

**Problems 4-11, 4 points each, no partial credit (32 points)**

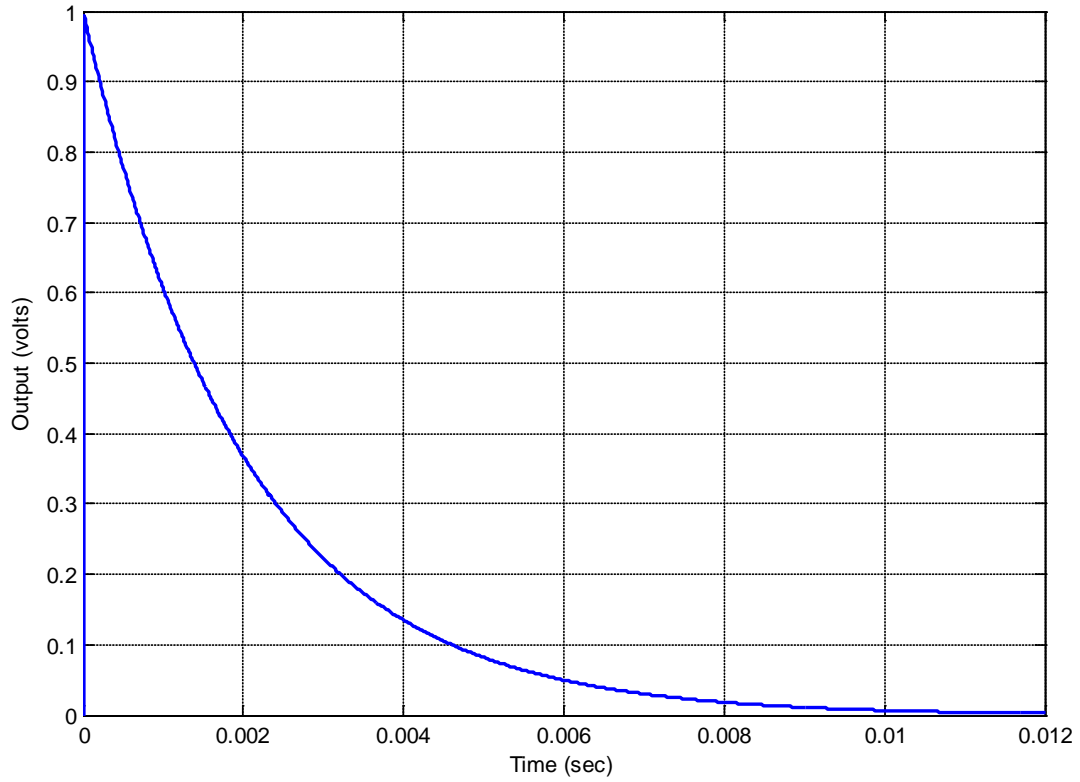
**4)** Consider the response of a first order circuit shown below. Of the following, which is the best estimate of the time constant?

- a) 0.001 sec   b) 0.002 sec   c) 0.003 sec   d) 0.004 sec   e) 0.005 sec   f) 0.006 sec

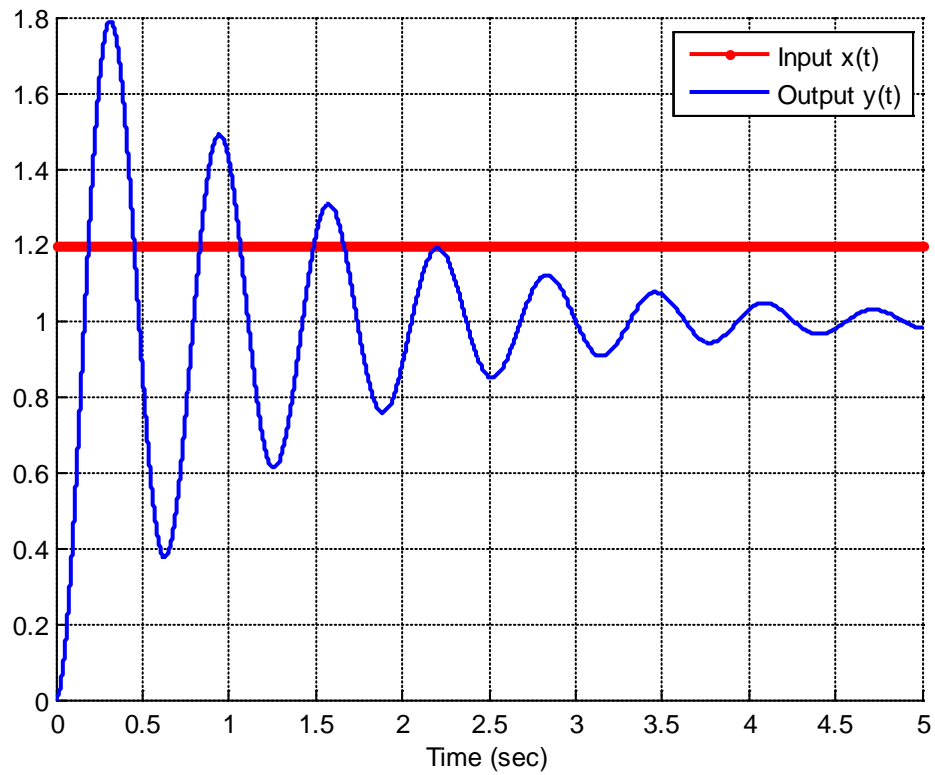


5) Consider the response of a first order circuit shown below. Of the following, which is the best estimate of the time constant?

- a) 0.001 sec   b) 0.002 sec   c) 0.003 sec   d) 0.004 sec   e) 0.005 sec   f) 0.006 sec



Problems 6 and 7 refer the following graph showing the response of a second order system to a step input.



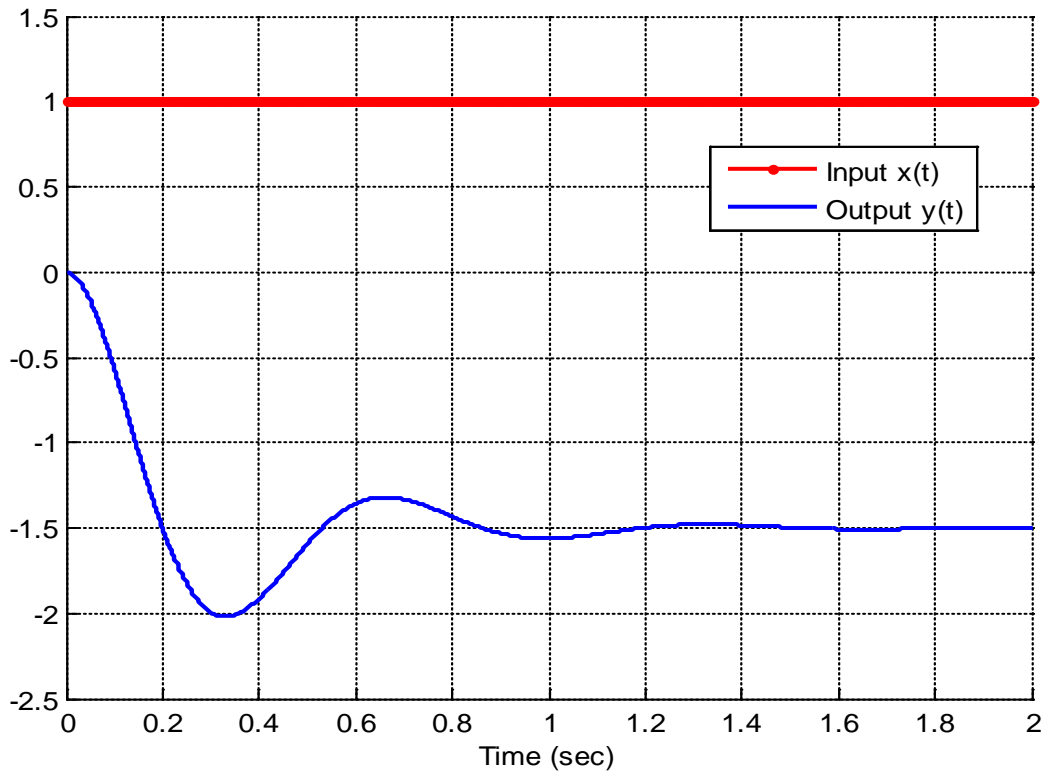
6) The percent overshoot for this system is best estimated as

- a) 180%   b) 150 %   c) 100%   d) 80 %   e) 60%   f) 50%

7) The static gain for this system is best estimated as

- a) 1.8   b) 1.2   c) 1.00   d) 0.83   e) 0.5

Problems 8-9 refer the following graph showing the response of a second order system to a step input.



8) The percent overshoot for this system is best estimated as

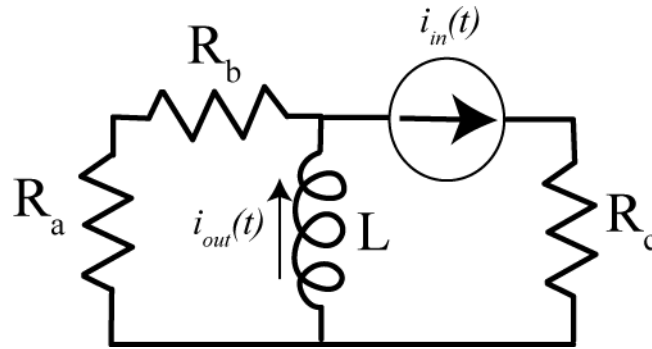
- a) 300%   b) -300 %   c) 200%   d) -200 %   e) 33%   f) -33%

9) The static gain for this system is best estimated as

- a) -3   b) 3   c) 2.5   d) -2.5   e) 1.5   f) -1.5



Problems 10 and 11 refer to the following circuit



**10)** The Thevenin resistance seen from the ports of the inductor is

- a)  $R_{th} = R_c \parallel (R_a + R_b)$    b)  $R_{th} = R_c$    c)  $R_{th} = R_a + R_b$    d)  $R_{th} = R_a + R_b + R_c$    e) none of these

**11)** The static gain for the system is

- a)  $K = 1$    b)  $K = \frac{R_a + R_b}{R_a + R_b + R_c}$    c)  $K = \frac{R_c}{R_a + R_b + R_c}$    d)  $K = \frac{R_c}{R_a + R_b}$    e) none of these

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