ECE-205 : Dynamical Systems

Homework #5

Due : Friday October 8 at High Noon

Exam 2, Tuesday October 12, 2010

1) Simplify the following expressions as much as possible

a) $g(t) = e^{-(t-1)}\delta(t)$ **b)** $g(t) = \int_{-1}^{t-5} e^{-2(t-\lambda)}\delta(\lambda-2)d\lambda$ **c)** $g(t) = \int_{-3}^{t-1} e^{-(t-\lambda)}\delta(\lambda+2)d\lambda$

d) $g(t) = \int_{-t-1}^{3} e^{-(t-\lambda)}\delta(\lambda-1)d\lambda$ **e)** $g(t) = \int_{t-1}^{2} e^{-(t-\lambda)}\delta(\lambda+2)d\lambda$ **f)** $g(t) = \int_{0}^{t} e^{-(t-\lambda)}\delta(\lambda-3)d\lambda$

g) $g(t) = \int_{-\infty}^{\infty} e^{-3\lambda}\delta(t-\lambda)d\lambda$ **h)** $g(t) = \int_{-\infty}^{\infty} e^{4\lambda}u(\lambda-2)\delta(t-\lambda+2)d\lambda$ **i)** $g(t) = \int_{-\infty}^{\infty} e^{\lambda}u(\lambda-1)\delta(t-\lambda-3)d\lambda$

Scrambled Answers:

$e^{-2(t-2)}u(t-7), e^{-(t+2)}u(-1-t), e^{-3t}, e^{t-3}u(t-4), e^{4(t+2)}u(t), e^{-(t-3)}u(t-3), e^{-(t+2)}u(t+1), e^{-(t-1)}u(t+2), e^{1}\delta(t)$

2) Determine the impulse response for each of the following systems

a) $y(t) = x(t-2) + \int_{-\infty}^{t} e^{-(t-\lambda)}x(\lambda)d\lambda$ **b)** $y(t) = \int_{-\infty}^{t} e^{-2(t-\lambda)}x(\lambda+1)d\lambda$ **c)** $y(t) = \int_{-\infty}^{\infty} e^{-\lambda}x(t+\lambda)d\lambda$

d) $\frac{1}{2}\dot{y}(t) + y(t) = 6\,x(t-1)$ **e)** $\dot{y}(t) - 3y(t) = x(t+2)$ **f)** $\dot{y}(t) = x(t-3)$

Scrambled Answers: $e^{-2(t+1)}u(t+1), e^{3(t+2)}u(t+2), u(t-3), e^{t}, \delta(t-2) + e^{-t}u(t), 12e^{-2(t-1)}u(t-1)$

3) For the following two systems,

a) $y(t) = \int_{-\infty}^{t} e^{-(t-\lambda)}x(\lambda+1)d\lambda$ **b)** $y(t) = \int_{-\infty}^{\infty} e^{2\lambda}u(\lambda)x(t-\lambda)d\lambda$

i) compute the impulse response directly

ii) compute the step response directly

ii) show that the derivative of the step response is the impulse response for these two systems. You should get two terms for the derivative, but one of the terms is zero (you should be able to show why this is true)!

Note that this is a general property of LTI systems, and is important since it is much easier to determine the step respones of a system than it is to determine the impulse response of an actual system.

4) For the following inpulse responses and inputs, determine the system output using analytical convolution. Be sure to include any required unit step functions in your answers.

a) $h(t) = e^{-t}u(t),\ x(t) = u(t\text{-}1)$

b) $h(t) = e^{-t}u(t) + \delta(t),\ x(t) = \mathrm{e}^{-(t-1)}u(t\text{-}1)$

c) $h(t) = \delta(t) + \delta(t-1),\ x(t) = \mathrm{e}^{-t}u(t)$

d) $h(t) = \delta(t-1),\ x(t) = \delta(t+2)$

Scrambled Answers $y(t) = \delta(t+1),\ y(t) = [1 - e^{-(t-1)}]u(t-1),\ y(t) = te^{-(t-1)}u(t-1),\ y(t) = e^{-t}u(t) + e^{-(t-1)}u(t-1)$ *:*

5) For LTI systems with the following impulse responses, determine if the system is BIBO stable.

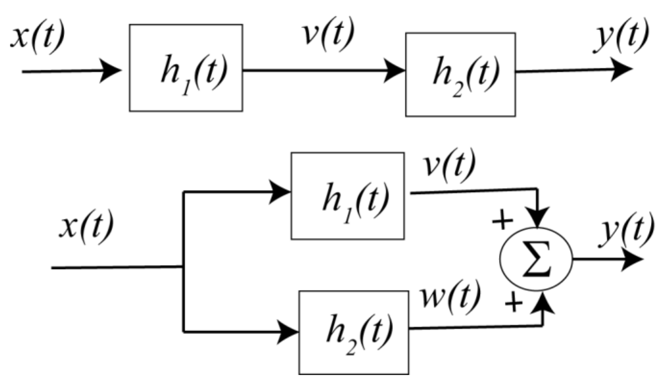
a) $h(t) = \delta(t)$ b) $h(t) = u(t)$ c) $h(t) = e^{-t}u(t)$ d) $h(t) = e^{-t^2}u(t)$

Answers: 3 are stable, one is unstable

6) For the following interconnected systems,

i) determine the overall impulse response (the impulse response between input *x(t)* and output *y(t)*) and

ii) determine if the system is causal.



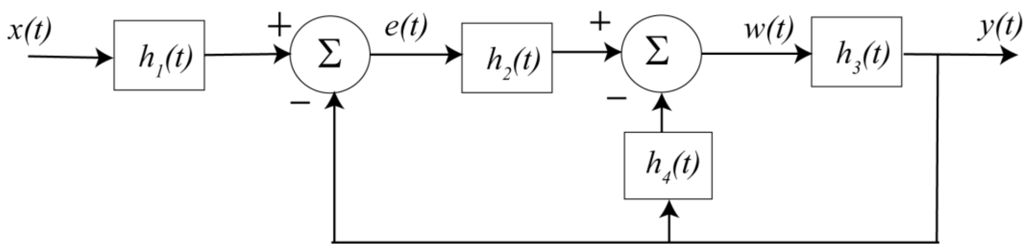
a) $h_1(t) = u(t\text{-}1),\ h_2(t) = u(t+1)$ **b)** $h_1(t) = u(t\text{-}1),\ h_2(t) = \delta(t+2)$ **c)** $h_1(t) = \mathrm{e}^{-(t-1)}u(t\text{-}1),\ h_2(t) = \delta(t) + u(t)$

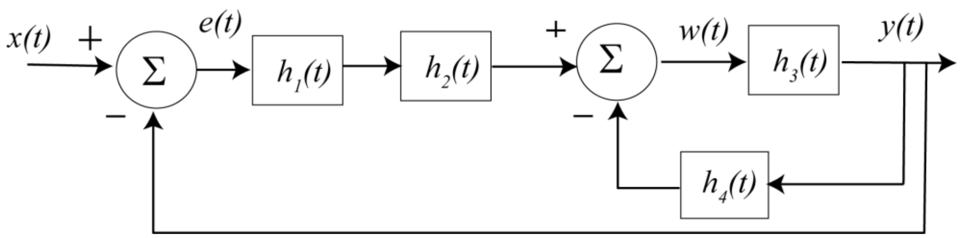
Scrambled Answers:

$h(t) = u(t-1),\ h(t) = tu(t),\ h(t) = u(t+1),\ h(t) = u(t+1) + u(t-1),\ h(t) = \delta(t) + u(t) + e^{-(t-1)}u(t-1),\ h(t) = u(t-1) + \delta(t+2)$

Three systems not causal

7) For the following systems, determine the relationship between the input and the output



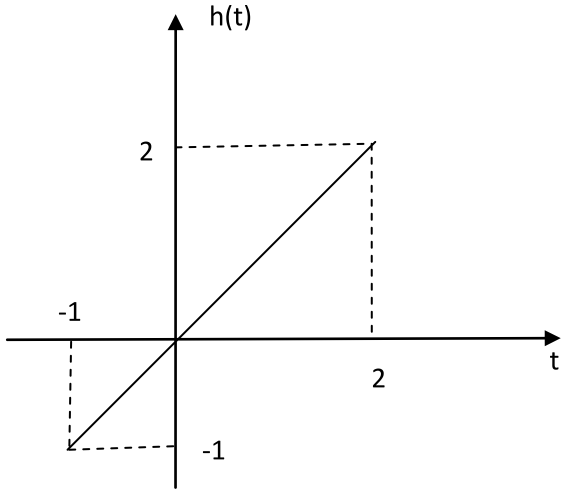


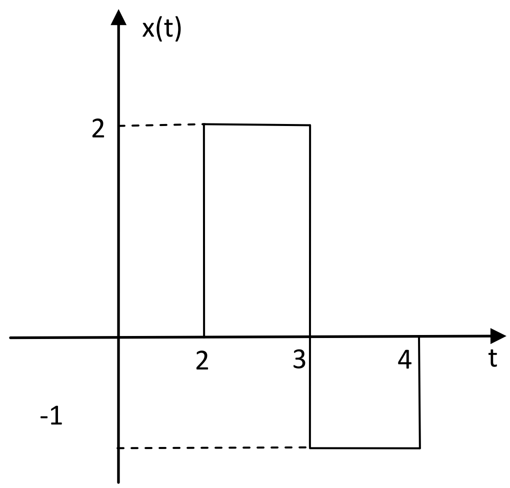
Answers:

$$y(t)*[\delta(t)+h_2(t)*h_3(t)+h_4(t)*h_3(t)]=x(t)*\left[h_1(t)*h_2(t)*h_3(t)\right]$$
$$y(t)*[\delta(t)+h_1(t)*h_2(t)*h_3(t)+h_3(t)*h_4(t)]=x*[h_1(t)*h_2(t)*h_3(t)]$$

8) Consider a linear time invariant system with impulse response given by

$h(t) = t\,[u(t+1) - u(t-2)]$ and input $x(t) = 2u(t-2) - 3u(t-3) + u(t-4)$, shown below





Using ***graphical convolution***, determine the output $y(t) = h(t) * x(t)$

Specifically, you must

a) Flip and slide $h(t)$, ***NOT*** $x(t)$

b) Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest

c) Determine the range of t for which each part of your solution is valid

d) Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.

e) ***DO NOT EVALUATE THE INTEGRALS!!***

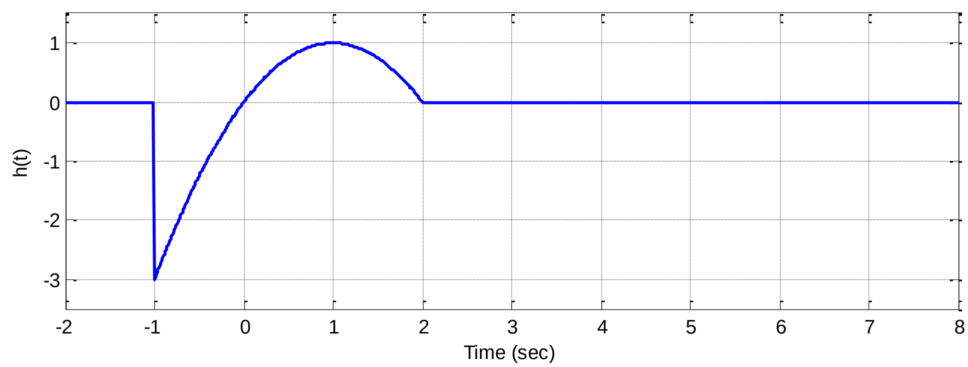
(The answer is at the end of the homework)

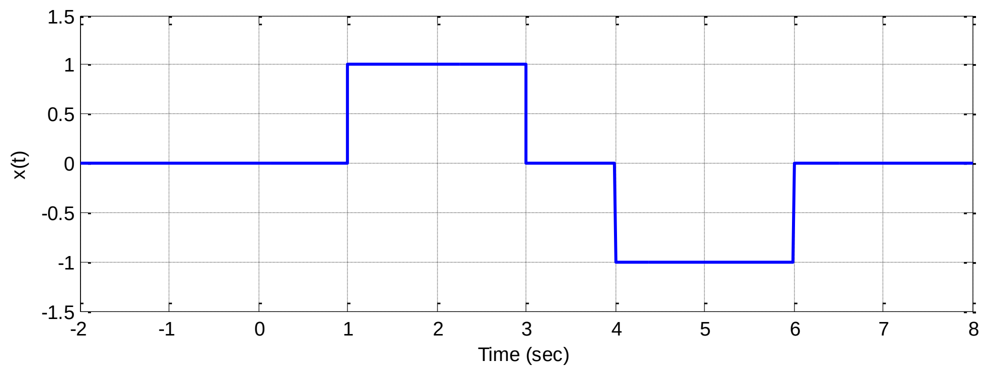
9) Consider a noncausal linear time invariant system with impulse response given by

$$h(t) = [1-(t-1)^2][u(t+1)-u(t-2)]$$

The input to the system is given by

$$x(t) = u(t-1) - u(t-3) - u(t-4) + u(t-6)$$





Using ***graphical convolution***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions

- ***<u>DO NOT EVALUATE THE INTEGRALS!!</u>***

(The answer is at the end of the homework)

Answer to problem 8:

$$y(t)=\begin{cases} 0 & t\le 1 \\ \int_{2}^{t+1}(t-\lambda)(2)d\lambda & 1\le t\le 2 \\ \int_{2}^{3}(t-\lambda)(2)d\lambda+\int_{3}^{t+1}(t-\lambda)(-1)d\lambda & 2\le t\le 3 \\ \int_{2}^{3}(t-\lambda)(2)d\lambda+\int_{3}^{4}(t-\lambda)(-1)d\lambda & 3\le t\le 4 \\ \int_{t-2}^{3}(t-\lambda)(2)d\lambda+\int_{3}^{4}(t-\lambda)(-1)d\lambda & 4\le t\le 5 \\ \int_{t-2}^{4}(t-\lambda)(-1)d\lambda & 5\le t\le 6 \\ 0 & t\ge 6 \end{cases}$$

Answer to problem 9:

$$y(t)=\begin{cases} 0 & t\le 0 \\ \int_{1}^{t+1}[1-(t-\lambda-1)^2](1)d\lambda & 0\le t\le 2 \\ \int_{1}^{3}[1-(t-\lambda-1)^2](1)d\lambda & 2\le t\le 3 \\ \int_{t-2}^{3}[1-(t-\lambda-1)^2](1)d\lambda+\int_{4}^{t+1}[1-(t-\lambda-1)^2](-1)d\lambda & 3\le t\le 5 \\ \int_{4}^{6}[1-(t-\lambda-1)^2](-1)d\lambda & 5\le t\le 6 \\ \int_{t-2}^{6}[1-(t-\lambda-1)^2](-1)d\lambda & 6\le t\le 8 \\ 0 & t\ge 8 \end{cases}$$