ECE-205 : Dynamical Systems

Homework #2

Due : Thursday September 16 at the beginning of class

1) Problem 2.5 from the Notes

2) Problem 2.6 from the Notes

3) For each of the following first order differential equations, use an integrating factor to write $y(t)$ as a function of its initial value $y(t_0)$ and an integral of the input (plus some other functions)

$$a\dot{y}(t)+by(t)=cx(t) \qquad \frac{1}{2}\dot{y}(t)-ty(t)=x(t+2)$$

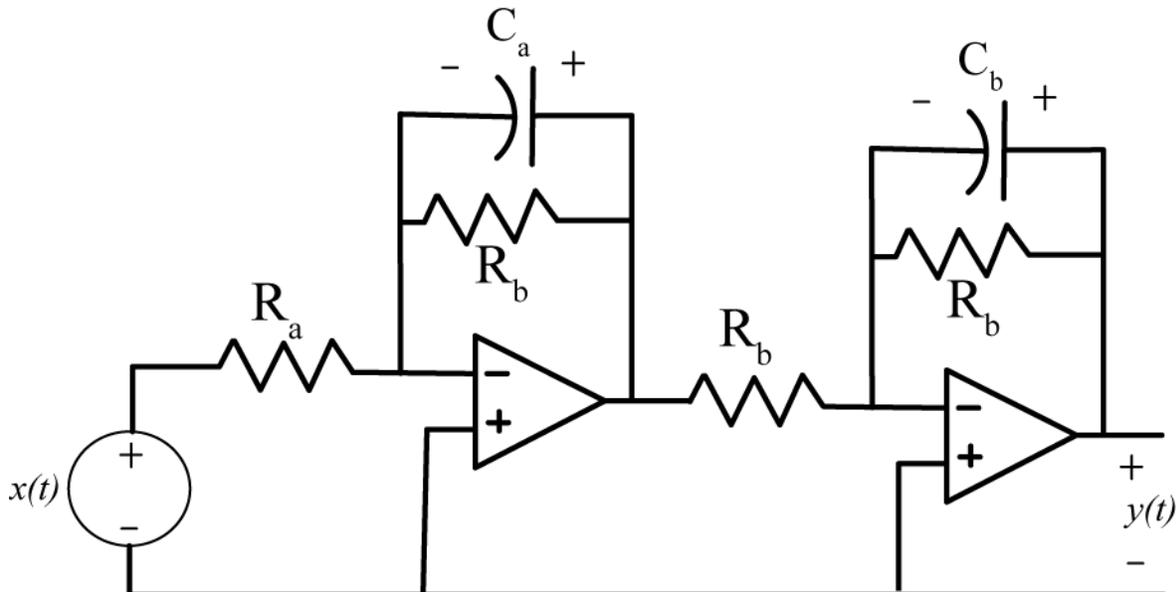$$\dot{y}(t)+\frac{1}{t}y(t)=x(t) \qquad \dot{y}(t)+\cos(t)y(t)=e^t x(t)$$

Answers:

$$y(t)=e^{-\frac{b}{a}(t-t_0)}y(t_0)+\int_{t_0}^{t}\frac{c}{a}e^{-\frac{b}{a}(t-\lambda)}x(\lambda)d\lambda \qquad y(t)=e^{t^2-t_0^2}y(t_0)+\int_{t_0}^{t}2e^{(t^2-\lambda^2)}x(\lambda+2)d\lambda$$

$$y(t)=\left(\frac{t_0}{t}\right)y(t_0)+\int_{t_0}^{t}\left(\frac{\lambda}{t}\right)x(\lambda)d\lambda \qquad y(t)=e^{-\sin(t)+\sin(t_0)}y(t_0)+\int_{t_0}^{t}e^{-\sin(t)}e^{\lambda+\sin(\lambda)}x(\lambda)d\lambda$$

4) Show that the governing differential equation for the following circuit is given by

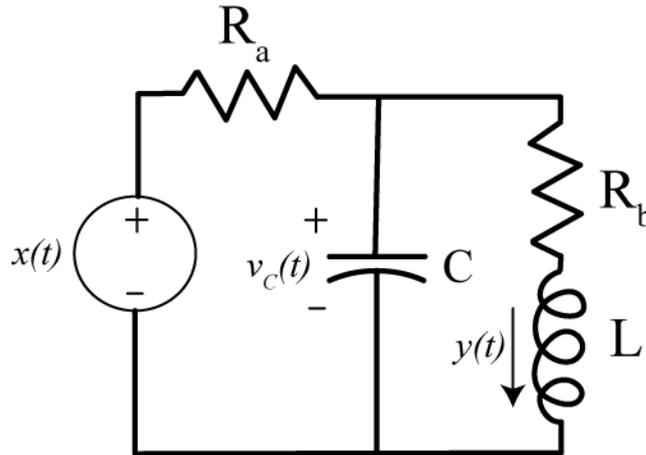$$R_b^2 C_a C_b \ddot{y}(t)+R_b(C_a+C_b)\dot{y}(t)+y(t)=\frac{R_b}{R_a}x(t)$$

and that for this circuit $\omega_n=\dfrac{1}{R_b\sqrt{C_a C_b}}$, $\zeta=\dfrac{C_a+C_b}{2\sqrt{C_a C_b}}$, and $K=\dfrac{R_b}{R_a}$

5) Show that the governing differential equation for the following circuit is given by

$$\left(\frac{LCR_a}{R_a+R_b}\right)\ddot{y}(t)+\left(\frac{L+R_aR_bC}{R_a+R_b}\right)\dot{y}(t)+y(t)=\left(\frac{1}{R_a+R_b}\right)x(t)$$
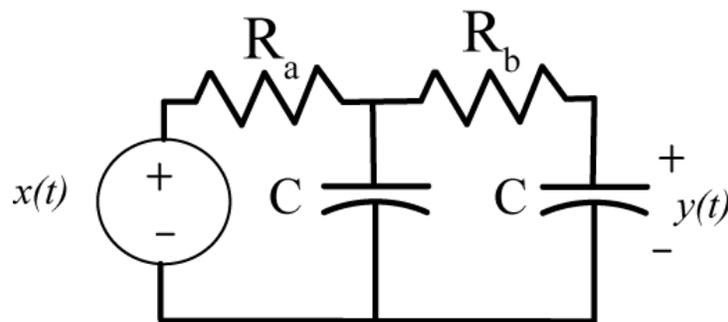
and that for this circuit $\omega_n=\sqrt{\dfrac{R_a+R_b}{LCR_a}}$, $\zeta=\dfrac{L+R_aR_bC}{2\sqrt{LCR_a}\sqrt{R_a+R_b}}$, and $K=\dfrac{1}{R_a+R_b}$.

6) Show that the governing differential equation for the following circuit is given by

$$\left(R_aR_bC^2\right)\ddot{y}(t)+\left(2R_aC+R_bC\right)\dot{y}(t)+y(t)=x(t)$$

and that for this circuit $\omega_n=\dfrac{1}{C\sqrt{R_aR_b}}$, $\zeta=\dfrac{2R_a+R_b}{2\sqrt{R_aR_b}}$, and $K=1$.

7) PreLab/Matlab/Simulink Problem

One of the standard forms for a second order system is

$$\ddot{y}(t) + 2\zeta\omega_n \dot{y}(t) + \omega_n^2 y(t) = \omega_n^2 K x(t)$$

where ζ is the damping ratio, ω_n is the natural frequency, and K is the static gain. Use this form of the standard second order system in the remainder of this problem.

We want to simulate a system described by this differential equation using a Matlab driver. Just as you did for a first order system, you need to solve for the highest power derivative (as a function of the input and lower power derivatives). If you then take $\ddot{y}(t)$ and run it though an integrator you will get $\dot{y}(t)$ and if you run this though an integrator you will get $y(t)$. Hence you will need two integrators and two feedback loops and one input into your summing block (click on the summing block and modify it to get three inputs). You may need to click on the gain block and then choose **flip block** to get the correct direction.

- We want to look at the step response, so the input to your system should be a step. The amplitude of the step should be controlled by the Matlab program, the length of the simulation should be controlled by the Matlab program, and the step should start at time zero.
- Your Simulink file should only contain variables (static gain, natural frequency, damping ratio, amplitude of the step, length of the simulation)
- Plot the transient output of the system and the steady state output KA on the same graph.
- If you use the parameters $\zeta = 0.1$, $\omega_n = 20$, $K = 2$, $A = 1$, and $Tf = 3$ (final simulation time), you should get results like that shown below. Print out your figure, your Matlab code, and your Simulink model and turn them in.