ECE-205
Exam 3
Fall 2010

Calculators and computers are not allowed. You must show your work to receive credit.

solutions

Problem 1 _________/30

Problem 2 _________/20

Problem 3 _________/20

Problem 4 _________/21

Problems 5-7_______/9

Total _____________

90-100	6
80-89	6
70-79	6
60-69	4
<60	3

median = 80

1) (30 points) For the following transfer functions, determine the *unit step response* of the system. *Do not forget any necessary unit step functions.*

a) $H(s) = \dfrac{e^{-2s}}{(s+1)(s+2)}$

b) $H(s) = \dfrac{1}{(s+2)^2}$

c) $H(s) = \dfrac{5}{s^2+4s+5}$

(a) $G(s) = \dfrac{1}{s(s+1)(s+2)} = \dfrac{A}{s} + \dfrac{B}{s+1} + \dfrac{C}{s+2}$ $\quad A = \frac{1}{2} \quad B = -1 \quad C = \frac{1}{2}$

$$\boxed{y(t) = \left[\frac{1}{2} - e^{-(t-2)} + \frac{1}{2}e^{-2(t-2)}\right]u(t-2)}$$

(b) $Y(s) = \dfrac{1}{s(s+2)^2} = \dfrac{A}{s} + \dfrac{B}{s+2} + \dfrac{C}{(s+2)^2}$ $\quad A = \frac{1}{4} \quad C = \frac{-1}{2} \quad \times s,$ let $s \to \infty \quad 0 = A + B \quad B = -1/4$

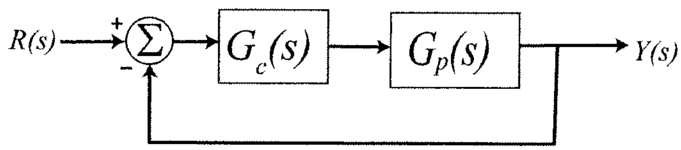
$$\boxed{y(t) = \left[\frac{1}{4} - \frac{1}{4}e^{-2t} - \frac{1}{2}te^{-2t}\right]u(t)}$$

(c) $Y(s) = \dfrac{5}{s(s^2+4s+5)} = \dfrac{5}{s[(s+2)^2+1]} = \dfrac{A}{s} + \dfrac{B}{(s+2)^2+1} + \dfrac{C(s+2)}{(s+2)^2+1}$

$A = 1 \quad \times s,$ let $s \to \infty \quad 0 = A + C \quad C = -1 \quad$ let $s = -2 \quad \frac{-5}{2} = \frac{-1}{2} + B \quad B = -2$

$$\boxed{y(t) = \left[1 - 2e^{-2t}\sin(t) - e^{-2t}\cos(t)\right]u(t)}$$

2) (20 points) Consider the following simple feedback control block diagram. The plant, the thing we want to control, has the transfer function $G_p(s) = \dfrac{4}{s+3}$



a) Determine the settling time of the plant alone (assuming there is no feedback)

b) For a proportional controller, $G_c(s) = k_p$, determine the closed loop transfer function $G_0(s)$ and then

i) the settling time, in terms of k_p

ii) the steady state error for a unit step, in terms of k_p

c) For and integral controller, $G_c(s) = \dfrac{k_i}{s}$, determine the closed loop transfer function $G_0(s)$ and the steady state error for a unit step in terms of k_i

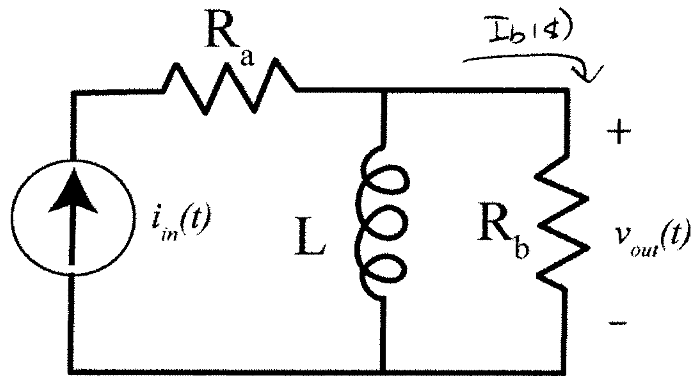
a) $\boxed{T_s = \frac{4}{3}}$

b) $G_0(s) = \dfrac{4K_p/s+3}{1 + 4K_p/s+3} = \dfrac{4K_p}{s+3+4K_p}$ $\quad \boxed{T_s = \dfrac{4}{3+4K_p}}$

$e_{ss} = 1 - G_0(0) = 1 - \dfrac{4K_p}{3+4K_p} = \dfrac{3+4K_p-4K_p}{3+4K_p} = \boxed{\dfrac{3}{3+4K_p} = e_{ss}}$

c) $G_0(s) = \dfrac{4K_i/s(s+3)}{1 + 4K_i/s(s+3)} = \dfrac{4K_i}{s^2+3s+4K_i}$ $\quad e_{ss} = 1 - G_0(0) = 1 - 1 = \boxed{0 = e_{ss}}$

3) (20 points) For the following circuit determine the *transfer function* and the corresponding *impulse response*.



$$I_b(s) = I_{in}(s)\,\frac{Ls}{R_b + Ls} \qquad V_{out}(s) = I_b(s)R_b = I_{in}(s)\,\frac{R_b L s}{R_b + Ls}$$

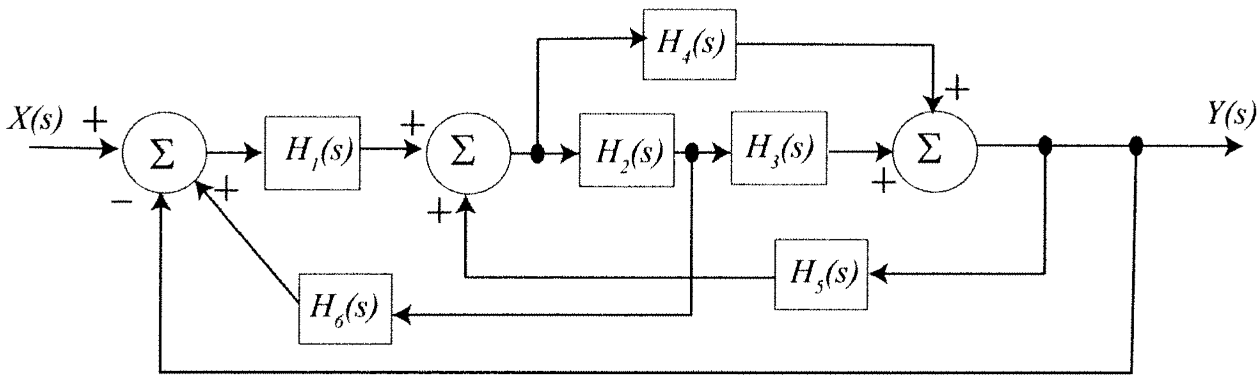
$$H(s) = \frac{V_{out}(s)}{I_{in}(s)} = \boxed{\frac{R_b L s}{Ls + R_b} = H(s)}$$

$$\begin{array}{r|l} & R_b \\ \hline Ls + R_b & R_b L s \\ & R_b L s + R_b^2 \\ \hline & -R_b^2 \end{array}$$

$$H(s) = R_b - \frac{R_b^2}{Ls + R_b} = R_b - \frac{R_b^2}{L}\,\frac{1}{s + R_b/L}$$

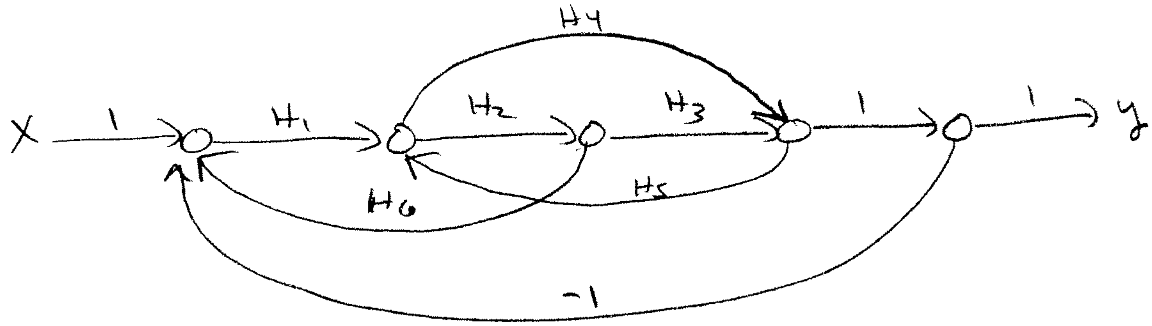
$$\boxed{h(t) = R_b\,\delta(t) - \frac{R_b^2}{L}\,e^{-\frac{R_b}{L}t}\,u(t)}$$

4) (20 points) For the following block diagram



a) Draw the corresponding signal flow graph, labeling each branch and direction. *Feel free to insert as many branches with a gain of 1 as you think you may need.*

b) Determine the system transfer function using Mason's gain rule. You must clearly indicate all of the paths, the loops, the determinant and the cofactors, but you do not need to simplify your final answer (it can be written in terms of the P_i, L_i, and Δ_i)



$P_1 = H_1 H_2 H_3$ $\quad P_2 = H_1 H_4$

$L_1 = H_1 H_2 H_6$ $\quad L_2 = H_4 H_5$ $\quad L_3 = -H_1 H_2 H_3$ $\quad L_4 = -H_1 H_4$ $\quad L_5 = H_2 H_3 H_5$

$\Delta = 1 - (L_1 + L_2 + L_3 + L_4 + L_5)$ $\quad \Delta_1 = 1$ $\quad \Delta_2 = 1$

$$G_o(s) = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

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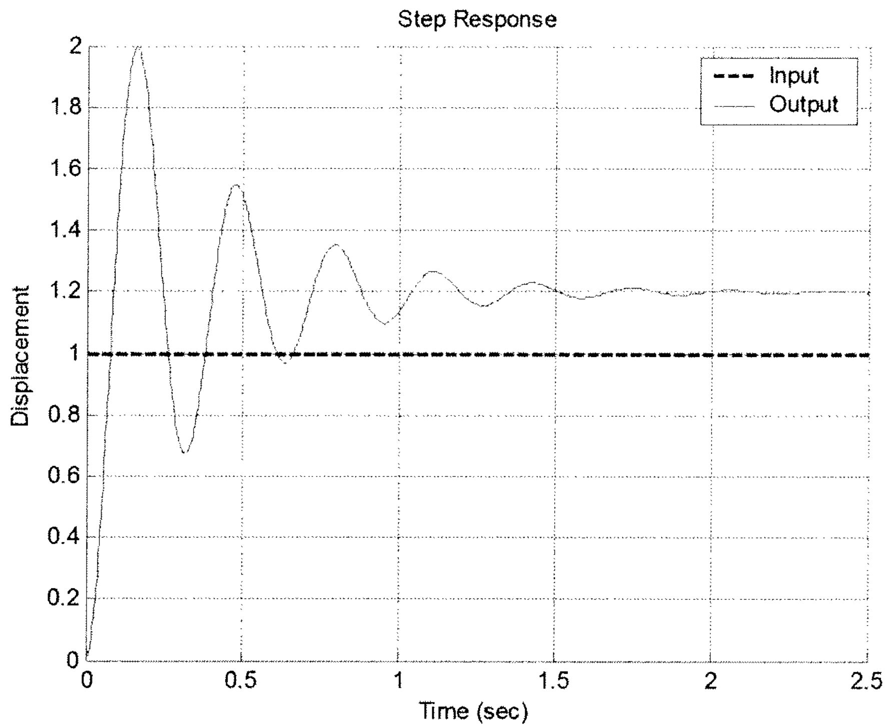
Problems 5-6 refer to the **unit step response** of a system, shown below



5) (3 points) The best estimate of the **steady state error** for a **unit step input** is

a) 0.20 b) -0.20 c) 1.0 d) -0.0 $e_{ss} = 1 - 1.2 = -0.2$

6) (3 points) The best estimate of the **percent overshoot** is $\frac{2-1.2}{1.2} = \frac{0.8}{1.2} \approx 2/3$

a) 200% b) 100% c) 67% d) 20%

7) (3 points) How many of the following transfer functions represents (asymptotically) **stable** systems?

$G_a(s) = \frac{s-1}{s+1}$ $G_b(s) = \frac{1}{s(s+1)}$ $G_c(s) = \frac{s}{s^2-1}$

$G_d(s) = \frac{s+1}{(s+1+j)(s+1-j)}$ $G_e(s) = \frac{(s-1-j)(s-1+j)}{s}$ $G_f(s) = \frac{(s-1-j)(s-1+j)}{(s+1-j)(s+1+j)}$

a) 0 b) 1 c) 1 d) 3 e) 4 f) 5 g) 6

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