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ECE-205

Exam 2

Fall 2010

Calculators and computers are not allowed. You must show your work to receive credit.

Problem 1 _________/15

Problem 2 _________/20

Problem 3 _________/35

Problems 4-13 _______/30 (3 points each)

Total _____________

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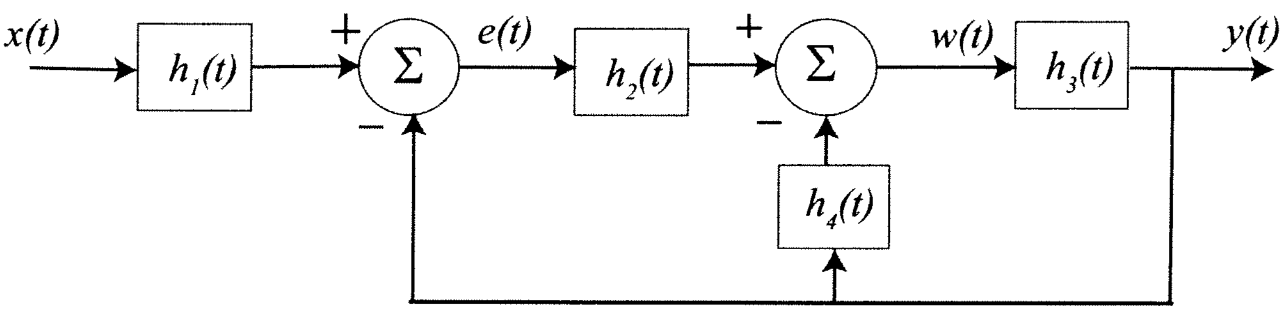
1) (15 points) The input-output relationship for the following system can be written as

$$y(t) * A(t) = x(t) * B(t)$$

Determine $A(t)$ and $B(t)$



$e(t) = x(t) * h_1(t) - y(t)$

$y(t) = [e(t) * h_2(t) - y(t) * h_4(t)] * h_3(t)$

$= e(t) * h_2(t) * h_3(t) - y(t) * h_3(t) * h_4(t)$

$= [x(t) * h_1(t) - y(t)] * h_2(t) * h_3(t) - y(t) * h_3(t) * h_4(t)$

$= x(t) * h_1(t) * h_2(t) * h_3(t) - y(t) * h_2(t) * h_3(t) - y(t) * h_3(t) * h_4(t)$

$y(t) + h_2(t) * h_3(t) * y(t) + h_3(t) * h_4(t) * y(t) = x(t) * h_1(t) * h_2(t) * h_3(t)$

$y(t) * \underbrace{[\delta(t) + h_2(t) * h_3(t) + h_3(t) * h_4(t)]}_{A(t)} = x(t) * \underbrace{[h_1(t) * h_2(t) * h_3(t)]}_{B(t)}$

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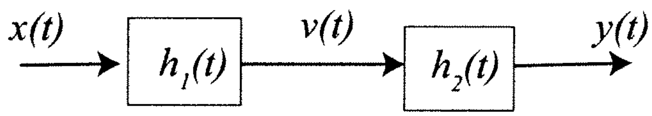
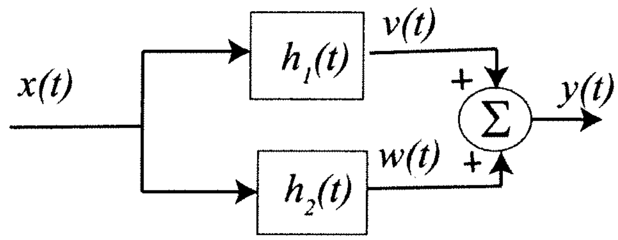
2) (20 points) For the following interconnected systems,

i) determine the overall impulse response (the impulse response between input *x(t)* and output *y(t)*) and

ii) determine if the system is causal.





a) $h_1(t) = \delta(t-2),\ h_2(t) = \delta(t+1)$

b) $h_1(t) = e^{-(t-2)}u(t-2),\ h_2(t) = u(t)$

(a) series $h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(\lambda)\, h_2(t-\lambda)\, d\lambda = \int_{-\infty}^{\infty} \delta(\lambda-2)\, \delta(t-\lambda+1)\, d\lambda = \delta(t-1)$

$\boxed{h(t) = \delta(t-1) \text{ causal}}$

parallel $h(t) = h_1(t) + h_2(t) = \boxed{\delta(t-2) + \delta(t+1) = h(t) \quad \text{non causal}}$

(b) series $h(t) = h_1(t) * h_2(t) = \int_{-\infty}^{\infty} h_1(t-\lambda)\, h_2(\lambda)\, d\lambda = \int_{-\infty}^{\infty} e^{-(t-\lambda-2)}\, u(t-\lambda-2)\, u(\lambda)\, d\lambda$

$= e^{-(t-2)} \int_{0}^{t-2} e^{\lambda}\, d\lambda = e^{-(t-2)} \left[e^{t-2} - 1\right] u(t-2) = \boxed{\left[1 - e^{-(t-2)}\right] u(t-2) = h(t) \quad \text{causal}}$

parallel $h(t) = h_1(t) + h_2(t) = \boxed{e^{-(t-2)} u(t-2) + u(t) = h(t) \quad \text{causal}}$

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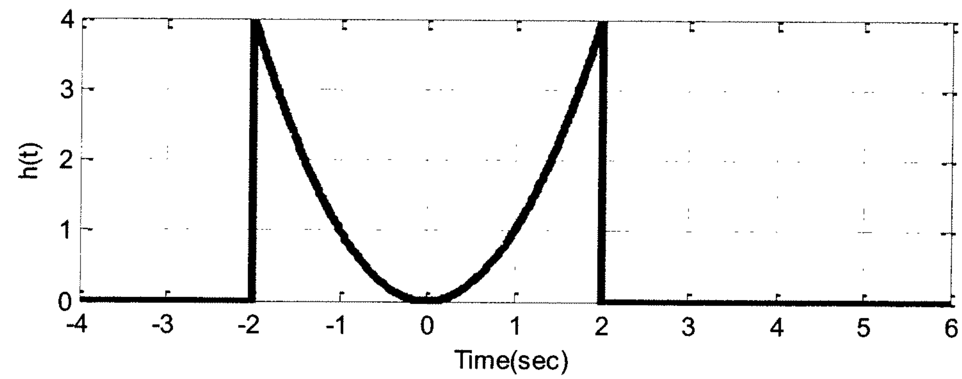
3) (35 points) Consider a linear time invariant system with impulse response given by

$$h(t) = t^2[u(t+2) - u(t-2)]$$

The input to the system is given by

$$x(t) = [u(t-1) - u(t-2)] - 2[u(t-3) - u(t-4)]$$



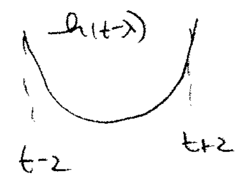
$h(2) = h(t-\lambda)$

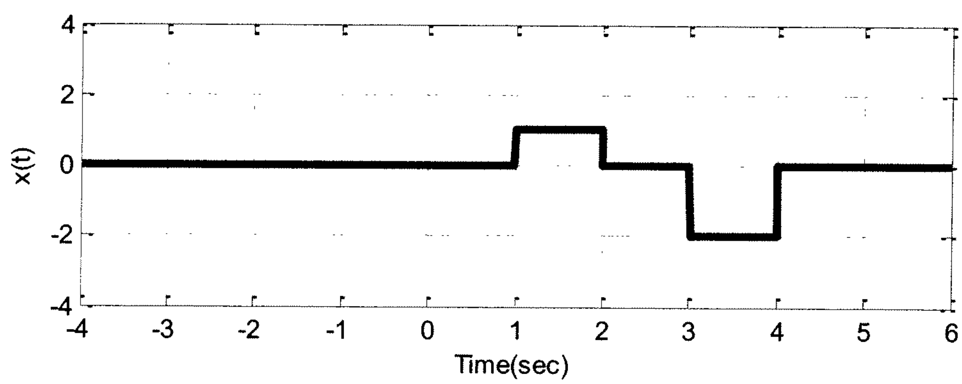
$2 = t - \lambda \quad \lambda = t-2$

$h(-2) = h(t-\lambda)$

$-2 = t - \lambda$

$\lambda = t+2$





Using ***graphical evaluation***, determine the output $y(t)$ Specifically, you must

- Flip and slide $h(t)$, ***NOT*** $x(t)$
- Show graphs displaying both $h(t-\lambda)$ and $x(\lambda)$ for each region of interest
- Determine the range of t for which each part of your solution is valid
- Set up any necessary integrals to compute $y(t)$. Your integrals must be complete, in that they cannot contain the symbols $x(\lambda)$ or $h(t-\lambda)$ but must contain the actual functions.
- Your integrals cannot contain any unit step functions
- ***DO NOT EVALUATE THE INTEGRALS!!***

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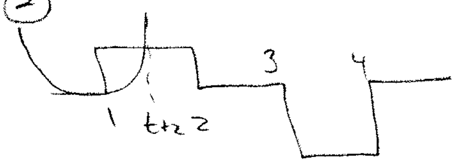
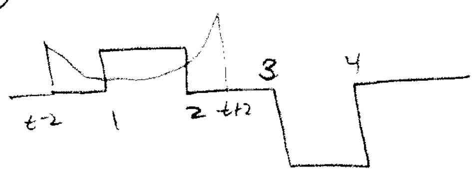
(1) $\boxed{y(t) = 0 \quad t \le -1}$

(2)



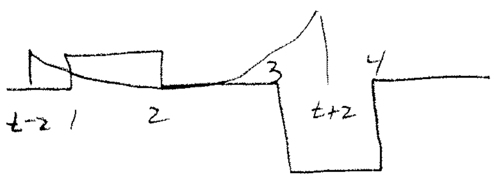
$$y(t) = \int_1^{t+2} (t-\lambda)^2 \, d\lambda \quad -1 \le t \le 0$$

(3)



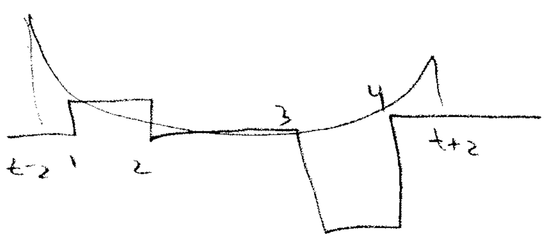
$$y(t) = \int_1^{2} (t-\lambda)^2 \, d\lambda \quad 0 \le t \le 1$$

(4)



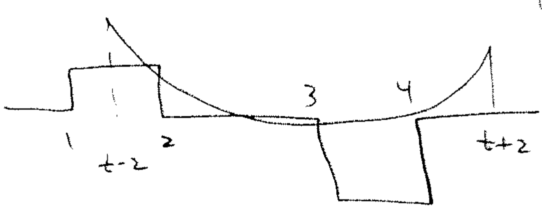
$$y(t) = \int_1^{2} (t-\lambda)^2 \, d\lambda + \int_3^{t+2} (t-\lambda)^2(-2) \, d\lambda \quad 1 \le t \le 2$$

(5)



$$y(t) = \int_1^{2} (t-\lambda)^2 \, d\lambda + \int_3^{4} (t-\lambda)^2(-2) \, d\lambda \quad 2 \le t \le 3$$

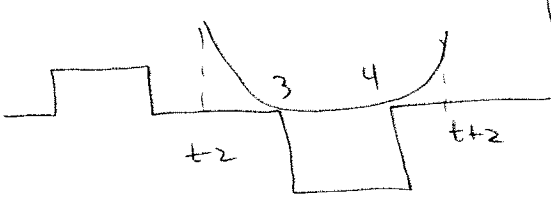
(6)



$$y(t) = \int_{t-2}^{2} (t-\lambda)^2 \, d\lambda + \int_3^{4} (t-\lambda)^2(-2) \, d\lambda \quad 3 \le t \le 4$$

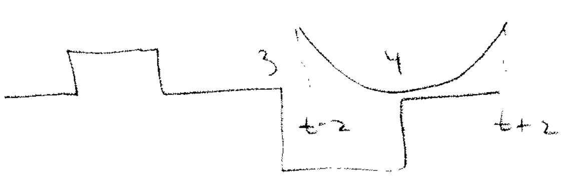
(7)

$$y(t) = \int_3^{4} (t-\lambda)^2(-2) \, d\lambda \quad 4 \le t \le 5$$



(8)

$$y(t) = \int_{t-2}^{4} (t-\lambda)^2(-2) \, d\lambda \quad 5 \le t \le 6$$



(9) $y(t) = 0 \quad t \ge 6$

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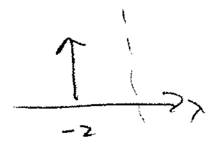
Multiple Choice Problems (30 points, 3 points each)

4) The **impulse response** for the LTI system $y(t) = 2x(t) + \int_{-\infty}^{t-2} e^{-(t-\lambda)} x(\lambda+2)\,d\lambda$ is

a) $h(t) = 2u(t) + e^{-(t+2)}u(t+1)$ b) $h(t) = 2\delta(t) + e^{-(t+2)}u(t+1)$

c) $h(t) = 2\delta(t) + e^{-(t+2)}u(t)$ d) $h(t) = 2\delta(t) + e^{-(t+2)}u(t-2)$

e) $h(t) = 2\delta(t) + e^{-(t+2)}u(t+2)$ f) none of these



$t-2 > -2$

$t > 0$

$h(t) = 2\delta(t) + e^{-(t+2)}u(t)$

5) The **impulse response** for the LTI system $\dot{y}(t) - y(t) = x(t-1)$ is

a) $h(t) = e^{t}u(t)$ b) $h(t) = e^{-t}u(t)$ c) $h(t) = e^{-(t-1)}u(t)$

d) $h(t) = e^{-(t-1)}u(t-1)$ e) $h(t) = e^{(t-1)}u(t-1)$ f) none of these

$\frac{d}{dt}(he^{-t}) = e^{-t}\delta(t-1) = e^{-1}\delta(t-1)$

$h(t)e^{-t} = e^{-1}u(t-1)$

$h(t) = e^{t-1}u(t-1)$

6) For a system with input $x(t)$ and output $y(t)$, is it necessary for $y(t_0) = 0$ in order for the system to be **linear**?

a) Yes b) No

7) For a system with input $x(t)$ and output $y(t)$, is it necessary for $y(t_0) = 0$ in order for the system to be **time-invariant**?

a) Yes b) No

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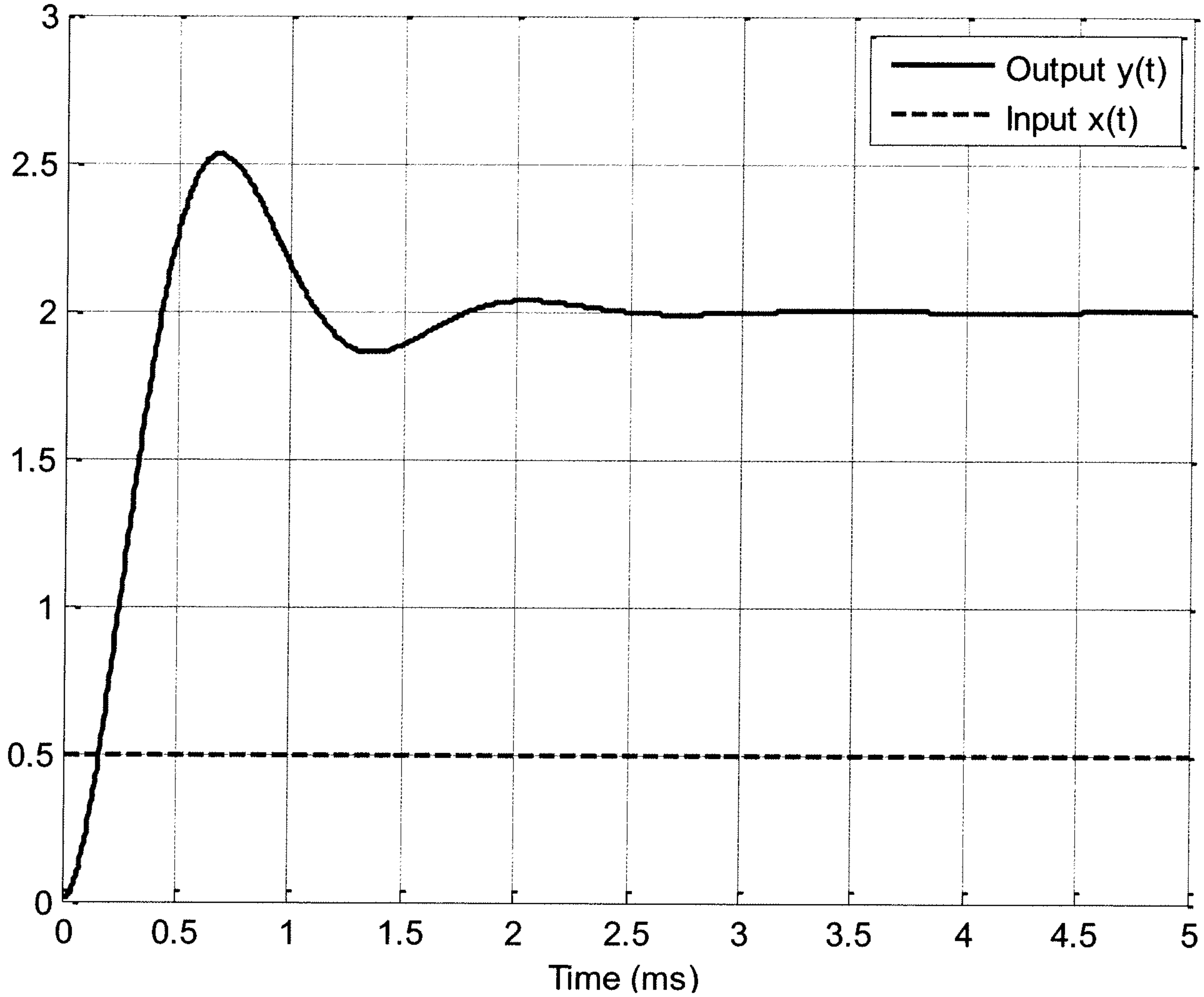
Problems 8-11 refer the following graph showing the response of a second order system to a step input.



8) The percent overshoot for this system is best estimated as $\frac{2.5-2}{2} = \frac{1/2}{2} = 1/4$

a) 400% b) 250 % c) 200% d) 150 % e) 100 % f) 25%

9) The (2%) settling time for this system is best estimated as

a) 1.5 ms b) 2.5 ms c) 4 ms d) 5 ms

10) The static gain for this system is best estimated as $K(\frac{1}{2}) = 2 \quad K = 4$

a) 1 b) 2 c) 3 d) 4

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11) Assume we have a first order system in standard form, and the input is a step. The usual form used to compute the response of the system is

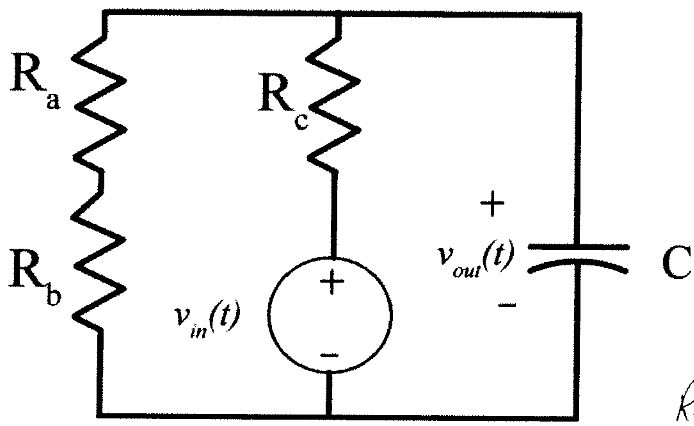
a) $y(t) = [y(0) - y(\infty)]e^{-t/\tau} + y(0)$ b) $y(t) = [y(\infty) - y(0)]e^{-t/\tau} + y(0)$

c) $y(t) = [y(\infty) - y(0)]e^{-t/\tau} + y(\infty)$ d) $y(t) = [y(0) - y(\infty)]e^{-t/\tau} + y(\infty)$

$y(t)\big|_{t=0} = y(0)$

$y(t)\big|_{t=\infty} = y(\infty)$

Problems 12 and 13 refer to the following circuit:



$R_{th} = R_c \,//\, (R_a + R_b)$

12) The Thevenin resistance seen from the ports of the capacitor is

a) $R_{th} = R_a + R_b$ b) $R_{th} = R_c$ c) $R_{th} = R_c \,\|\, (R_a + R_b)$ d) $R_{th} = R_a + R_b + R_c$ e) none of these

13) The static gain for the system is

a) $K = 1$ b) $K = \dfrac{R_c}{R_a + R_b + R_c}$ c) $K = \dfrac{R_a + R_b}{R_a + R_b + R_c}$ d) $K = \dfrac{R_c}{R_a + R_b}$ e) none of these

$$V_c(\infty) = \frac{V_{in}(\infty)(R_a + R_b)}{R_c + R_a + R_b}$$