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ECE-205

Exam 1

Fall 2010

Calculators can only be used for simple calculations. Solving integrals, differential equations, systems of equations, etc. does not count as a simple calculation.

You must show your work to receive credit.

Problem 1	______/10
Problem 2	______/20
Problem 3	______/30
Problem 4-13	______/40

Total ____________

90-100 7
80-89 10
70-79 4
60-69 2
<60 4

median = 83

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1) (10 points) For a first order system described by the differential equation

$$2\dot{y}(t) + 3ty(t) = x(t-1)$$

with $t_0 = 0$ and $y(t_0) = 1$, use integrating factors to solve the differential equation. Include the initial conditions in your solution.

$\dot{y}(t) + \frac{3}{2}t\,y(t) = \frac{1}{2}x(t-1)$ $\qquad \frac{d\phi(t)}{dt} = \frac{3}{2}t \qquad \phi(t) = \frac{3}{4}t^2$

$$\frac{d}{dt}\left(y(t)e^{\frac{3}{4}t^2}\right) = \frac{1}{2}x(t-1)e^{\frac{3}{4}t^2}$$

$$y(t)e^{\frac{3}{4}t^2} - \overset{1}{\cancel{y(t_0)}}\,e^{\overset{0}{\cancel{\frac{3}{4}t_0^2}}} = \int_{t_0}^{t} \frac{1}{2}x(\lambda-1)e^{\frac{3}{4}\lambda^2}\,d\lambda$$

$$y(t)e^{\frac{3}{4}t^2} - 1 = \int_0^t \frac{1}{2}x(\lambda-1)e^{\frac{3}{4}\lambda^2}\,d\lambda$$

$$\boxed{y(t) = e^{-\frac{3}{4}t^2} + e^{-\frac{3}{4}t^2}\int_0^t \frac{1}{2}x(\lambda-1)e^{\frac{3}{4}\lambda^2}\,d\lambda}$$

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2) (20 points) Assume we have a first order system with the governing differential equation

$$0.4\dot{y}(t) + y(t) = 3x(t)$$

The system has the initial value of 1, so $y(0) = 1$. The input to this system is

$$x(t) = \begin{cases} 0 & t < 0 \\ -1 & 0 \le t < 1 \\ 3 & 1 \le t < 2 \\ -2 & 2 < t \end{cases}$$

$\tau = 0.4$

$K = 3$

Determine the output of the system in each of the above time intervals. *Simplify your final answer as much as possible and box it. Be sure to include the correct initial value in the first interval!*

$y(t) = [y(0) - y(\infty)]e^{-t/\tau} + y(\infty)$

(a) $0 \le t < 1$ $\quad y(0) = 1 \quad y(\infty) = -3$

$y(t) = [1 - (-3)]e^{-t/0.4} - 3 = \boxed{4e^{-t/0.4} - 3 = y(t)}$

(b) $1 \le t < 2$ $\quad$ "$y(0)$" $= y(1) = 4e^{-1/0.4} - 3 = -2.67166 \quad y(\infty) = +9$

$y(t) = [-2.67166 - (9)]e^{-(t-1)/0.4} + 9 = \boxed{-11.67166\, e^{-(t-1)/0.4} + 9 = y(t)}$

(c) $2 \le t$ $\quad$ "$y(0)$" $= y(2) = -11.67166\, e^{-(2-1)/0.4} + 9 = 8.04193 \quad y(\infty) = -6$

$y(t) = [8.04193 - (-6)]\, e^{-(t-2)/0.4} - 6 = \boxed{14.04193 e^{-(t-2)/0.4} - 6 = y(t)}$

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3) (30 points) For the following three differential equations, assume the input is $x(t) = 4u(t)$ (the input is equal to four for time greater than zero), and the initial conditions are $y(0) = \dot{y}(0) = 0$

Determine the solution to each of the following differential equations and put your final answer in a box. Be sure to use the initial conditions to solve for all unknowns. You must show all your work to receive credit.

a) $\ddot{y}(t) + 3\dot{y}(t) + 2y(t) = x(t)$ $2y_f = 4$ $\boxed{y_f = 2}$ $r^2 + 3r + 2 = 0$ $(r+1)(r+2) = 0$

$y(t) = c_1 e^{-t} + c_2 e^{-2t} + 2$ $y(0) = 0 = c_1 + c_2 + 2$ adding $\rightarrow$ $-c_2 + 2 = 0$

$\dot{y}(t) = -c_1 e^{-t} - 2c_2 e^{-2t}$ $\dot{y}(0) = 0 = -c_1 - 2c_2$ $\boxed{c_2 = 2}$ $\boxed{c_1 = -4}$

$$\boxed{y(t) = -4e^{-t} + 2e^{-2t} + 2}$$

b) $\ddot{y}(t) + 4\dot{y}(t) + 4y(t) = 8x(t)$ $4y_F = 8 \cdot 4$ $y_f = 8$ $r^2 + 4r + 4 = 0$ $(r+2)(r+2) = 0$

$y(t) = c_1 e^{-2t} + c_2 t e^{-2t} + 8$ $y(0) = 0 = c_1 + 8$ $\boxed{c_1 = -8}$

$\dot{y}(t) = -2c_1 e^{-2t} + c_2 e^{-2t} - 2c_2 t e^{-2t}$ $\dot{y}(0) = 0 = -2c_1 + c_2$ $c_2 = 2c_1 = \boxed{-16 = c_2}$

$$\boxed{y(t) = -8e^{-2t} - 16te^{-2t} + 8}$$

c) $\ddot{y}(t) + 4\dot{y}(t) + 16y(t) = 4x(t)$ $16y_F = 4 \cdot 4$ $y_F = 1$ $r^2 + 4r + 16 = 0$

$r = \frac{-4 \pm \sqrt{16-64}}{2} = -2 \pm j\sqrt{12}$

$y(t) = 1 + ce^{-2t}\sin(\sqrt{12}t + \theta)$

$\dot{y}(t) = -2ce^{-2t}\sin(\sqrt{12}t + \theta) + ce^{-2t}\sqrt{12}\cos(\sqrt{12}t + \theta)$

$\dot{y}(0) = 0 = -2\sin(\theta) + \sqrt{12}\cos(\theta)$ $\tan(\theta) = \frac{\sqrt{12}}{2}$ $\boxed{\theta = 60^\circ}$

$y(0) = 0 = 1 + c\sin(\theta)$ $c = \frac{-1}{\sin(\theta)} = -1.15470$

$$\boxed{y(t) = 1 - 1.15470e^{-2t}\sin(\sqrt{12}t + 60^\circ)}$$

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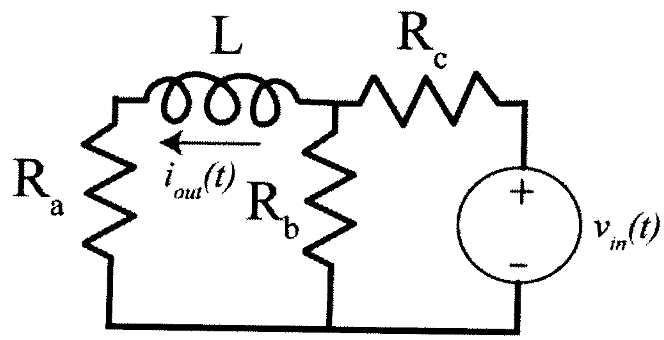
Problems 4-13, 4 points each, no partial credit (40 points)

Problems 4 and 5 refer to the following circuit



4) The Thevenin resistance seen from the ports of the inductor is

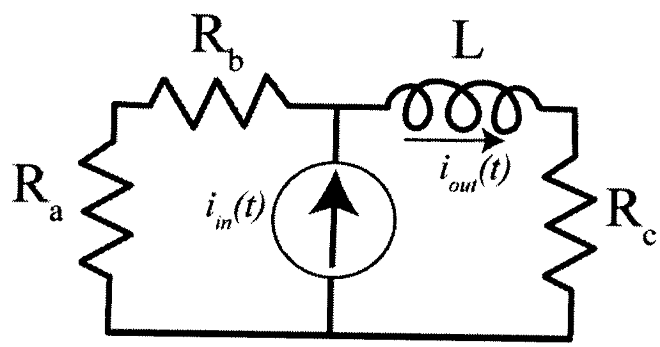
a) $R_{th} = R_a + R_b \| R_c$ b) $R_{th} = R_c + R_a \| R_b$ c) $R_{th} = R_a + R_b$ d) $R_{th} = R_a + R_c$ e) none of these

5) The static gain for the system is

a) $K = 1$ b) $K = \dfrac{R_b}{R_a + R_b}$ c) $K = \dfrac{R_a}{R_a + R_b}$ d) $K = \dfrac{R_b}{R_c + R_b}$ e) none of these

$i_{out} = \dfrac{v_{in}}{R_c + R_a \| R_b} \cdot \dfrac{R_b}{R_a + R_b}$

Problems 6 and 7 refer to the following circuit



6) The Thevenin resistance seen from the ports of the inductor is

a) $R_{th} = R_c \| (R_a + R_b)$ b) $R_{th} = R_c$ c) $R_{th} = R_a + R_b$ d) $R_{th} = R_a + R_b + R_c$ e) none of these

7) The static gain for the system is

a) $K = 1$ b) $K = \dfrac{R_a + R_b}{R_a + R_b + R_c}$ c) $K = \dfrac{R_c}{R_a + R_b + R_c}$ d) $K = \dfrac{R_c}{R_a + R_b}$ e) none of these

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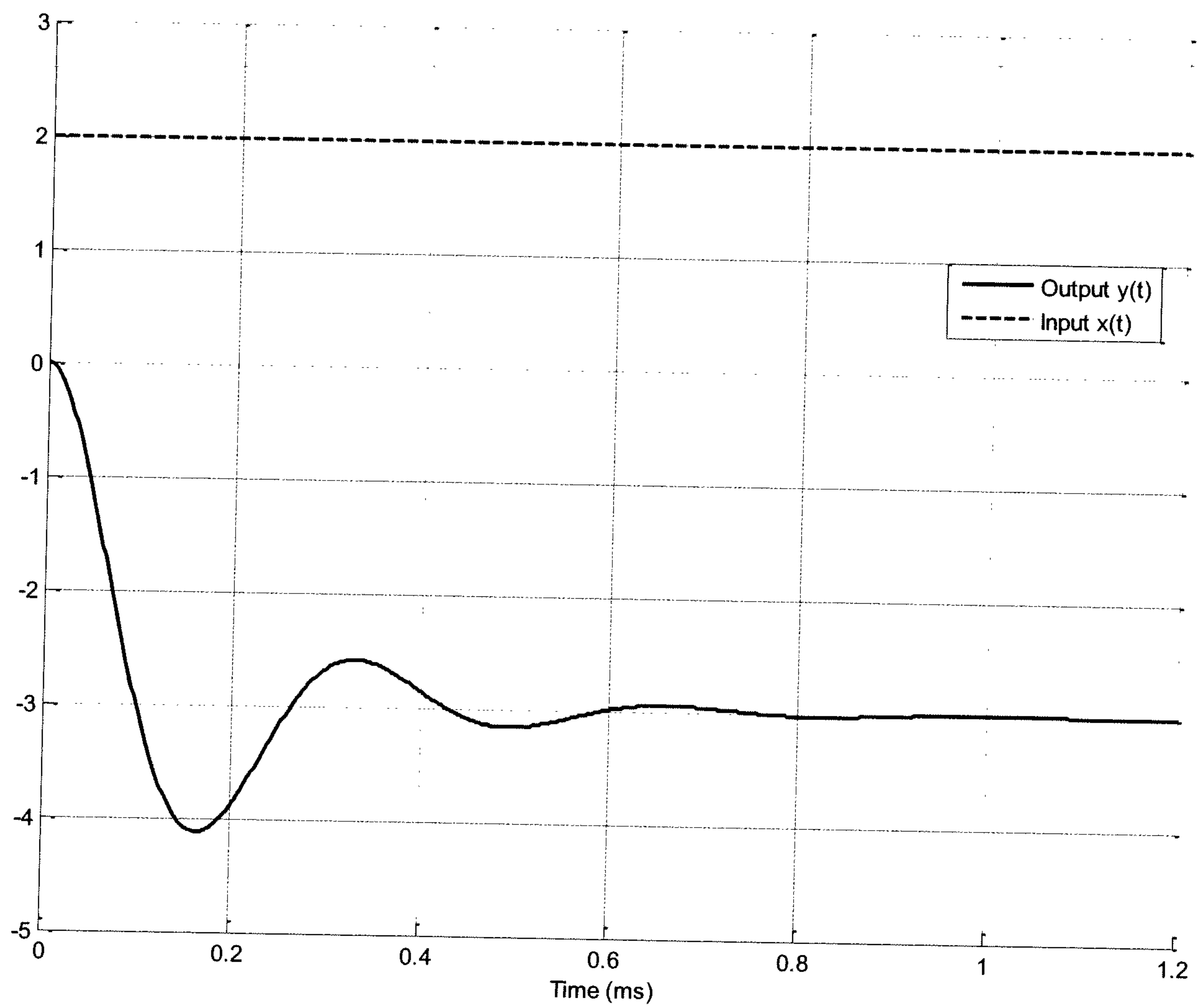
Problems 8-10 refer the following graph showing the response of a second order system to a step input.



8) The percent overshoot for this system is best estimated as

$\frac{-4-(-3)}{-3} = \frac{-1}{-3} = 33\%$

a) 400% b) -400 % c) 300% d) -300 % e) -33% f) 33%

9) The (2%) settling time for this system is best estimated as

a) 0.3 ms b) 0.6 ms c) 1.0 ms d) 1.2 ms

10) The static gain for this system is best estimated as

$K2 = -3$

$K = -3/2$

a) 1.5 b) 3 c) -1.5 d) -3

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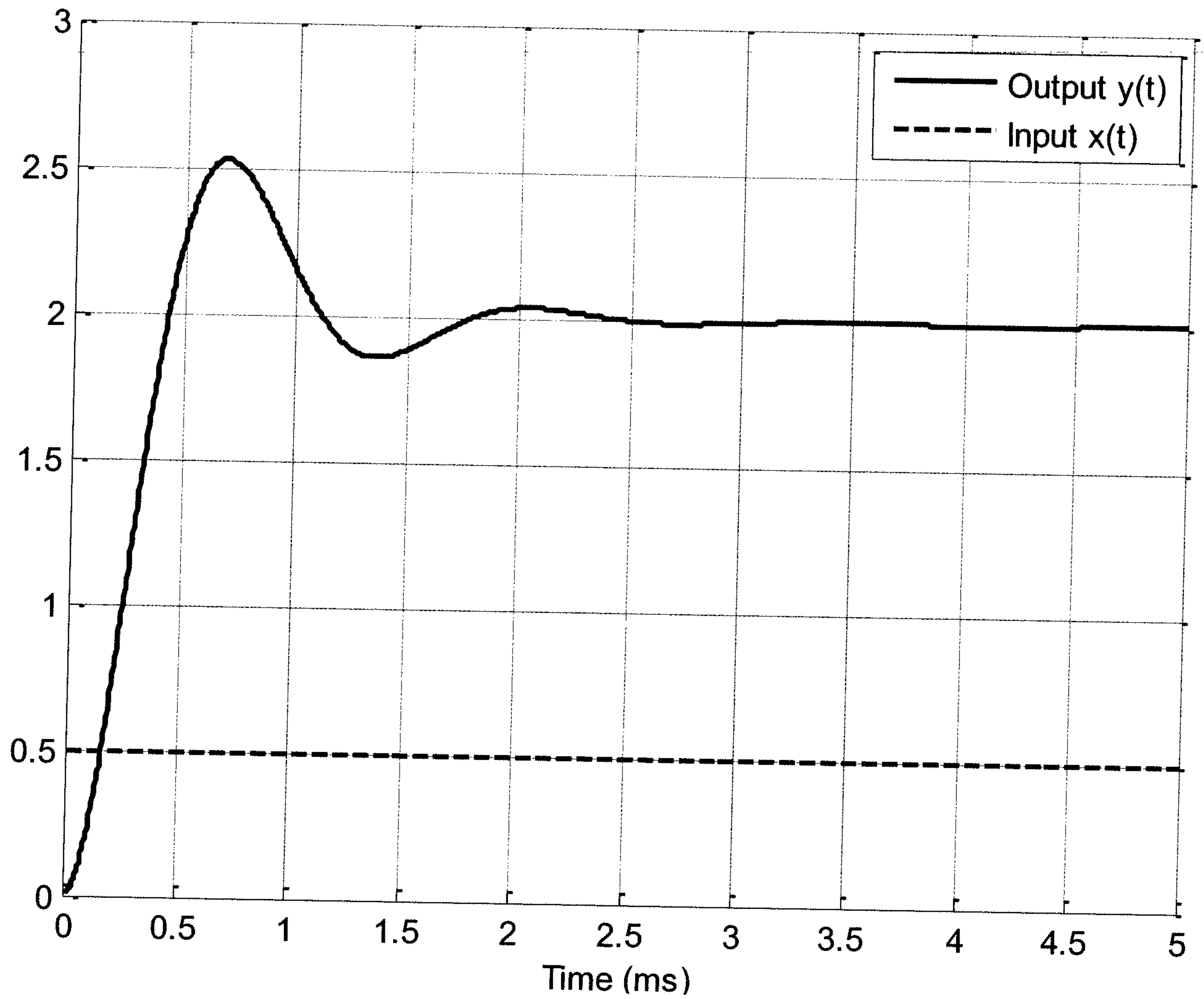
Problems 11-13 refer the following graph showing the response of a second order system to a step input.



11) The percent overshoot for this system is best estimated as

a) 400% b) 250 % c) 200% d) 150 % e) 100 % f) 25%

$\frac{2.5-2}{2} = \frac{0.5}{2} = 25\%$

12) The (2%) settling time for this system is best estimated as

a) 1.5 ms b) 2.5 ms c) 4 ms d) 5 ms

$K(0.5) = 2$

13) The static gain for this system is best estimated as

a) 1 b) 2 c) 3 d) 4

$K = \frac{2}{0.5} = 4$