ECE-497/BME-491: Applied Biomedical Signal Processing

Homework #7
Due at the beginning of class, February 16, 2007

In this problem we will investigate solving one form of the *inverse problem of electrocardiography* using truncated singular value decomposition and zero order Tikhonov regularization. A catheter has been inserted (through a vein in the leg) into the left ventricle of the heart, and a small balloon on the end of the catheter has been inflated. There are 64 electrodes on the outside of the probe which record electrical potentials. A finite element model has been generated from which the transfer matrix from the endocardium (the inside of the heart surface) to the probe has been generated. The goal is to estimate the endocardial potentials from the potentials measured on the probe and the knowledge of the transfer matrix. In order to check ourselves, nine endocardial potentials have been measured. Our estimates will be compared to these estimates. The estimated electrograms will be plotted on the actual (measured) electrograms, the correlation coefficient between the estimated and measured electrograms will be computed, and the average correlation coefficient will be computed. A plot of the singular values will also be produced, and you will see that in this problem the singular values do not fall off very rapidly.

1) The program *probe_tsvd.m* uses the TSVD method for estimating the endocardial potentials. The only argument to this program is the number of modes (from 1 to 64) to use in the TSVD method. The program *probe_tiki.m* uses zero order Tikhonov regularization to determine the endocardial potentials. The only argument to this program is the regularization parameter μ.

a) Use all 64 modes in the TSVD method, turn in your plot.

b) Determine the number of modes to use in the TSVD expansion to produce the good results (correlation coefficient above 0.9, at least). Turn in your plot. Does the correlation coefficient look like a good measure to use to determine if the estimated electrograms are close to the measured electrograms?

c) Use the program *probe_tiki.m* to produce a good estimate of the endocardial electrograms. Turn in your plot.

In the next two problems we will look at the use of zero and first order Tikhonov regularization for solving a different version of the *inverse problem of electrocardiography.* In these problems electrical potentials have been measured on the epicardium (the outer heart surface) of a pig. A finite element model for the torso (from the epicardium to the body surface) of the pig has been generated and a transfer matrix $\mathbf{Z}$ was constructed. The heart potential data was multiplied by the transfer matrix to construct body surface potentials. Although we have generated the system output data (the $\underline{y}$ vector), due to the poor condition number of the transfer matrix (approximately 5×10^8 in this case) we will not be able to accurately estimate the epicardial potentials, even with a very large SNR. There are six different cardiac activation patterns (or protocols) since the heart was paced from six different sites. These protocols were labeled 0-5. The input to the files are the SNR on the output signal, the protocol to use, and the regularization parameter μ. The output is a figure displaying both true (measured) and estimated electrograms on the epicardial surface that we computed using one of the Tikhonov algorithms, the relative error for each electrogram, and the average relative error (written to the screen). The relative error is one of the common methods of estimating the errors, and is computed as

$$RE = \frac{||\underline{x} - \hat{\underline{x}}||}{||\underline{x}||}$$

where $\underline{x}$ represents the true (measured) epicardial potentials and $\hat{\underline{x}}$ represents the estimated epicardial potentials. A small relative error value is good.

2) In this problem we will use the program *epi_tiki.m*, which solves the inverse problem using zero order Tikhonov regularization. Zero order Tikhonov regularization penalizes the amplitude of the estimated epicardial electrograms. Choose two different protocols to use for each part of this problem.

a)Set the SNR to 50 dB and the regularization parameter to zero. Run the programs and record your average relative errors. As you will see there is no real reason to turn in the plots.

b) Keep the SNR at 50 dB, and set the regularization parameter to 1.0. You should have what are clearly over regularized solutions. Describe what you see.

c) Keep the SNR at zero and try to find a good solution. This entails two things: (1) a good value for the average relative error (a good value is a small value), and (2) estimated electrograms that look like the real electrograms (as much as possible). It should be clear a good value for μ is between 0 and 1. *Turn in your plots.*

3) In this problem we will use the program *epi_tikk.m*, which solve the inverse problem using first order Tikhonov regularization. First order Tikhonov regularization penalizes the spatial slope of the estimated epicardial electrograms. This program is *slow....*, so be patient. For a SNR of 50 dB, determine values of the regularization parameter that produces good estimates of the epicardial electrograms for the two protocols used in problem 2. *Turn in your plots.*

4) In this problem we will show how to use generalized singular value decomposition for *higher order* Tikhononv regularization. For zero order Tikhonov regularization we penalize the amplitude of the estimated value, while for higher order regularization we penalize some function of the estimated value. Typically for first order regularization we penalize the slope, for second order regularization we penalize the second derivative, and so on. Higher order Tikhonov regularization can be formulated as finding the estimate $\underline{x}$ to minimize

$$\min \Pi = ||\underline{y} - \mathbf{Z}\underline{x}||^2 + \mu||\mathbf{R}\underline{x}||^2$$

where $\mathbf{R}$ is a regularization operator, and μ is the regularization parameter.

a) By going through the mathematics, show that the solution is given by

$$\underline{x} = \left(\mathbf{Z}^T\mathbf{Z} + \mu\mathbf{R}^T\mathbf{R}\right)^{-1}\mathbf{Z}^T\underline{y}$$

b) Using the generalized singular value decomposition (gsvd), we can write

$$\begin{aligned}\mathbf{Z} &= \mathbf{U}\mathbf{C}\mathbf{X}^T \\ \mathbf{R} &= \mathbf{V}\mathbf{S}\mathbf{X}^T\end{aligned}$$

where $\mathbf{U}$ and $\mathbf{V}$ are *unitary* matrices, $\mathbf{C}$ and $\mathbf{S}$ are *diagonal* matrices with elements s_i and c_i on the diagonals, and

$$\mathbf{C}^T\mathbf{C} + \mathbf{S}^T\mathbf{S} = \mathbf{I}$$

Assuming $\mathbf{X}^{-1}$ exists, and defining

$$\begin{aligned}\underline{\tilde{y}} &= \mathbf{U}^T\underline{y} \\ \underline{\tilde{x}} &= \mathbf{X}^T\underline{x} \\ \underline{x} &= \mathbf{X}^{-T}\underline{\tilde{x}}\end{aligned}$$

show that we can write

$$\tilde{x}_i = \frac{c_i\tilde{y}_i}{c_i^2 + \mu s_i^2}$$

The *generalized singular values* are given by $\gamma_i = \frac{c_i}{s_i}$. Although Matlab has routines to compute the gsvd, it is a bit more involved than it appears here to actually implement this.