ECE 130 HW\#5 - Due Tuesday, March 23

1. Consider the following truth table where the four-bit number $\mathrm{A}(\mathrm{A} 3, \mathrm{~A} 2, \mathrm{~A} 1, \mathrm{~A} 0)$ is input and the four-bit number $\mathrm{X}(\mathrm{X} 3, \mathrm{X} 2, \mathrm{X} 1, \mathrm{X} 0)$ is output:

| $\mathbf{A 3}$ | $\mathbf{A 2}$ | $\mathbf{A 1}$ | $\mathbf{A} \mathbf{0}$ | $\mathbf{X 3}$ | $\mathbf{X 2}$ | $\mathbf{X 1}$ | $\mathbf{X 0}$ |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 0 | 0 | 0 | 0 | 1 | 1 | 1 | 1 |
| 0 | 0 | 0 | 1 | 1 | 1 | 0 | 0 |
| 0 | 0 | 1 | 0 | 1 | 0 | 0 | 1 |
| 0 | 0 | 1 | 1 | 0 | 1 | 1 | 0 |
| 0 | 1 | 0 | 0 | 0 | 1 | 0 | 1 |
| 0 | 1 | 0 | 1 | 0 | 0 | 1 | 1 |
| 0 | 1 | 1 | 0 | 0 | 0 | 1 | 0 |
| 0 | 1 | 1 | 1 | 0 | 0 | 1 | 0 |
| 1 | 0 | 0 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 0 | 1 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 0 | 0 | 0 | 0 | 1 |
| 1 | 0 | 1 | 1 | 0 | 0 | 0 | 1 |
| 1 | 1 | 0 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 0 | 1 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 0 | 0 | 0 | 0 | 0 |
| 1 | 1 | 1 | 1 | 0 | 0 | 0 | 0 |

a. Write down the correct functional representation for each of the four outputs in the canonical sigma notation:

$$
\begin{aligned}
& \mathrm{X} 3(\mathrm{~A} 3, \mathrm{~A} 2, \mathrm{~A} 1, \mathrm{~A} 0)=\Sigma( \\
& \mathrm{X} 2(\mathrm{~A} 3, \mathrm{~A} 2, \mathrm{~A} 1, \mathrm{~A} 0)=\Sigma( \\
& \mathrm{X} 1(\mathrm{~A} 3, \mathrm{~A} 2, \mathrm{~A} 1, \mathrm{~A} 0)=\Sigma( \\
& \mathrm{X} 0(\mathrm{~A} 3, \mathrm{~A} 2, \mathrm{~A} 1, \mathrm{~A} 0)=\Sigma(
\end{aligned}
$$

b. Write down the correct functional representation for each of the four outputs in the canonical pi notation:

$$
\begin{aligned}
& \mathrm{X} 3(\mathrm{~A} 3, \mathrm{~A} 2, \mathrm{~A} 1, \mathrm{~A} 0)=\Pi( \\
& \mathrm{X} 2(\mathrm{~A} 3, \mathrm{~A} 2, \mathrm{~A} 1, \mathrm{~A} 0)=\Pi( \\
& \mathrm{X} 1(\mathrm{~A} 3, \mathrm{~A} 2, \mathrm{~A} 1, \mathrm{~A} 0)=\Pi( \\
& \mathrm{X} 0(\mathrm{~A} 3, \mathrm{~A} 2, \mathrm{~A} 1, \mathrm{~A} 0)=\Pi(
\end{aligned}
$$

c. Expand the canonical sigma notation for X 3 into a canonical sum-of-products equation (do not attempt to simplify)
d. Expand the canonical pi notation for X 0 into a canonical product-of-sums equation (do not attempt to simplify)
e. Using Boolean algebra, simplify the canonical sum-of-products equation for X3 (from part c) into a sum-of-products equation that has the fewest possible literals and operators. (Note that there may be a more simplified form that is not sum-of-products.)
2. Prove Theorem Nine ...

$$
A *(A+B)=A
$$

a. ... using only other theorems (justify each step with the theorem number, like T1):
b. ... using only postulates

