

ECE 351 Lab 7

Frequency Response of Networks

In this lab we will measure the frequency response of the passive networks shown in Figure 1. For capacitors, assume $\pm 20\%$ tolerance for ceramic and polyester capacitors and -20% , $+80\%$ tolerance for electrolytic capacitors.

V.A.Pole-Zero Calculations

The calculations for Circuit 1 in Figure 1 are straight forward and will not be discussed. Circuit 2 has high and low frequency poles and is a little more involved. There are two methods for finding the poles of Circuit 2.

V.A.1.Separation of Poles

In Circuit 2 we note that $C_1 \gg C_2$ so that the poles due to C_1 and C_2 may be far apart. We also note that C_1 is in series with the signal path and looks like a coupling capacitor. Thus, we shall treat C_1 as a coupling capacitor and assume that it rolls off the gain at low frequency. We see that C_2 is in parallel with R_2 and rolls off the gain at high frequency. C_2 looks like c_π in a common-emitter amplifier circuit.

We shall treat C_1 as a low frequency capacitor and C_2 as a high frequency capacitor. **This is an approximation.** When we are finding the pole due to C_1 , we will assume that C_2 is so small that it is an open circuit. When we are finding the pole due to C_2 , we shall assume that C_1 is so large that it is a short. This is the method we use to find the poles in an amplifier. When we find the poles due to coupling and bias capacitors (low frequency poles) we assume that c_π and c_μ are open circuits. When we find the poles due to c_π and c_μ we assume that all coupling and bias capacitors are short circuits.

To find the pole due to C_1 , we assume that C_2 is an open circuit and obtain the circuit of Figure 2. The pole due to C_1 is at

$$\omega_1 \cong \frac{1}{C_1(R_1 + R_2)} = 500r/s$$

To find the pole due to C_2 , we assume that C_1 is a short circuit and obtain the circuit of Figure 3. The pole due to C_2 is at

$$\omega_1 \cong \frac{1}{C_2(R_1 \parallel R_2)} = 200,000r/s$$

V.A.2.Laplace Transforms

Next we will use Laplace transforms to find the poles and zeros. When using Laplace transforms the impedance of a capacitor is $Z_C = 1/sC$. Thus, the impedances of capacitors C1 and C2 are $Z_{C1} = 1/sC_1$ and $Z_{C2} = 1/sC_2$. Let $Z_x = Z_{C2} \parallel R_2$, so

$$\begin{aligned} Z_{C2} &= Z_{C2} \parallel R_2 \\ &= \frac{R_2 \left(\frac{1}{sC_2} \right)}{R_2 + \left(\frac{1}{sC_2} \right)} \\ &= \frac{R_2}{1 + sC_2R_2} \end{aligned}$$

The voltage gain of Circuit 2 can be calculated from the voltage divider equation,

$$\frac{v_o}{v_s} = \frac{Z_x}{Z_x + Z_{C1} + R_1}$$

Substituting, we get

$$\begin{aligned} \frac{v_o}{v_s} &= \frac{Z_x}{Z_x + Z_{C1} + R_1} \\ &= \frac{\left(\frac{R_2}{1 + sC_2R_2} \right)}{\left(\frac{R_2}{1 + sC_2R_2} \right) + R_1 + \frac{1}{sC_1}} \\ &= \frac{sC_1R_2}{sC_1R_2 + sC_1R_1(1 + sC_2R_2) + (1 + sC_2R_2)} \\ &= \frac{sC_1R_2}{s^2(C_1C_2R_1R_2) + s(C_1R_1 + C_1R_2 + C_2R_2) + 1} \\ &= \frac{s/C_2R_1}{s^2 + s \left[\frac{C_1R_1 + C_1R_2 + C_2R_2}{C_1C_2R_1R_2} \right] + \left[\frac{1}{C_1C_2R_1R_2} \right]} \end{aligned}$$

To find the zeros, we set the numerator to zero and solve for s ,

$$\frac{s}{C_2 R_1} = 0$$

Thus, there is one zero at $s_o = 0$, or $\omega = 0$.

To find the poles, we set the denominator to zero and solve for s ,

$$s^2 + s \left[\frac{C_1 R_1 + C_1 R_2 + C_2 R_2}{C_1 C_2 R_1 R_2} \right] + \left[\frac{1}{C_1 C_2 R_1 R_2} \right] = 0$$

This is a quadratic equation in s . Solving for s we get,

$$s = \frac{1}{2} \left[- \left[\frac{C_1 R_1 + C_1 R_2 + C_2 R_2}{C_1 C_2 R_1 R_2} \right] \pm \sqrt{\left(\left[\frac{C_1 R_1 + C_1 R_2 + C_2 R_2}{C_1 C_2 R_1 R_2} \right] \right)^2 - \left[\frac{4}{C_1 C_2 R_1 R_2} \right]} \right]$$

Plugging in the numerical values we get, $s = -498.75$ r/s and $s = -200,501.25$ r/s. These correspond to poles at $\omega = 498.75$ r/s and $\omega = 200,501.25$ r/s. These numerical results agree closely with the approximate answers we found using separation of poles.

V.B. Pre-Lab Calculations

We will be using the circuits of Figure 1. For these circuits make the following calculations and plots before coming to lab.

1. Obtain the transfer function $\frac{v_o}{v_s}(j\omega)$ or $\frac{v_o}{v_s}(s)$ for each circuit. Give the function in terms of the circuit variables (no numbers — use labels such as R_1 , R_2 , C_1 , and C_2). Identify the poles and zeros from your transfer function.
2. Plug in circuit component values and obtain numerical values for the maximum and minimum values of the poles and zeros. Record the results in **Table 3** and **Table 4**.
3. Using MATLAB, generate magnitude Bode plots of the two transfer functions. Plot each transfer function on a separate plot. Find the numerical values of the poles and zeros from the Bode plot and record the values in **Table 3** and **Table 4**.

- Use the PSpice AC analysis to obtain Bode magnitude plots for each circuit. Find the numerical values of the poles and zeros from the Bode plot and record the values in **Table 3** and **Table 4**. Use the cursors to find and label the numerical values of the poles on the plot. (See pages 232 to 240 of the PSpice manual for help.)

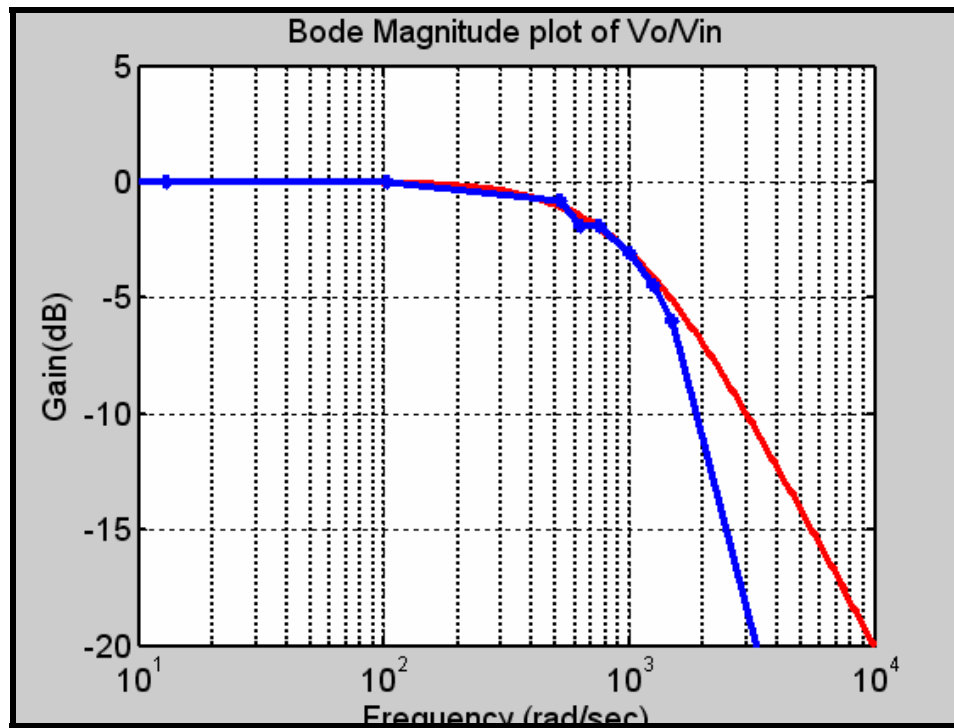
V.C.Laboratory Procedure

Wire Circuit 1 shown in Figure 1.

- With channel A measure v_s . With channel B measure v_o . For each gain measurement, measure both the input and output waveforms because the signal generator's output may change with frequency.
- Measure v_s and v_o for several (**at least 20**) frequencies and fill in **Table 1**. **To make measurements easier, you can use a large input (as large as you want) because this is a linear circuit.**
- Specifically attempt to measure the location of the poles and zeros. Remember that a pole occurs when the gain is down to 0.707 times the original gain and a zero occurs when the gain is up to 1.414 times the original gain. Record the location of the poles and zeros. Find the frequency of the poles and zeros and record the values in **Table 3**.
- Using MATLAB, plot the measured data and the theoretical transfer curve in a Bode magnitude plot. Plot the theoretical transfer curve with a solid blue line. Plot the measured data with red x's at each individual measured point. The Matlab code segment below generates a Bode Magnitude plot of the function $\frac{V_o}{V_{in}} = \frac{1000}{1000 + j\omega}$ and a plot of the measured data on a single graph.

```
set(0,'defaultlinewidth',3);
set(0,'defaultaxesfontname','Arial');
set(0,'defaultaxesfontsize',14);
set(0,'defaultaxeslinewidth',2);
set(0,'defaulttextfontname','Arial');
omega=logspace(1,4,200);
mag=20*log10(abs(1000./(i*omega+1000)));
measured_Omega=[1, 13,103,523,628, 750,1000, 1250, 1500,10000];
measured_gain=[1 1 1 0.91 0.8 0.8 0.7 0.6 0.5 0.01];
measured_mag=20*log10(measured_gain);
semilogx(omega, mag, 'r', measured_Omega, measured_mag, 'b*-');
axis([10,10000,-20 5]);
grid on;
```

```
ylabel('Gain(dB)');  
xlabel('Frequency (rad/sec)');  
title('Bode Magnitude plot of Vo/Vin');
```



Repeat the above procedure for Circuit 2 in Figure 1. Record the measured data in **Table 2** and **Table 4**.

Table 1: Measured data for Circuit 1 of Figure 1

Frequency (Hz)	$\omega \left(\frac{r}{s} \right)$	v_s (Can be large.)	v_o	Gain $\left(\frac{v_o}{v_s} \right)$	Gain in Decibels $20 \text{Log}_{10} \left(\frac{v_o}{v_s} \right)$

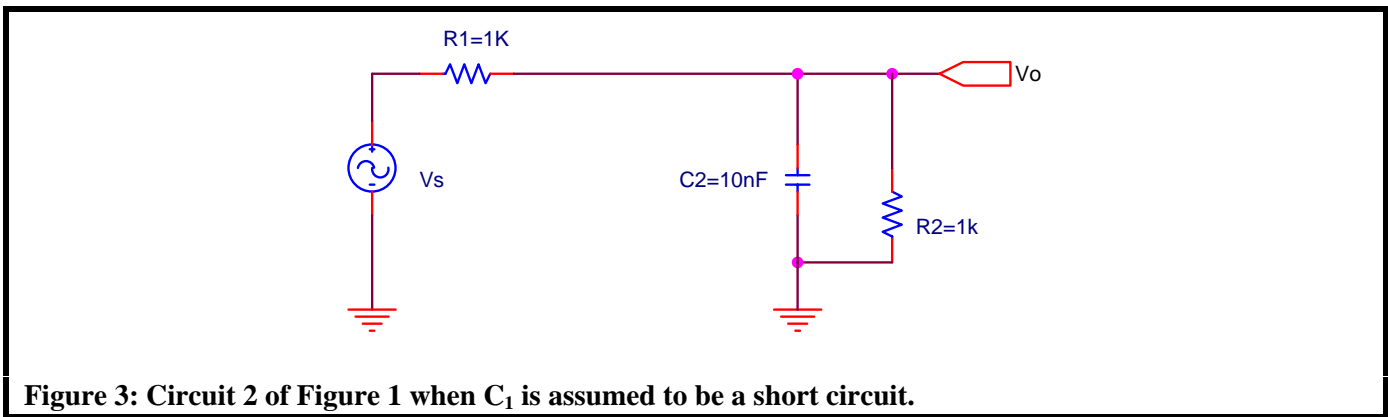
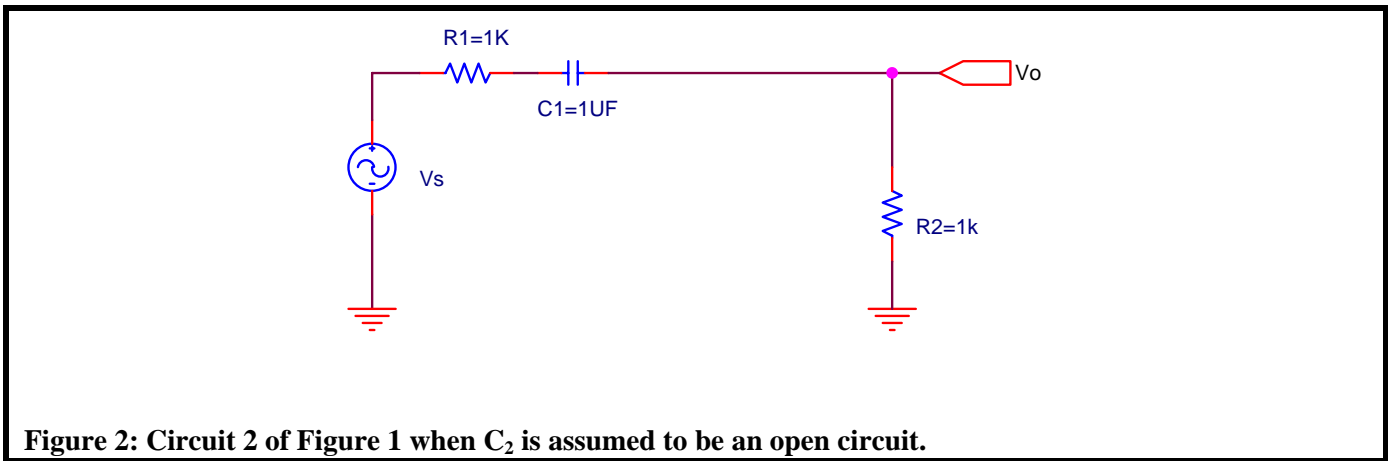
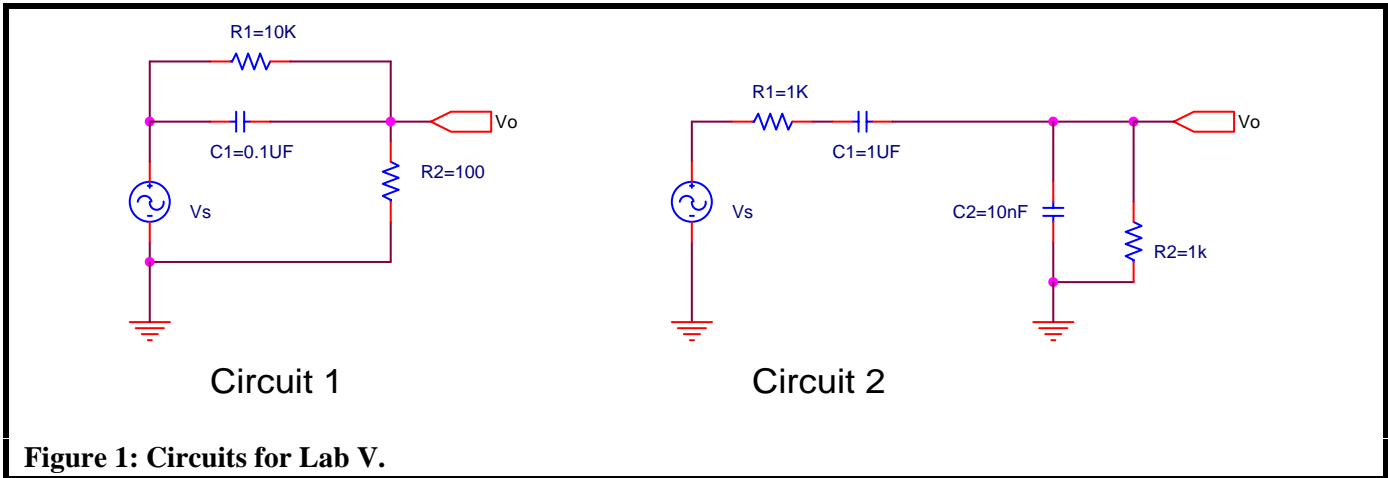


Table 3: Results Summary for Circuit 1				
	Frequency of Pole		Frequency of Zero	
	Radians per second	Hz	Radians per second	Hz
Calculated Maximum				
Calculated Minimum				
Numerical Value from MATLAB				
Numerical Value from PSpice				
Measured Value				

Table 4: Results Summary for Circuit 2				
	Low Frequency Pole		High Frequency Pole	
	Radians per second	Hz	Radians per second	Hz
Calculated Maximum				
Calculated Minimum				
Numerical Value from MATLAB				
Numerical Value from PSpice				
Measured Value				