Department of Electronic Engineering
ELE2EMI
Electronic Measurements & Instrumentation

# 2 Bridge Circuits

Chapter references:

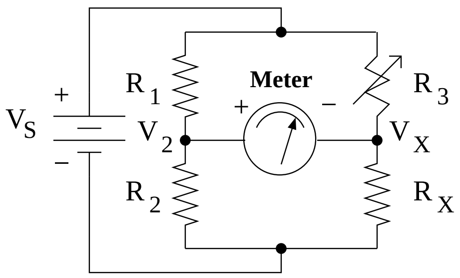
- *Berlin & Getz*, chapter 9.
- *Carr*, chapter 4.

Definition: Bridges are electrical circuits for performing null measurements on resistances in DC (direct current) and general impedances in AC (alternating current).

## 2.1 DC Bridges

### 2.1.1 Wheatstone Bridge

The DC bridge shown below is known as the Wheatstone bridge. It is used to obtain a precise measurement (accurate to about $0.1\%$) of a resistor $R_X$. (Usually it will be represented in a diamond shape, but it's easier for me to draw the resistors vertically or horizontally, and i think the circuit is slightly easier to understand this way.)




This bridge consists of two resistor branches: on the left are two precision resistors $R_1$ and $R_2$. On the right are a precisely calibrated variable resistor $R_3$ and an unknown (or imprecisely known) resistor $R_X$. If the current flowing between the mid-points of the branches happens to be zero, then the branches will function as accurate voltage dividers, in which case the voltage across $R_2$ will be

$$V_2 = \frac{R_2}{R_1 + R_2} V_S$$

and this will equal the voltage across the unknown resistor $R_X$ which will be

$$V_X = \frac{R_X}{R_X + R_3}$$

We obtain this equality by carefully adjusting $R_3$.

In the diagram a sensitive current meter, a *galvanometer,* is used to detect equality of the voltages. In order not to risk damage to the meter, it's advisable to obtain approximate balance before inserting the meter: we can estimate a safe starting value for $R_3$ by first measuring $R_X$ approximately using a less precise instrument such as digital multimeter.

A *zero-centre* galvanometer is the most suitable type, as it can measure small changes in either direction as we adjust $R_3$.

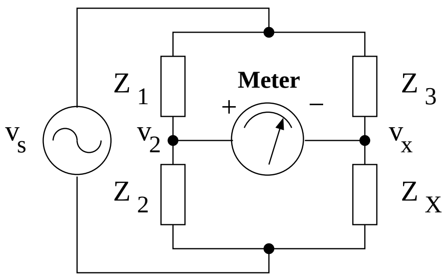
If the meter has no significant *zero error*, then when it reads zero current the ratios of the resistors in both dividers must agree, leading to the *null condition* formula:

$$R_1 R_X = R_2 R_3$$

As *Carr* section 4-5 explains, the current through the galvanometer is proportional to any small change in the value of one of the resistors. This enables the indirect measurement of quantities that affect the resistivity of certain materials. Such *transducers* are described in more detail in a subsequent lecture.

## 2.2 AC Bridges

The general alternating current (AC) bridge is used to measure inductors and capacitors. It has the same form as a Wheatstone bridge, but the voltage source is now AC, general impedances replace the four resistances, and the null detector is no longer a galvanometer but an instrument suited to AC measurement.




The form of the null condition remains the same:

$$Z_1 Z_X = Z_2 Z_3$$

but these are now complex quantities, so this leads to two balance equations.

The resistive component is usually balanced in the same way as in the Wheatstone bridge using a precisely calibrated variable resistance, whereas the reactive component is balanced by a similar impedance (capacitance for capacitance, or inductance for inductance) in an adjacent arm of the bridge, or by an opposite impedance (inductance balancing capacitance) in an opposite arm.

Variable inductors and capacitors are not so easy to make as variable resistors, so to obtain the effect of a variable precision reactance, most circuits use a fixed precision capacitor with a variable resistance (either in series or in parallel with it).

### 2.2.1 AC Null Detectors

For precise measurement of a null condition in the presence of time-varying voltages and currents, the galvanometer must be replaced by instruments such as AC voltmeters or oscilloscopes. (As many of you will recall from Electronics 1, this school makes much use of oscilloscopes for AC measurements.)

Differential amplifiers (in addition to those already contained in the oscilloscope) can be employed to increase the sensitivity of the measurement.

### 2.2.2 AC Isolation

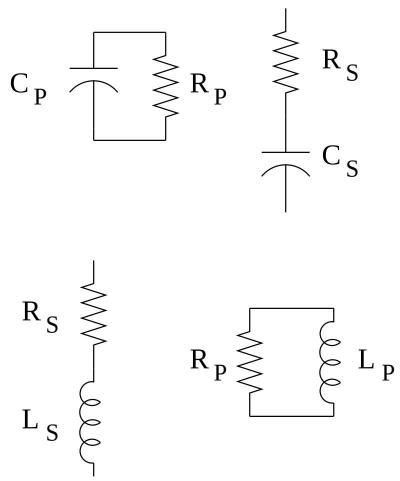
A difficulty you may already have encountered with AC signals is that they are sensitive to the presence of objects that are not in obvious electrical contact with them, for example a human standing perhaps 0.3 metre away. This is because the changing electric fields in AC are not confined to conductive wires, but extend through space. In effect, a nearby human contributes capacitance to the AC circuit.

The human operator needs to be near the controls and the meter, but perhaps these can be isolated from the AC bridge. *Carr* (page 97 and Figure 4-5 on page 98) suggests a transformer or a differential amplifier to bring the signal to a coaxial cable and then to the meter.

### 2.2.3 Variations

Reference: *Berlin & Getz*, section 9-4.

Capacitors are not ideal capacitances, but contain a resistive component; likewise for resistors. A typical capacitor model is a capacitance $C_P$ in parallel with a leakage resistance $R_P$. Likewise a realistic inductor is commonly modelled as an inductor $L_S$ in series with a resistance $R_S$. However, the opposite models can also be used, as shown below.




It's important to recognise that the values of the components in the two types of model are not equal, though they are related. For the capacitor,

$$R_S = R_P \frac{D^2}{1 + D^2}$$

$$C_S = C_P(1 + D^2)$$

The *dissipation factor*

$$D = \frac{X_P}{R_P} = \frac{1}{\omega R_P C_P} = \frac{R_S}{X_S} = \omega C_S R_S$$

is proportional to the power dissipated (lost as heat) through the capacitor each cycle of the AC signal. A good (nearly ideal) capacitor has a *low* dissipation factor.

For the inductor, the relations are:

$$R_P = R_S(1 + Q^2)$$
$$L_P = L_S(1 + \frac{1}{Q^2})$$

where the *quality factor* $Q$ *is*

$$Q = \frac{X_S}{R_S} = \frac{\omega L_S}{R_S} = \frac{R_P}{X_P} = \frac{R_P}{\omega L_P}$$

A good inductor has a *high* quality factor.

Note that the formulas for $D$ and $Q$ are reciprocal in form: this is a consequence of how engineers chose to define them. For both capacitors and inductors, the ideal is that $R_S$ should be small and $R_P$ should be large.

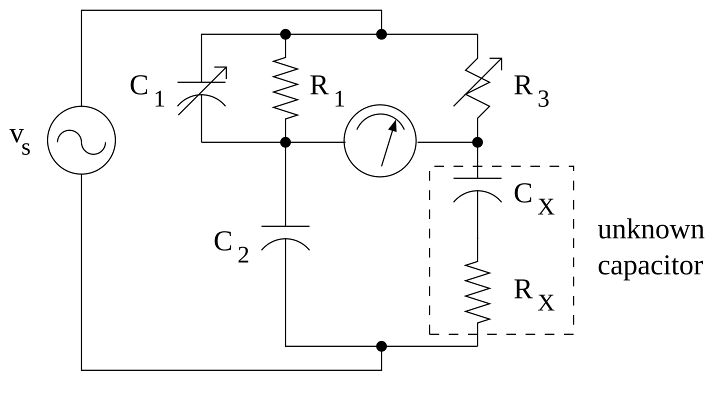
Incidentally, in frequency dependent circuits, the quality factor $Q$ describes how well-tuned the circuit is. This is useful to know when designing, for example, radio receivers. A circuit with a high $Q$ can more easily filter out unwanted radio stations while giving good reception of the desired station. Since a real inductor has a nonzero resistance, it is a simple tuned circuit in its own right (as are real capacitors for the same reason).

It's important to note that the series and parallel models have limited ranges of applicability, and the model inductances $L_S$ and $L_P$ and capacitances $C_S$ and $C_P$ may themselves vary with frequency in ways that can only be determined experimentally.

We shall now describe a few of the more common AC bridges.

### 2.2.4 Schering's Bridge

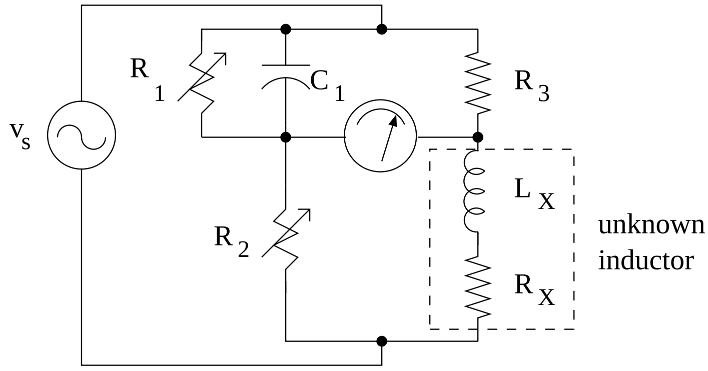
Schering's bridge is used to measure *capacitance*. It may also be used to measure the capacitor's dissipation factor, and the resistances of cables and equipment. (Note that 'Schering' and 'capacitor' both contain the letter "C".)




Its balance equations are independent of frequency:

$$R_X = R_1 \frac{C_1}{C_2}$$
$$C_X = C_2 \frac{R_1}{R_3}$$

### 2.2.5 Maxwell's Bridge




In Maxwell's bridge, resistors $R_2$ and $R_3$ remain as in Wheatstone's bridge, but a variable resistor $R_1$ is placed in parallel with a standard (precisely known) capacitor $C_1$. A series model is used for the unknown inductor.

Note that what amounts to an appreciable resistance depends heavily on the frequency range. At low frequencies, resistance dominates inductance, not capacitance, but at high frequencies it dominates capacitance, not inductance.

In Maxwell's bridge, the resistive null condition is identical to that in the Wheatstone bridge:

$$R_X = \frac{R_2 R_3}{R_1}$$

and the reactive null condition relates $L_X$ to $C_1$:

$$L_X = C_1 R_2 R_3$$

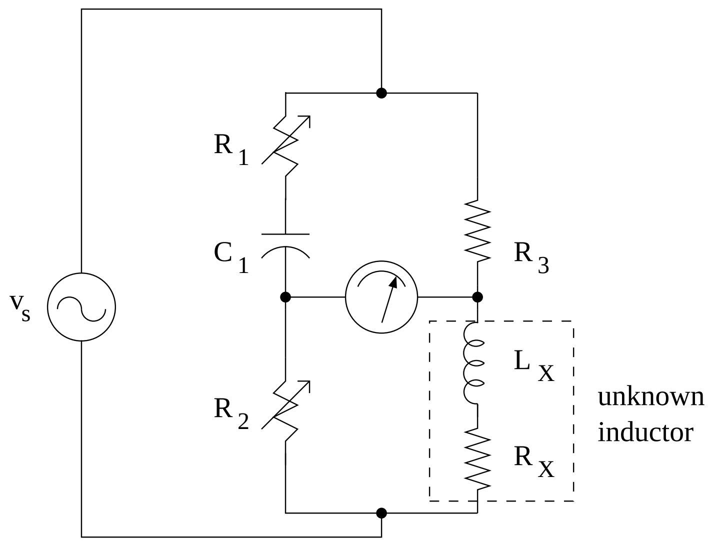
Conveniently, the frequency does not appear in either balance equation, so one does not need to make a frequency measurement to measure the inductance.

The Maxwell bridge can be used to measure the $Q$ of the unknown inductor ($L_X$ in series with $R_X$) provided that the frequency is determined:

$$Q = \omega R_1 C_1$$

Maxwell's bridge is suited to measuring *low-Q inductors* ($Q$ between 1 and 10). It helps to remember that both 'Maxwell' and 'low' contain the letter 'L'.

### 2.2.6 Hay's Bridge




In Hay's bridge, $R_1$ is in series with $C_1$. Hay's bridge is less convenient for measuring inductances, as its balance equations

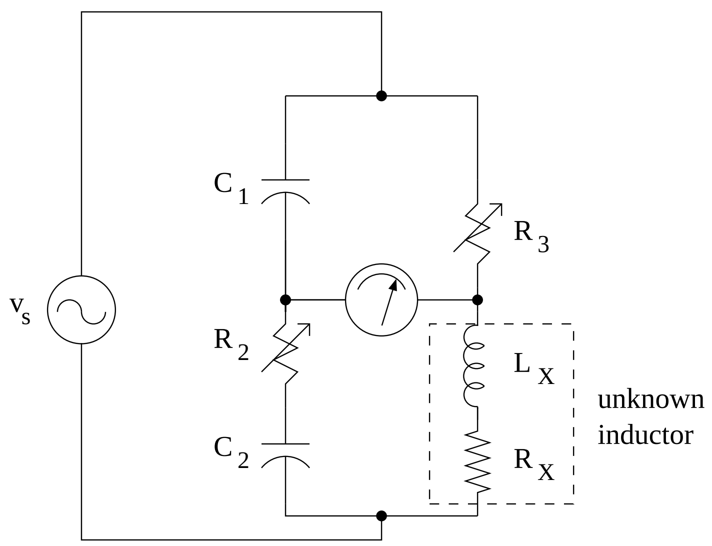
$$L_X = \frac{C_1 R_2 R_3}{1 + (\omega R_1 C_1)^2}$$

$$R_X = \frac{(\omega C_1)^2 R_1 R_2 R_3}{1 + (\omega R_1 C_1)^2}$$

would usually require us to make frequency measurements. However, for inductors with high $Q$, this is less significant.

Hay's bridge is suited to measuring *high-Q inductors* ($Q$ above 10). Think 'H' for 'Hay' and 'high'), and therefore complements Maxwell's bridge.

### 2.2.7 Owen's Bridge




The balance equations for Owen's bridge are frequency independent:

$$L_X = C_1 R_2 R_3$$

$$R_X = R_2 \frac{C_1}{C_2}$$

The Owen bridge is suited to measuring a *wide* range of inductances. (There's a 'W' in both 'Owen' and 'wide'!)

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2007 July 25