Fault Analysis

Previously we examined methods for evaluating how power flows around a system in steady-state. Now we will examine methods for evaluating fault levels in a power system. Fault level analysis is important for a number of reasons:

- Equipment needs to be sized so that it will not be damaged by fault currents.
- Overvoltages can occur during faults that will degrade insulation.
- System security often depends on how fast a fault is cleared, which in turn depends on fault location and severity.
- Minimum fault levels need to be determined to ensure that relays recognize and clear the fault.

It is worth noting that *faults* are usually caused by short circuits due to equipment failure, lightning, or contact with foreign bodies. They should not be confused with *overloads*, which mean that the design load has been exceeded and can be tolerated for a limited time. Although both conditions produce currents that are greater than those under normal load, faults will be much more damaging and require prompt clearance, often with no intentional delay.

ASSUMPTIONS & SIMPLIFICATIONS

Because of the level of difficulty associated with fault analysis, the following simplifications are usually made:

- The system is represented by its one-line diagram.
- Unlike load-flow, fault analysis is usually not performed on very large systems.
- Shunt paths to ground are neglected, e.g. non-motor loads, transformer magnetizing reactances and core losses, line shunt capacitances etc.
- Large motors are modeled as a constant current source for the first few cycles of the fault.
- All generator excitation voltages are equal and each generator is represented by its excitation voltage behind a reactance.
- All transformers are at their nominal tap positions.
- On high voltage systems, the series resistance of lines and transformers is neglected because their X/R ratio is high. This approximation becomes less valid as system voltage levels reduce and so has to be applied with care.
- Faults are usually considered to be either:
  - *Bolted* i.e. having no fault resistance, or
  - *Arcing* in which energy is dissipated in the arc.

TYPES OF FAULT

The types of fault we will be considering are:

- Balanced three-phase (3ϕ or $3\phi - G$).
- Single-phase to ground ($1\phi - G$).
- Phase to phase to ground ($2\phi - G$).
- Phase to phase (2ϕ).

The first three categories are capable of producing the largest fault currents, while last one will produce the smallest. It is usual to evaluate all four as both the largest and smallest fault levels are important.

BALANCED FAULTS

Balanced faults (3ϕ or 3ϕ – G) are readily evaluated using the [Y_{bus}] matrix that we saw previously in load flow analysis. On small and medium sized systems it is inverted to produce the *Bus Impedance Matrix* [Z_{bus}] i.e.

$$[Z_{bus}] = [Y_{bus}]^{-1}$$

For larger systems there are more efficient methods for producing [Z_{bus}] and one is presented in "Solution of Large Networks by Matrix Methods" by H.E. Brown, Wiley.

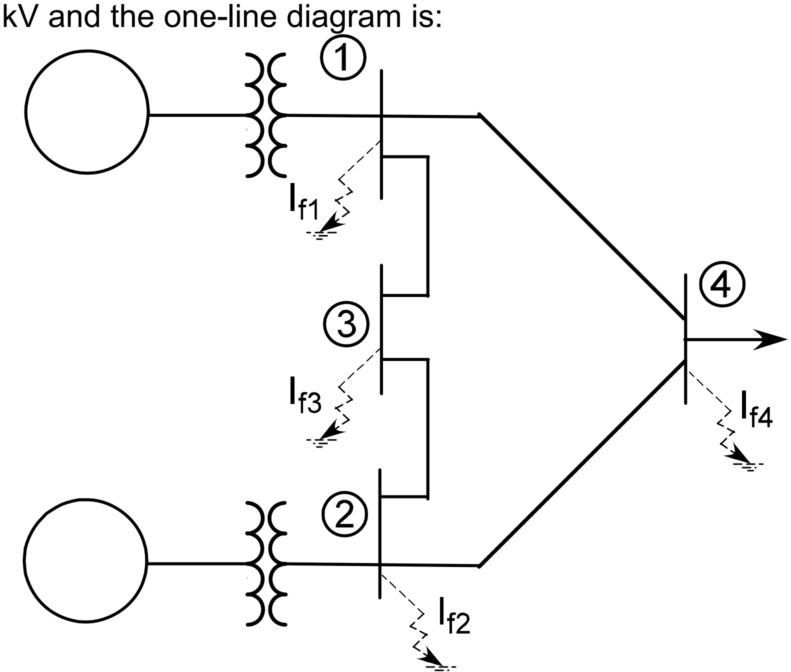
An important feature of the [Z_{bus}] matrix is that it's ***main diagonal elements are the Thevenin impedances of the associated bus and neutral.***

This makes evaluation of 3ϕ fault currents very easy if the following procedure is used:

- Draw the one-line diagram for the system being studied and indicate all points where fault levels are to be evaluated. Be sure to assign a bus number to all fault points, even if you have to introduce artificial buses e.g. to get the fault level in the middle of a line.
- Select appropriate base quantities and evaluate all voltages and impedances in pu on the common base. Be sure to select the correct reactance values for synchronous machines. Fault calculations should always use the sub-transient reactance.
- Convert Thevenin machines to their Norton equivalents and construct [Y_{bus}] for the system.
- Invert [Y_{bus}] to produce [Z_{bus}] and select the diagonal terms corresponding to the buses where fault levels are to be evaluated.
- The nominal symmetrical per-unit fault level is the reciprocal of the Thevenin impedance (1.0 pu voltage being assumed.)
- Apply multiplication factors as appropriate to account for dc off-set.

EXAMPLE

Let's go to one of the systems we looked at previously. The system bases are 100 MVA, 13.8 kV and the one-line diagram is:

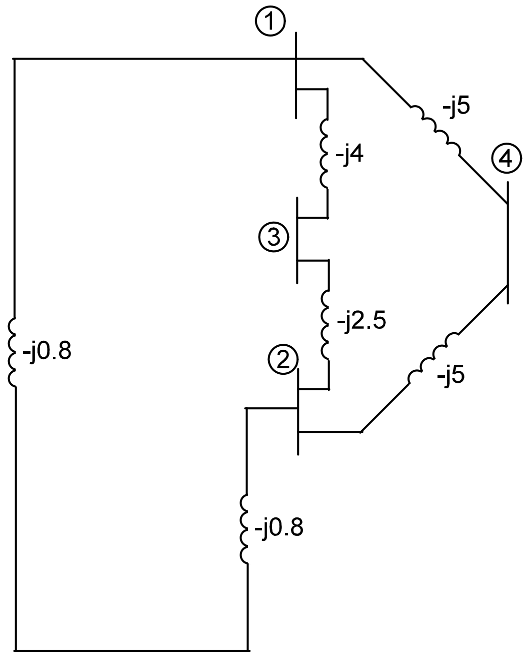


$\mathbf{I_{f1}}$, $\mathbf{I_{f2}}$ etc are fault currents that flow from the faulted bus into the fault. They are evaluated one at a time in pu.

The base current is:

$$I_B = \frac{100 \times 10^6}{\sqrt{3} \times 13.8 \times 10^3} = 4.184 \text{ kA}$$

Applying the approximations we saw earlier means we ignore the load (assuming it is non-motor) and the admittance diagram is:



The generators have also been omitted because we are only interested in getting the admittance matrix.

Following the same procedure as previously produces:

$$[Y_{bus}] = -j\begin{bmatrix} 9.8 & 0 & -4 & -5 \\ 0 & 8.3 & -2.5 & -5 \\ -4 & -2.5 & 6.5 & 0 \\ -5 & -5 & 0 & 10 \end{bmatrix}$$

Inverting this produces:

$$[Z_{bus}] = j\begin{bmatrix} 0.6813 & 0.5687 & 0.6380 & 0.6250 \\ 0.5687 & 0.6813 & 0.6120 & 0.6250 \\ 0.6380 & 0.6120 & 0.7818 & 0.6250 \\ 0.6250 & 0.6250 & 0.6250 & 0.7250 \end{bmatrix}$$

The off-diagonal terms are not used to calculate the fault currents flowing into the fault (they are used to calculate branch currents.) The main diagonal terms are the Thevenin Impedances. If we assume 1.0 pu voltage then each fault current is the reciprocal of the Thevenin Impedance, resulting in:

$$I_{f1} = I_{f2} = \frac{1}{0.6813} = 1.4678 \text{ pu} = 6.14 \text{ kA}$$

$$I_{f3} = \frac{1}{0.7818} = 1.2791 \text{ pu} = 5.35 \text{ kA}$$

$$I_{f4} = \frac{1}{0.7250} = 1.3793 \text{ pu} = 5.77 \text{ kA}$$

These are the symmetrical RMS fault currents (**I_{SYM}**). When sizing equipment it is important to allow for dc off-set and determining the asymmetrical current (**I_{ASYM}**) from:

$$\mathbf{I_{ASYM}} = \mathbf{I_{SYM}} \times K_O$$

Where K_O depends on the speed of protection employed and typically varies from 1.1 (slow) to 1.6 (fast).

To calculate branch flows, the off-diagonal terms in [Z_{bus}] are used. For a fault at bus “k”, the current flowing in a branch connecting buses “i” and “j” is given by:

$$I_{ij}^{k} = I_{fk}\left(\frac{Z_{jk} - Z_{ik}}{Z_{branch}}\right)$$

UNBALANCED FAULTS

Unbalanced faults (1ϕ, 2ϕ – G, or 2ϕ) are much more common than balanced faults and can have higher currents. They are obviously much harder to analyze.

Any unbalanced set of voltages (or currents) can be factorized into three balanced set of voltages (or currents) called *SYMMETRICAL COMPONENTS*, described as follows:

1. Positive-sequence components, consisting of three phasors equal in magnitude, displaced from each other by 120° in phase, and having the same phase sequence as the original (unbalanced) phasors. These are associated with the number "one".
2. Negative-sequence components, consisting of three phasors equal in magnitude, displaced from each other by 120° in phase, and having the opposite phase sequence to the original (unbalanced) phasors. These are associated with the number "two".
3. Zero-sequence components, consisting of three phasors equal in magnitude, with zero phase displacement from each other. These are associated with the number "zero".

These are illustrated in figure 43, for an arbitrary set of unbalanced voltages, having the phase sequence abc:

$$V_a = V_{a1} + V_{a2} + V_{a0}$$

$$V_b = V_{b1} + V_{b2} + V_{b0}$$

$$V_c = V_{c1} + V_{c2} + V_{c0}$$

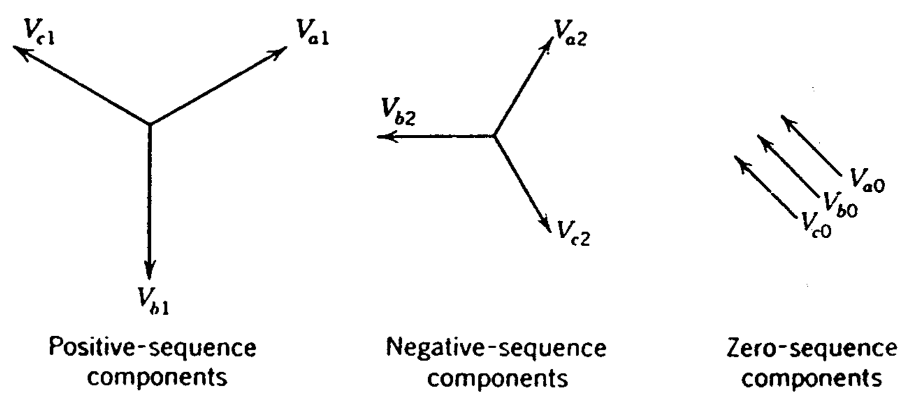


Figure 43 - Phasor relationships for symmetrical components

It can be seen, in figure 44, that the sequence components sum to represent the unbalanced voltages.

The question that arises at this point is "where did the sequence components come from?" i.e., how did we know the magnitudes and phase angles associated with V_{a1}, V_{a2}, V_{b1}, V_{co} etc. The method for finding the symmetrical

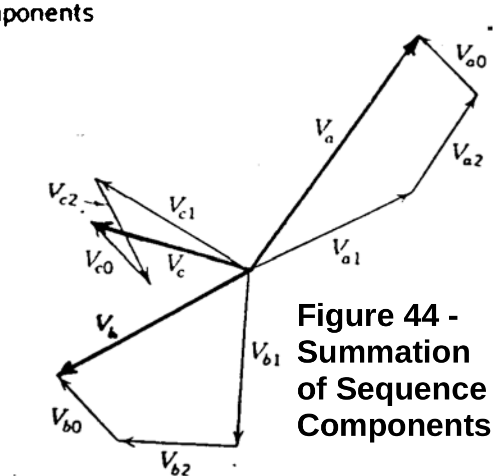


Figure 44 - Summation of Sequence Components

components is very simple if the complex arithmetic is performed in terms of the **"a"** operator.

When a phasor is multiplied by the operator **a**, it's magnitude is unaltered but it's phase angle is turned 120° counterclockwise as shown in figure 45:

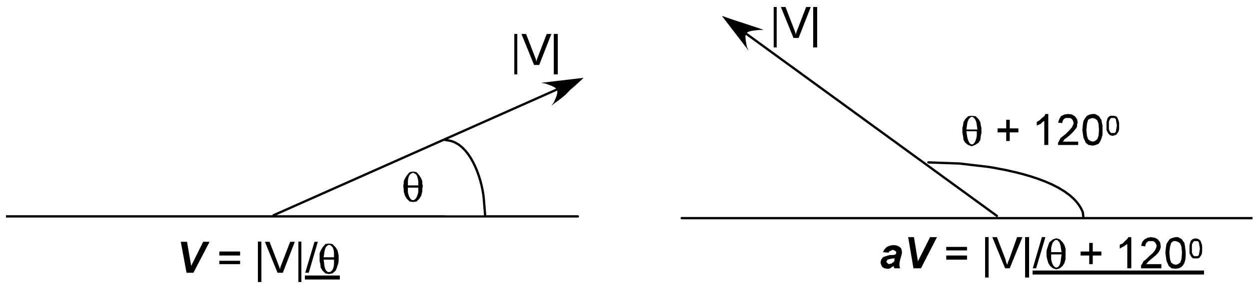


Figure 45 - "a" operator acting on a phasor

Some properties of the **a** operator can be seen in figure 46:

Notice that $\mathbf{a}^3 = 1.0$, and $\mathbf{a}^2 + \mathbf{a} + 1 = 0$

The **a** operator enables us to write all of the sequence voltages in terms of the 'a' phase:

$$\begin{aligned} V_{b1} &= a^2V_{a1} & \quad V_{c1} &= aV_{a1} \\ V_{b2} &= aV_{a2} & \quad V_{c2} &= a^2V_{a2} \\ V_{b0} &= V_{a0} & \quad V_{c0} &= V_{a0} \end{aligned}$$

The above set of equations will be referred to collectively as equation (11).

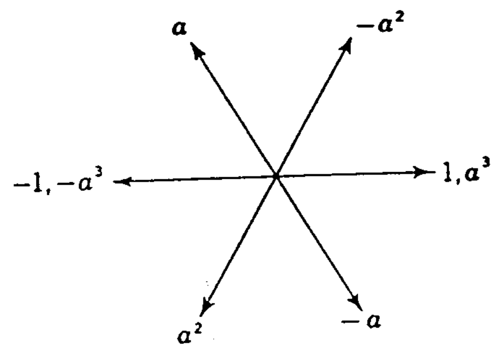


Figure 46 - "a" operator properties

The original unbalanced phase voltages can now be written as:

$$\begin{aligned} V_{an} &= V_{a0} + V_{a1} + V_{a2} \\ V_{bn} &= V_{a0} + a^2V_{a1} + aV_{a2} \\ V_{cn} &= V_{a0} + aV_{a1} + a^2V_{a2} \end{aligned}$$

or in matrix form:

$$\begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} \qquad \textbf{(12)}$$

Since we know V_{an}, V_{bn} and V_{cn}, the above is three equations in three unknowns, which can be solved by any appropriate method. After matrix inversion the result is:

$$\begin{bmatrix} V_{a0} \\ V_{a1} \\ V_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix} \qquad \textbf{(13)}$$

Equations (12) and (13) correspond to equations (11.9) and (11.16) in Yamayee. The preceding analysis could, of course, have been done in terms of line currents instead of phase voltages, in which case the results would have been:

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 1 & 1 & 1 \\ 1 & a^2 & a \\ 1 & a & a^2 \end{bmatrix} \begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} \quad \textbf{(14)}$$

$$\begin{bmatrix} I_{a0} \\ I_{a1} \\ I_{a2} \end{bmatrix} = \frac{1}{3} \begin{bmatrix} 1 & 1 & 1 \\ 1 & a & a^2 \\ 1 & a^2 & a \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} \quad \textbf{(15)}$$

You may be thinking "why are we doing all this?" Many power system analysis problems involve starting with voltages and having to find currents (or vice versa) by using circuit analysis. Analyzing unbalanced systems directly is very difficult, especially when Y-Δ transformers are present. Symmetrical components permit us to use balanced system analysis as shown in the following diagram:

Given unbalanced voltages

↓ Convert to produce

Sequence voltages (balanced)

↓ Analyze using sequence networks to produce

Sequence currents (also balanced)

↓ Convert back to produce

Actual unbalanced currents

We know that in a four-wire system the neutral current is the sum of the line (or phase) currents. The first row of equation (15) tells us that this sum is three times the zero sequence current. From this we conclude:

$I_n = I_a + I_b + I_c = 3 \cdot I_{ao}$ or $I_{ao} = ⅓ \cdot I_n$

Therefore in a three-wire system (Δ or ungrounded Y) I_n will be zero and hence there will be no zero sequence component of current. No matter how the system is unbalanced **the zero sequence current must be zero if there is no neutral**.