ECE471 INDUSTRIAL POWER SYSTEMS
Homework Set 1

1. A balanced Δ-connected load having an impedance of 24 + *j*18 Ω/ph is connected in parallel with a balanced Y-connected load having an impedance, of 20/0 Ω/ph. The paralleled loads are fed from a line having an impedance of 0.1 + j0.5 Ω/ph. The magnitude of the line-to-neutral voltage of the Y-load is 680 V.
a) Calculate the magnitude of the current in the line feeding the loads.
b) Calculate the magnitude of the phase current in the Δ-connected load.
c) Calculate the magnitude of the phase current in the Y-connected load.
d) Calculate the magnitude of the voltage at the sending end of the line.

Assume the system is fully balanced. If it is balanced then the currents for each load can be calculated using Ohm's Law, then summed to calculate the line currents.

The phase current in the Y connected load is $I_Y := \frac{V_{AN}}{Z_Y}$

The phase current for phase AB of the delta load is $I_{AB} := \frac{V_{AB}}{Z_\Delta}$

The phase current for phase CA is $I_{CA} := I_{AB} \cdot e^{j \cdot 120 \cdot deg}$

The current in the line of phase A is $I_A := I_Y + I_{AB} - I_{CA}$

The line - neutral voltage at the sending end is $V_s := V_{AN} + I_A \cdot Z_{line}$

a)	$\lvert I_A \rvert = 97.361\ A$	$arg(I_A) = 24.775 \cdot deg$
b)	$\lvert I_{AB} \rvert = 39.26\ A$	$arg(I_{AB}) = 66.87 \cdot deg$
c)	$\lvert I_Y \rvert = 34\ A$	$arg(I_Y) = 0 \cdot deg$
d)	$\lvert V_s \rvert =$ ~~752.814~~ V 710.4 V	$arg(V_s) =$ ~~6.483~~ $\cdot deg$ 3.2

2. The three pieces of computer equipment described below are installed as part of a computer center. Each piece of equipment is a balanced three-phase load rated at 208 V. Calculate:

(a) the magnitude of the line current supplying these three devices, and

(b) the power factor of the combined load.

Disk: 6.157 kVA at 0.79 pf lag.
Drum: 16.93 kW at 0.96 pf lag.
CPU: line current 73.8 A, 22.694 kW.

First calculate the complete CPU load. $S_{CPU\_mag} := \sqrt{3} \cdot V_{line} \cdot I_{CPU}$

$P_{CPU} := \sqrt{S_{CPU\_mag}^{2} - (|Q_{CPU}|)^2}$ then $S_{CPU} := P_{CPU} + Q_{CPU}$

Calculate the total load $S_{total} := S_{disk} + S_{Drum} + S_{CPU}$

Calculate the line current from $S = \sqrt{3} \cdot V_{line} \cdot \overline{I_{line}}$

Therefore $I_{line} := \overline{\frac{S_{total}}{V_{line} \cdot \sqrt{3}}}$

Calculate the power factor of the combined load $pf_{load} := \cos(\arg(S_{total}))$

a)	$\lvert I_{line} \rvert = 139.827$ A
b)	$pf_{load} = 0.892$ lag

3. A three-phase, 5 hp, 208 V, 60 Hz induction motor runs at 1164 rpm when it delivers rated output power.

(a) Determine the number of poles of the machine.
(b) Determine the slip at full load.
(c) Determine the frequency of the rotor current.
(d) Determine the speed of the rotor field with respect to:
  (i) Stator structure.
  (ii) Stator rotating field.

a) n_s = 1200 rpm, so the machine is 6 pole

b) $s = \frac{n_s - n_m}{n_s} = \frac{1200 - 1164}{1200} = 0.03$

c) $f_R = sf_s = 0.03 \times 60 = 1.8$ Hz

d)
  i) The rotor field travels at synchronous speed and the stator structure is stationary, so the relative speed is 1200 rpm.
  ii) The stator rotating field also travels at synchronous speed, so they are in step and their relative speed is zero rpm.

4. A three-phase, 460 V, 100 hp, 60 Hz, eight-pole induction machine operates at 3% slip (positive) at full load.

(a) Determine the speed of the motor and its direction relative to the rotating field.
(b) Determine the rotor frequency.
(c) Determine the speed of the stator field.
(d) Determine the speed of the air gap field.
(e) Determine the speed of the rotor field relative to:
  (i) the rotor structure
  (ii) the stator structure
  (iii) the stator rotating field.

a) $n_s = \frac{3600}{p/2} = \frac{3600}{8/2} = 900 \text{ rpm}$

and $n_m = (1 - s)n_s$ = 873 rpm (in the same direction as the rotating magnetic field)

b) $f_R = sf_s = 0.03 \times 60 = 1.8$ Hz

c) and d) Both the stator and air-gap fields travel at synchronous speed, 900 rpm.

e)
  i) The rotor field travels at synchronous speed (900 rpm) and the rotor structure travels at 873 rpm, so the relative speed is 27 rpm.
  ii) The rotor field travels at synchronous speed and the stator structure is stationary, so the relative speed is 900 rpm.
  iii) The stator rotating field also travels at synchronous speed, so they are in step and their relative speed is zero rpm.

5. A 40 hp motor has a full-load speed of 1150 rpm. Calculate its full-load torque.

$$T = \frac{hp \text{ x } 5252}{rpm} = \frac{40 \text{ x } 5252}{1150} = 182.7 \text{ lb.ft} \quad (247.8 \text{ Nm})$$

6. Calculate the synchronous speed of a 60 Hz, eight-pole motor in (a) rpm and (b) rad/s.

$$n_s = \frac{3600}{p/2} = \frac{3600}{8/2} = 900 \text{ rpm} \qquad \omega_s = \frac{2\pi \text{ x } 900}{60} = 94.2 \text{ rad/s}$$

7. A 60 Hz, six-pole induction motor has a full-load slip of 3.0%. Calculate its rated speed.

$$n_s = \frac{3600}{p/2} = \frac{3600}{6/2} = 1200 \text{ rpm} \qquad n_m = (1 - s)n_s = 1164 \text{ rpm}$$

8. Calculate the full-load efficiency of a 50 hp, three-phase, 460 V motor that has a rated line current of 65 A at a power factor of 90%. (Use Equation 1.12 for power input.)

$P_{in} = \sqrt{3}\ V_L\ I_L\ pf = \sqrt{3} \text{ x } 460 \text{ x } 65 \text{ x } 0.9 = 46.6 \text{ kW}$

$P_{out} = 746 \text{ x } hp = 746 \text{ x } 50 = 37.3 \text{ kW}$

$$\eta = \frac{P_{out}}{P_{in}} \text{ x } 100\% = \frac{37.3}{46.6} \text{ x } 100\% = 80\%$$