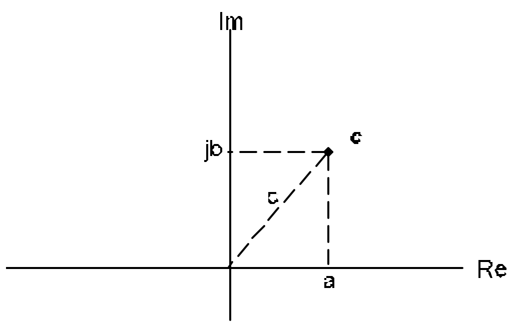
Complex Numbers

Phasor analysis is used to find the steady-state response of systems to sinusoidal inputs. Phasor analysis requires the use of complex numbers and so one should become comfortable with adding, subtracting, multiplying, and dividing complex numbers. Also, one should be able to solve systems of equations where the coefficients and variables are complex—we solving systems use a calculator or laptop.

> *Note*: In electrical systems, the imaginary element, $i = \sqrt{-1}$, is renamed. $j = \sqrt{-1}$ is used since j will be less likely to be confused with current.

A complex number can be represented in the plane—the complex plane



c = a + jb (rectangular form)
c = $c\angle\theta_c$ (polar form, represents the analytic form $c\,e^{j\theta}$)

Note: $c\angle\theta$ is a notation for $ce^{j\theta}$. The exponental form is the mathematical form $c\angle\theta$.

conversion from polar to rectangular

$$a = c\cos\theta$$
$$b = c\sin\theta$$

from *rectangular to polar*

$$c^2 = a^2 + b^2$$
$$\theta = \tan^{-1}(b/a)$$

Euler's Identity $e^{j\theta} = \cos\theta + j\sin\theta$

This relation can be proved by expressing each term as a power series.

$$e^{jq} = 1 + jq + \frac{(jq)^2}{2!} + \frac{(jq)^3}{3!} + \frac{(jq)^4}{4!} + \ldots$$

$$\cos q = 1 - \frac{q^2}{2!} + \frac{q^4}{4!} - \ldots$$

$$\sin q = q - \frac{q^3}{3!} + \frac{q^5}{5!} - \ldots$$

Addition, Subtraction, Multiplication, and Division

Using the two complex numbers, ***c*** and ***d***.

$\boldsymbol{c} = a + jb = c\angle\theta_c$ $\boldsymbol{d} = e + jf = d\angle\theta_d$

addition / subtraction

$\boldsymbol{c} \pm \boldsymbol{d} = (a \pm e) + j\,(b \pm f)$

multiplication

$\boldsymbol{c}\,\boldsymbol{d} = (c\angle\theta_c)\,(d\angle\theta_d) = cd\angle(\theta_c + \theta_d)$

division

$\boldsymbol{c} / \boldsymbol{d} = (c\angle\theta_c) / (d\angle\theta_d) = c/d\angle(\theta_c - \theta_d)$

miscellaneous

$j = 1\angle 90°$

$-j = 1\angle -90°$

$1 / j = 1 / 1\angle 90° = 1\angle 0° / 1\angle 90° = 1/1\angle(0° - 90°) = 1\angle -90° = -j$

$-1 = 1\angle 180° = 1\angle -180°$

Notation

- The notation Re (***c***) refers to the real part of the complex number ***c***.
- The notation Im (***c***) refers to the imaginary part of the complex number ***c***.
- ***c***** refers to the complex conjugate of the complex number ***c***.

--For $\boldsymbol{c} = a + jb = c\angle\theta$, $\boldsymbol{c}^* = a - jb = c\angle -\theta$.
--note that $\boldsymbol{c}\,\boldsymbol{c}^* = c^2$