ECE370 POWER & ENERGY SYSTEMS
Homework Set 7 - Solutions

7.1 A three-phase induction motor is supplied with power from a 60-Hz source. At full load the motor speed is 1728 rpm, and at no load the speed is nearly 1800 rpm. At full-load conditions, determine the following:

a) Number of poles
b) Slip
c) Frequency of the rotor voltages
d) Speed of the rotor field with respect to the rotor
e) Speed of the rotor field with respect to the stator
f) Speed of the rotor field with respect to the stator field

(a) $$P = \frac{120f}{n_s} = \frac{(120)(60)}{1800} = 4 \text{ poles}$$

(b) $$s = \frac{n_s - n}{n_s} = \frac{1800 - 1728}{1800} = 0.04$$

(c) $$f_r = sf = (0.04)(60) = 2.4 \text{ Hz}$$

(d) $$\text{slip rpm} = n_s - n = 1800 - 1728 = 72 \text{ rpm}$$

(e) $$n_s = 1800 \text{ rpm}$$

(f) $$n_s - n_s = 0 \text{ rpm}$$

7.2 A three-phase, 440-V, 60-Hz, wye-connected induction motor takes a stator current of 50 A at 0.8 power factor while running at its rated speed of 855 rpm. The stator loss is 2500 W, and the rotational losses are 3200 W. Calculate the efficiency of the motor.

$$s = \frac{900 - 855}{900} = 0.05$$

$$P_{in} = \sqrt{3}\, V_L I_L \cos\theta = \sqrt{3}\,(440)(50)(0.8) = 30{,}484 \text{ W}$$

$$P_{ag} = P_{in} - SCL = 30{,}484 - 2500 = 27{,}984 \text{ W}$$

$$P_{dev} = (1-s)\,P_{ag} = (1 - 0.05)(27{,}984) = 26{,}585 \text{ W}$$

$$P_{out} = P_{dev} - P_{rotational} = 26{,}585 - 3200 = 23{,}385 \text{ W}$$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{23{,}385}{30{,}484}\, 100\% = 76.7\ \%$$

7.3 A three-phase, 480 V, 60 Hz, wye-connected induction motor draws a ***reactive power*** of 18 kVAR at 0.8 lag pf and runs at 1158 rpm. The stator losses are 500 W and the rotational losses are 350 W. Determine:

a) Rotor copper loss.
b) Shaft output torque and horsepower.
c) Efficiency.
d) Input current magnitude.

$$s = \frac{1200 - 1158}{1200} = 0.035 \qquad \text{pf} = 0.8 \quad \therefore \ \text{rf} = 0.6 \ \text{and} \ \text{S} = 30 \ \text{KVA}$$

(a) $P_{in} = S \times PF = (30{,}000)(0.8) = 24{,}000 \ W$

$P_{ag} = P_{in} - SCL = 24000 - 500 = 23{,}500 \ W$

$RCL = s P_{ag} = (0.035)(23{,}500) = 822.5 \ W$

(b) $P_{dev} = (1-s) P_{ag} = (1-0.035)(23{,}500) = 22{,}677.5 \ W$

$P_{out} = P_{dev} - P_{rotational} = 22{,}677.5 - 350 = 22{,}327.5 \ W = 29.9 \ hp$

$n_s = \frac{120 f}{P} = \frac{(120)(60)}{6} = 1200 \ rpm$

$n = (1-s) n_s = (1-0.035)(1200) = 1158 \ rpm$

$\omega_m = \frac{2\pi n}{60} = \frac{(2\pi)(1158)}{60} = 121.3 \ rad/sec$

$T_{out} = \frac{P_{out}}{\omega_m} = \frac{22{,}327.5}{121.3} = 184 \ N\text{-}m$

(c) $\eta = \frac{P_{out}}{P_{in}} = \frac{22{,}327.5}{24000} 100\% = 93 \%$

d) $$|I| = \frac{S_{3\theta}}{\sqrt{3} V_L} = \frac{30000}{\sqrt{3} \ x \ 480} = 36.1 \ A$$

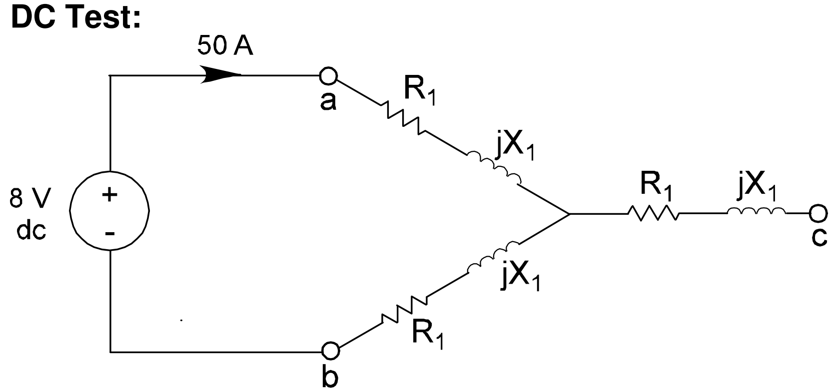
7.4 The following test results were obtained for a three-phase, 400 hp, 2.4 kV, eight-pole, wye-connected induction motor.

a) Determine the exact equivalent circuit.

Test performed	Voltage (V)	Current (A)	Power (kW)
No-load	2400	9.3	29.1
Locked-rotor (performed at 15 Hz)	112.6	100	7.5
Rotational loss	-	-	19
DC (measured between phases a – b)	80	400	-

b) **Estimate** the efficiency when rated load is supplied.

DC Test:



Voltage across one phase = 40 V

$\therefore\ R_1 = \dfrac{40}{400} = \boxed{0.1\ \Omega/\text{phase.}}$

Locked Rotor Test: $pf_{LR} = \dfrac{P_\theta}{V_\theta I_\theta} = \dfrac{7500/3}{112.6/\sqrt{3}\ x100} = 0.3846 \text{ lag}$ $\therefore\ \theta_{LR} = 67.4^0$

$\therefore\ Z_{eq} = \dfrac{112.6/\sqrt{3}}{100} = 0.65\ \Omega$ $\therefore\ \boldsymbol{Z_{eq}} = 0.65\underline{/67.4} = 0.25 + j0.6$

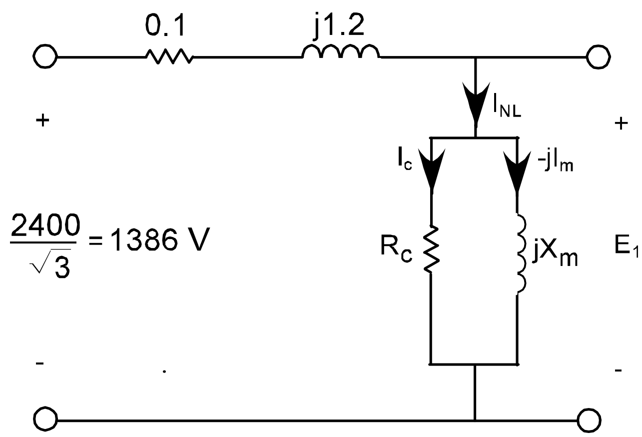
Since $R_1 = 0.1$ then $\boxed{R'_2 = 0.15\ \Omega/\text{phase}}$

And $X = X_{test}\dfrac{f_{rated}}{f_{test}} = 0.6\dfrac{60}{15} = 2.4\ \Omega$

It is usual to assume $X_1 = X'_2$ $\boxed{\therefore\ X_1 = X'_2 = 1.2\ \Omega}$

No-load Test: $pf_{NL} = \frac{P_\theta}{V_\theta I_\theta} = \frac{29100/3}{2400/\sqrt{3} \times 9.3} = 0.753 \text{ lag}$ $\therefore$ $\theta_{NL} = 41.2^0$

and $I_{NL} = 9.3\angle{-41.2}$. This splits according to the following diagram:



$E_1 = 1386\angle 0 - (0.1 + j1.2) \times 9.3\angle{-41.2}$
$= 1377 \angle{-0.3}$ V

and $I_m = 9.3 \times \sin 41.2 = 6.13$ A

$\therefore X_m = \frac{1377}{6.13} = \boxed{225\ \Omega}$

$P_{Rc} = P_{NL} - P_{rot} - 3|I_{NL}|^2 R_1 = 29100 - 19000 - 3 \times 9.3^2 \times 0.1 = 10074$ W

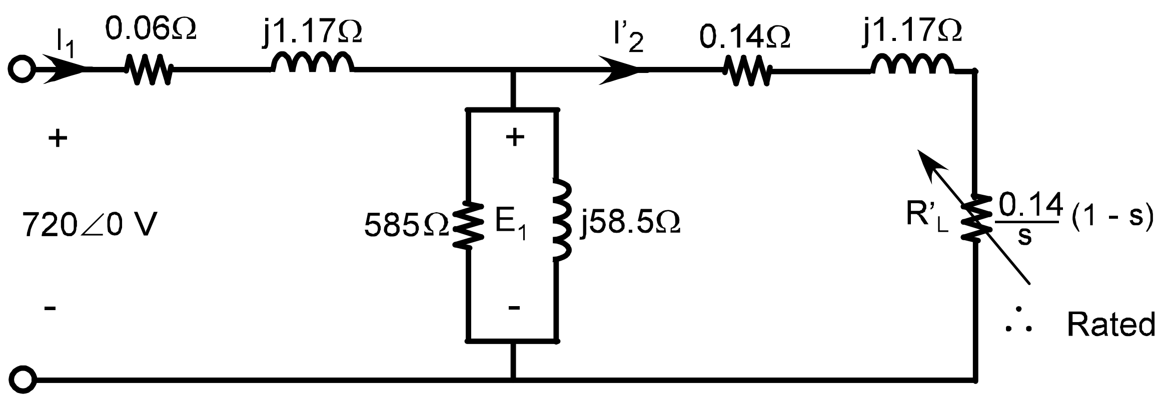
and $P_{RC} = \frac{3|E_1|^2}{R_c}$ $\therefore R_c = \frac{3 \times 1377^2}{10074} = \boxed{565\ \Omega}$

b) $\eta = \frac{P_{out}}{P_{out} + P_{loss}}$ and $P_{out} = 746 \times 400 = 298.4$ kW, while $P_{loss} = P_{LR} + P_{NL}$

$\therefore \eta = \frac{298.4}{298.4 + 29.1 + 7.5} x 100\% = \boxed{89.1\ \%}$

7.5 The equivalent circuit parameters of a three-phase, Y-connected induction motor are, $R_1 = 0.06\ \Omega$, $R_2' = 0.14\ \Omega$, $X_1 = X_2' = 1.17\ \Omega$, $R_c = 585\ \Omega$, $X_m = 58.5\ \Omega$. The nameplate states: 1.247 kV, 60 Hz, 1176 rpm, with rotational losses of 6644 W. Determine:

a) Magnitude of rated input current.
b) Rated power factor.
c) Rated shaft horsepower.
d) Rated output torque.
e) Rated efficiency.
f) Starting current.
g) Starting pf.



Motor must be 6 pole

$\therefore\ n_s = 1200$ rpm

$\therefore$ Rated $s = \dfrac{1200 - 1176}{1200} = 0.02$

a) The load resistor: $R_L' = \dfrac{0.14 \times 0.98}{0.02} = 6.86\ \Omega/\text{ph}$

The impedance seen by the source is:

$\boldsymbol{Z_{in}} = (0.06 + j1.17) + 585 /\!/ j58.5 /\!/ (7 + j1.17) = 6.629 + j3.053\Omega/\text{ph}$

$$\therefore \quad \boldsymbol{I_1} = \frac{720\angle 0}{6.629 + j3.053} = \boxed{98.66\angle\text{-}24.7\ \text{A}}$$

b) pf = cos(24.7) = $\boxed{0.908\ \text{lag}}$

c) $\boldsymbol{E_1} = 720\angle 0 - 98.66\angle\text{-}24.7 \times (0.06 + j1.17) = 674.1\angle\text{-}8.7\ \text{V}$

$$\therefore \quad \boldsymbol{I_2'} = \frac{674.1\angle -8.7}{7 + j1.17} = 94.98\angle -18.2\ \text{A}, \quad \text{and } P_D = 3 \times 94.98^2 \times 6.86 = 185.7\ \text{kW}$$

$P_{out} = P_D - P_{rot} = 179\ \text{kW} = \boxed{240\ \text{hp}}$

d) $\omega_m = 2\pi \times {}^{1176}/_{60} = 123.2$ rad/s and $T_{out} = \dfrac{P_{out}}{\omega_m} = \dfrac{179 \times 10^3}{123.2} = \boxed{1454\ \text{Nm}}$

e) $\eta = \frac{P_{out}}{P_{in}}$ and $P_{in} = \sqrt{3} V_L I_L \cos\theta = \sqrt{3} \times 1247 \times 98.66 \times 0.908 = 193.5$ kW

$$\therefore \quad \eta = \frac{179}{193.5} \times 100\% = \boxed{92.5\ \%}$$

f) At start-up the slip is unity and R'_L is zero. The impedance seen by the source is:
$\boldsymbol{Z_{in}} = (0.06 + j1.17) + 585 /\!/ j58.5 /\!/ (0.14 + j1.17) = 0.1968 + j2.317\ \Omega$/ph

$$\therefore \quad \boldsymbol{I_{start}} = \frac{720\angle 0}{0.1968 + j2.317} = \boxed{309.7\angle\text{-}85.1\ \ \text{A}}$$

g) $pf_{start} = \cos(85.1) = \boxed{0.085\ \text{lag}}$