

ECE370 POWER & ENERGY SYSTEMS Homework Set 5 - Solutions

- 5.1 A six-pole dc machine has an armature connected as a lap winding. The armature has 32 slots with six conductors per slot. The armature is rotated at 600 rpm, and the flux per pole is 30 mWb. Calculate the induced voltage.

$$Z = 32 \times 6 = 192 \text{ conductors}$$

$$\omega_m = \frac{2\pi n}{60} = \frac{2\pi(600)}{60} = 62.83 \text{ rad/sec}$$

$$E_a = \frac{PZ\phi_p\omega_m}{2\pi a} = \frac{(6)(192)(30 \times 10^{-3})(62.83)}{2\pi(6)} = 57.6 \text{ V}$$

- 5.2 A four-pole dc generator has a wave-wound armature containing 48 slots with eight conductors per slot. The generator is driven at 1180 rpm and generates a voltage of 480 V. What is the flux per pole?

$$Z = 48 \times 8 = 384 \text{ conductors}$$

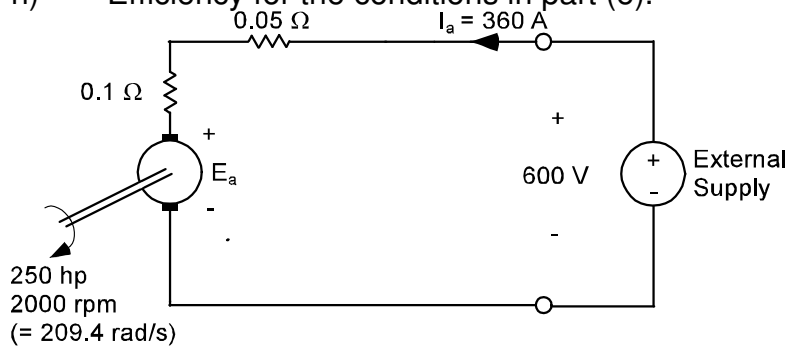
$$K_a = \frac{PZ}{2\pi a} = \frac{(4)(384)}{2\pi(2)} = 122.23$$

$$\omega_m = \frac{2\pi n}{60} = \frac{2\pi(1180)}{60} = 123.57 \text{ rad/sec}$$

$$\phi_p = \frac{E_a}{K_a\omega_m} = \frac{480}{(122.23)(123.57)} = 31.8 \text{ mWb}$$

5.3 A 600 V dc series motor is rated at 250 hp, 2000 rpm. It has an armature resistance of 0.1Ω and a field resistance of 0.05Ω . It draws a current of 360 A from the supply when delivering rated load. Ignore magnetic saturation and determine:

- Rated output torque.
- Rated developed torque.
- Rated efficiency.
- Rotational losses at rated speed.
- Speed when the load is changed, causing the line current to drop to 90 A.
- Developed torque for the conditions in part (e).
- Horsepower output for the conditions in (e) if the rotational losses are proportional to speed (not speed²).
- Efficiency for the conditions in part (e).



$$P_{\text{out}} = 250 \times 746 = 186.5 \text{ kW}$$

$$T_{\text{out}} = \frac{186.5 \times 10^3}{209.4} = \boxed{890.5 \text{ Nm}}$$

To get developed torque we need the developed power, $P_D = E_a I_a$

$$E_a = 600 - 0.15 \times 360 = 546 \text{ V}$$

$$P_D = 546 \times 360 = 196.56 \text{ kW}$$

b) $\therefore T_D = \frac{196.56 \times 10^3}{209.4} = \boxed{938.5 \text{ Nm}}$

c) $P_{\text{in}} = 600 \times 360 = 216 \text{ kW} \therefore \eta = \frac{186.5}{216} \times 100\% = \boxed{86.3\%}$

d) $P_{\text{rot}} = P_D - P_{\text{out}} = \boxed{10.06 \text{ kW}}$

e) For rated conditions: $(K_a \Phi) = \frac{E_a}{\omega_m} = \frac{546}{209.4} = 2.607$

$$\therefore (K_a \Phi)' = 2.607 \times \frac{90}{360} = 0.6517$$

and $E'_a = 600 - 0.15 \times 90 = 586.5 \text{ V} \therefore \omega'_m = \frac{E'_a}{(K_a \Phi)'} = \frac{586.5}{0.6517} = 900 \text{ rad/s}$

$\therefore \boxed{n' = 8593 \text{ rpm}}$ notice that the speed has increased with reduced load.

f) $T_D' = (K_a \Phi)' I_a' = 0.6517 \times 90 = \boxed{58.66 \text{ Nm}}$

g) $P_D' = 586.5 \times 90 = 52.79 \text{ kW}$ and $P'_{\text{rot}} = 10.06 \times 10^3 \times \left(\frac{8593}{2000}\right) = 43.25 \text{ kW}$

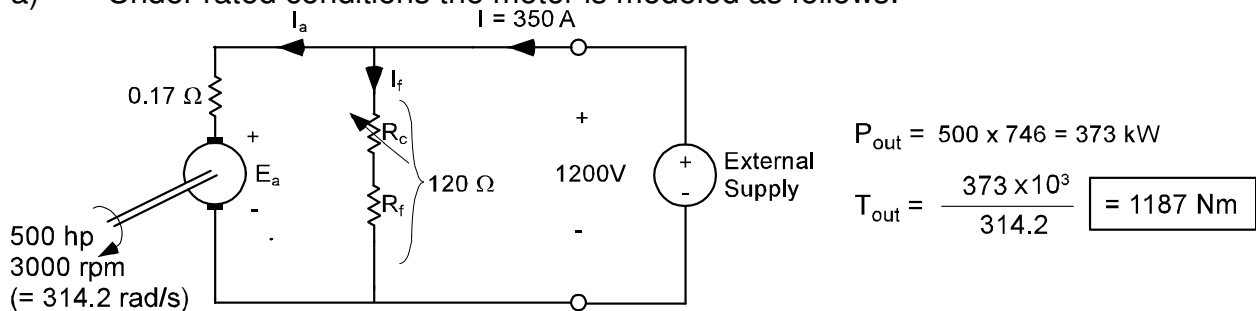
$$P'_{\text{out}} = P_D' - P'_{\text{rot}} = 9.56 \text{ kW} = \boxed{12.82 \text{ hp}}$$

h) $P_{\text{in}} = 600 \times 90 = 54 \text{ kW} \therefore \eta = \frac{9.56}{54} \times 100\% = \boxed{17.7\%}$

5.4 A 1200 V dc shunt motor is rated at 500 hp, 3000 rpm. It has an armature resistance of 0.17Ω and a total field circuit resistance of 120Ω . It draws a current of 350 A from the supply when delivering rated load. Ignore magnetic saturation and determine:

- Rated output torque.
- Rated developed torque.
- Rated efficiency.
- Rotational losses at rated speed.
- Speed when the load is changed, causing the line current to drop to 70 A. (Field resistance is unaltered.)
- HP output for the conditions in (e) if the rotational losses are proportional to speed².
- Efficiency for the conditions in part (e).

a) Under rated conditions the motor is modeled as follows:



b) To get developed torque we need the developed power, $P_D = E_a I_a$

$$I_f = \frac{V}{(R_f + R_c)} = \frac{1200}{120} = 10 \text{ A} \quad \therefore I_a = 350 - 10 = 340 \text{ A}$$

$$E_a = 1200 - 0.17 \times 340 = 1142 \text{ V} \quad \therefore P_D = 1142 \times 340 = 388.3 \text{ kW}$$

$$\therefore T_D = \frac{388.3 \times 10^3}{314.2} = 1236 \text{ Nm}$$

c) $P_{in} = 1200 \times 350 = 420 \text{ kW} \quad \therefore \eta = \frac{373}{420} \times 100\% = 88.8\%$

d) $P_{rot} = P_D - P_{out} = 15.3 \text{ kW}$

e) For rated conditions: $(K_a \Phi) = \frac{E_a}{\omega_m} = \frac{1142}{314.2} = 3.64$ and this will not change.

and $I'_a = 70 - 10 = 60 \text{ A} \quad E'_a = 1200 - 0.17 \times 60 = 1189.8 \text{ V}$

$$\therefore \omega'_m = \frac{E'_a}{(K_a \Phi)} = \frac{1189.8}{3.64} = 327.3 \text{ rad/s} \quad \therefore n' = 3125 \text{ rpm}$$

f) $P'_{out} = P'_D - P'_{rot}$ and $P'_D = 1189.8 \times 60 = 71.39 \text{ kW}$

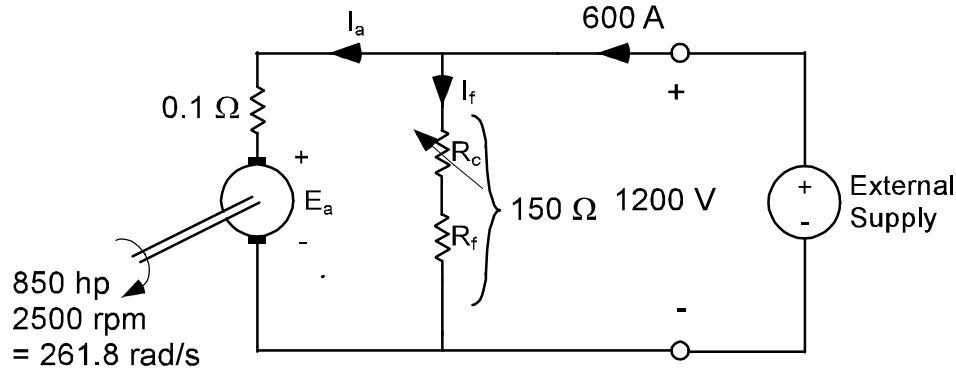
$$P'_{rot} = 15.3 \times \left(\frac{3125}{3000} \right)^2 = 16.6 \text{ kW} \quad P'_{out} = P'_D - P'_{rot} = 54.79 \text{ kW} = 73.44 \text{ hp}$$

g) $P'_{in} = 1200 \times 70 = 84 \text{ kW} \quad \therefore \eta' = \frac{54.79}{84} \times 100\% = 65.22\%$

5.5 A 1200 V dc shunt motor is rated at 850 hp, 2500 rpm. It has an armature resistance of 0.1Ω and a total field circuit resistance of 150Ω . It draws a current of 600 A from the supply when delivering rated load. Ignore magnetic saturation and determine:

- Rated output torque.
- Rated developed torque.
- Rated efficiency.
- Rotational losses at rated speed.
- Line current when the total field circuit resistance is changed to 50Ω , while the developed torque remains constant.
- Speed for the conditions in part (e).
- Horsepower output for the conditions in (e) if the rotational losses are proportional to speed².
- Efficiency for the conditions in (e).

Under rated conditions the motor is modeled as follows:



a) $P_{\text{out}} = 850 \times 746 = 634.1 \text{ kW} \quad \therefore T_{\text{out}} = \frac{634.1 \times 10^3}{261.8} = \boxed{2422 \text{ Nm}}$

b) To get developed torque we need the developed power, $P_D = E_a I_a$ and to get I_a we need I_f since $I_a = I - I_f$.

$$I_f = \frac{V}{(R_f + R_c)} = \frac{1200}{150} = 8 \text{ A} \quad \therefore I_a = 600 - 8 = 592 \text{ A}$$

$$E_a = 1200 - 0.1 \times 592 = 1140.8 \text{ V} \quad \therefore P_D = 1140.8 \times 592 = 675.4 \text{ kW}$$

$$\therefore T_D = \frac{675.4 \times 10^3}{261.8} = \boxed{2580 \text{ Nm}}$$

c) To get rated efficiency.

$$P_{\text{in}} = 1200 \times 600 = 720 \text{ kW} \quad \therefore \eta = \frac{634.1}{720} \times 100\% = \boxed{88.1\%}$$

d) To get rotational losses at rated speed.

$$P_{\text{rot}} = P_D - P_{\text{out}} = \boxed{41.25 \text{ kW}}$$

e) Since the resistance has changed, the field current is $I_f = \frac{1200}{50} = 24 \text{ A}$

$$\text{For rated conditions: } (K_a\Phi) = \frac{E_a}{\omega_m} = \frac{1140.8}{261.8} = 4.358$$

$$\text{For new conditions: } (K_a\Phi)' = \frac{24}{8} \times 4.358 = 13.07$$

$$\text{Developed torque is unaltered } \therefore T_D = 2580 = (K_a\Phi)'I_a'$$

$$\therefore I_a' = \frac{2580}{13.07} = 197.3 \text{ A} \quad \therefore \boxed{I' = 221\frac{1}{3} \text{ A}}$$

f) To get speed we need to know E_a' and $(K_a\Phi)'$

$$\therefore E_a' = 1200 - 0.1 \times 197.3 = 1180.3 \text{ V}$$

$$\therefore \omega_m' = \frac{E_a'}{(K_a\Phi)'} = \frac{1180.3}{13.07} = 90.29 \text{ rad/s} \quad \therefore \boxed{n' = 862.2 \text{ rpm}}$$

g) To get the output power we need the developed power and the rotational losses.

$$P_D' = E_a'I_a' = 1180.3 \times 197.3 = 232.9 \text{ kW}$$

and
$$P_{\text{rot}}' = 41.25 \times 10^3 \times \left(\frac{862.2}{2500}\right)^2 = 4.91 \text{ kW}$$

$$\therefore P_{\text{out}}' = P_D' - P_{\text{rot}}' = 228 \text{ kW} = \boxed{305.6 \text{ hp}}$$

h)
$$P_{\text{in}}' = 1200 \times 221\frac{1}{3} = 265.6 \text{ kW} \quad \therefore \eta' = \frac{228}{265.6} \times 100\% = \boxed{85.9\%}$$

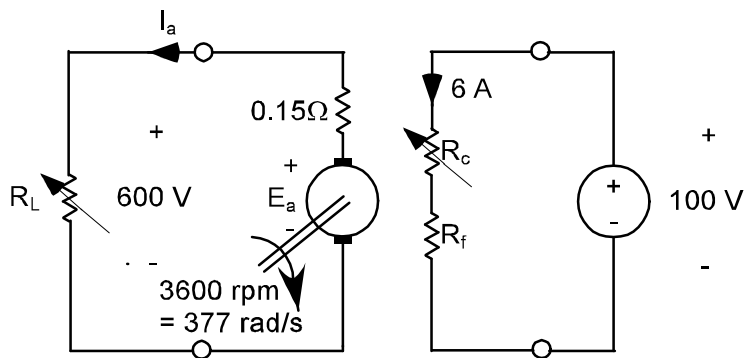
5.6 The following information is taken from the nameplate of a separately-excited dc generator.

Armature: 120 kW, 600 V, 0.15 Ω.
 Field: 100 V, 6.0 A

At its rated speed of 3600 rpm the no-load torque is measured at 16 Nm. Ignore armature reaction and magnetic saturation.

- a) For rated load conditions determine:
- Rated Shaft Horsepower
 - Rated Efficiency
 - Load Impedance
- b) Determine the terminal voltage, kW output and the efficiency if the field current is reduced to 4.5 A and the speed is reduced to 3000 rpm, causing the rotational losses to become 4 kW. The load impedance is unchanged.

a) i)



$$I_a = \frac{P_{out}}{V} = \frac{120 \times 10^3}{600} = 200 \text{ A}$$

$$E_a = 600 + 0.15 \times 200 = 630 \text{ V}$$

$$P_D = E_a I_a = 630 \times 200 = 126 \text{ kW}$$

$$\omega_m = 2\pi \times 3600/60 = 377 \text{ rad/s}$$

$$P_{rot} = 16 \times 377 = 6032 \text{ W} \quad P_{shaft} = P_D + P_{rot} = 126 + 6.032 = 132 \text{ kW} = \boxed{177 \text{ hp}}$$

ii) $P_{in} = P_{shaft} + P_{field} = 132 \times 10^3 + 100 \times 6 = 132.6 \text{ kW}$

$$\eta = \frac{P_{out}}{P_{in}} = \frac{120 \times 10^3}{132.6 \times 10^3} \times 100\% = \boxed{90.5\%}$$

ii) $R_L = \frac{V}{I_a} = \frac{600}{200} = \boxed{3 \Omega}$

b) $E_a = K_a \Phi \omega_m$ or $630 = K_a \Phi \times 377 \quad \therefore K_a \Phi = 1.6711$

$$(K_a \Phi)' = 1.6711 \times \frac{4.5}{6} = 1.2533 \quad \text{and} \quad \omega_m' = 2\pi \times 3000/60 = 314.2 \text{ rad/s}$$

$$\therefore E_a' = 1.2533 \times 314.2 = 393.7 \text{ V}$$

$$I_a = \frac{E_a}{R_a + R_L} = \frac{393.7}{0.15 + 3} = 125 \text{ A} \quad \text{and} \quad V' = I_a R_L = \boxed{375 \text{ V}}$$

$$P'_{out} = V' I_a = 375 \times 125 = \boxed{46.87 \text{ kW}}$$

$$P_D' = 393.7 \times 125 = 49.22 \text{ kW} \quad \therefore P'_{shaft} = 49.22 + 4 = 53.22 \text{ kW}$$

$$\therefore P'_{in} = 53.22 \times 10^3 + 100 \times 4.5 = 53.67 \text{ kW}$$

$$\therefore \eta = \frac{46.87}{53.67} \times 100\% = \boxed{87.3\%}$$