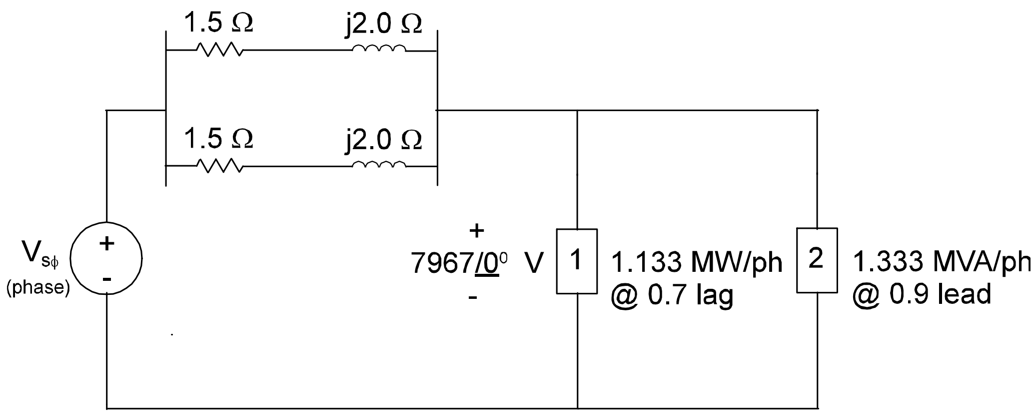
ECE370 POWER & ENERGY SYSTEMS
Homework Set 4 - Solutions

1. Two 3ϕ loads are fed from a 60 Hz supply via two parallel feeders each with impedance 1.5 + j2 Ω/phase. Load 1 is Y-connected, 3.4 MW, 0.7 pf **lag** and Load 2 is Δ-connected, 4 MVA, 0.9 pf **lead**. The voltage at the load is 13.8 kV. Determine:
   a) the real and reactive power of the combined loads,
   b) the power factor of the combined loads, (0.9709 lag)
   c) the percent voltage regulation when both feeders are in service, and
   d) the percent voltage regulation when one feeder is out of service. (7.49%)

The 1ϕ equivalent circuit is:



L1: $\frac{1.133}{0.7}\angle\cos^{-1}0.7 = 1.619\angle 45.6 = 1.133 + j1.1562\ \mathrm{MVA/ph}$

L2: $1.333\angle -\cos^{-1}0.9 = 1.333\angle -25.8 = 1.20 - j0.5812\ \mathrm{MVA/ph}$

Sum: $\boldsymbol{S} = 2.333 + j0.575 = 2.403\angle 13.8$ MVA/ph

a) $\boldsymbol{S_{tot}} = 3\boldsymbol{S} = 7.21\angle 13.8 =$ **7.0 + j1.725 MVA**

b) $pf_L = \cos(13.8) =$ **0.9709 lag**

c) With both feeders in service: $Z_f = 0.75 + j1.0$ Ω/ph

$$I = \frac{2.404\mathrm{x}10^6\angle -13.8}{7967\angle 0} = 301.6\angle -13.8\ \mathrm{A}$$

$$\boldsymbol{V_{s\phi}} = 7967\angle 0 + (0.75 + j1.0)\ \mathrm{x}\ 301.6\angle -13.8 = 8263\angle 1.7,\quad |V_{SL}| = 14.31\ \mathrm{kV}$$

$$VR = \frac{14.31 - 13.8}{13.8}\mathrm{x}100\% = \boxed{3.71\%}$$

d) With only one feeder in service: $Z_f = 1.5 + j2.0$ Ω/ph

$$I = \frac{2.404\mathrm{x}10^6\angle -13.8}{7967\angle 0} = 301.6\angle -13.8\ \mathrm{A}$$

$$\boldsymbol{V_{s\phi}} = 7967\angle 0 + (1.5 + j2.0)\ \mathrm{x}\ 301.6\angle -13.8 = 8564\angle 3.2,\quad |V_{SL}| = 14.83\ \mathrm{kV}$$

$$VR = \frac{14.83 - 13.8}{13.8}\mathrm{x}100\% = \boxed{7.49\%}$$

2. A factory is supplied at 69 kV from a 3ϕ, 60 Hz supply. It draws a continuous load of 72 MW with a pf of 0.6 lag. Determine:

a) The reactive power rating (kVAR) and capacitance (μF/phase) of a Y-connected bank that will improve the pf to 0.96 lag. (75 MVAR)

b) The annual saving in demand charge, if the monthly demand charge is \$10.00/kVA. (5.4 M\$/yr)

c) The annual saving in energy charge, if the energy charge is 8 ¢/kWh and the feeder resistance is 1.5 Ω/ph.

a)

$$S_L = \frac{72}{0.6} \times 10^6 \angle \cos^{-1} 0.6 = 72 + j96 \text{ MVA}$$

$$S_R = \frac{72}{0.96} \times 10^6 \angle \cos^{-1} 0.96 = 72 + j21 \text{ MVA}$$

$$\left.\right\} Q_C = j75 \text{ MVAR} = j25 \text{ MVAR/ph}$$

$$V_\phi = \frac{69}{\sqrt{3}} = 39.84 \text{ kV}$$

For a Y-connected bank: $X_{CY} = \dfrac{(39.84 \times 10^3)^2}{25 \times 10^6} = 63.48\ \Omega / ph$

$$C_Y = \frac{1}{377 \times 63.48} = \boxed{41.8\ \mu\text{F/ph}}$$

b) $|S_L| = 120 \times 10^3$ kVA $\qquad |S_R| = \dfrac{72}{0.96} \times 10^3 = 75 \times 10^3$ kVA

$\Delta S = 45 \times 10^3$ kVA $\quad \$_{saved} = 45 \times 10^3 \times 10 \times 12 = \boxed{5.4 \text{ M\$/yr}}$

c) Energy saving results from loss reduction, where: $\quad P_{loss} = 3 \times |I|^2 \times R$

Before: $\quad |I| = \dfrac{40 \times 10^6}{39.84 \times 10^3} = 1004$ A $\qquad P_{loss} = 3 \times 1004^2 \times 1.5 = 4.537$ MW

After: $\quad |I| = \dfrac{25 \times 10^6}{39.84 \times 10^3} = 627.6$ A $\qquad P_{loss} = 3 \times 625.6^2 \times 1.5 = 1.772$ MW

$\Delta P_{loss} = 2.765 \times 10^3$ kW $\quad \$_{saved} = 2.765 \times 10^3 \times 8760 \times 0.08 = \boxed{1.938 \text{ M\$/yr}}$

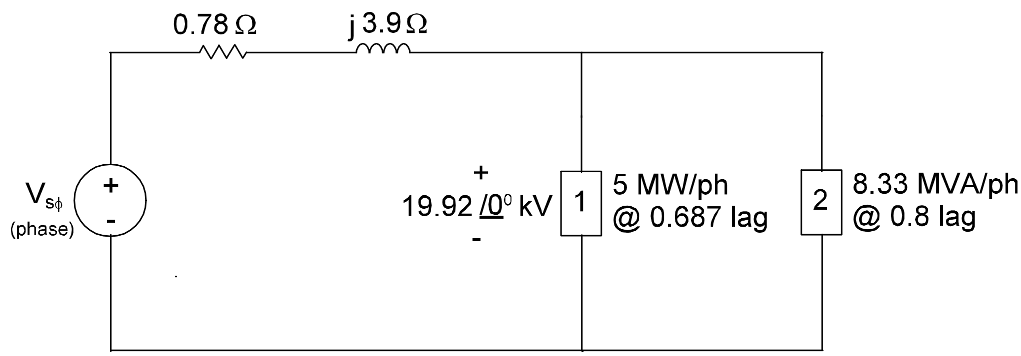
3. Two three-phase loads are supplied at 34.5 kV via a feeder rated at 540 A with an impedance of 0.78 + j3.9 Ω/phase. The first load is 15 MW @ 0.687 lag, and the second load is 25 MVA @ 0.8 lag. The line is found to be overloaded and it is decided that some load should be transferred to another feeder. Determine:

a) The % of the combined load current that should be transferred, so that the feeder will satisfy the rating given. (The pf of the load remains constant.)

b) The % voltage regulation after the load has been transferred. (8.78%)

c) The value of capacitance/phase of a Y-connected capacitor bank needed to improve the new load pf to 0.98 lag.

d) The % voltage regulation after the pf has been improved. (3.5%)



L1: $\frac{5}{0.687}\angle\cos^{-1}0.687 = 7.278\angle 46.6 = 5.0 + j5.289 \text{ MVA/ph}$

L2: $8.33\angle\cos^{-1}0.8 = 8.333\angle 36.9 = 6.67 + j5 \text{ MVA/ph}$

Sum: $\boldsymbol{S} = 11.667 + j10.289 = 15.56\underline{/41.4}$ MVA/ph

a) $\boldsymbol{I} = \frac{15.56x10^6\angle -41.4}{19.92x10^3\angle 0} = 780.9\angle -41.4\ A \quad \%_{trans} = \frac{780.9 - 540}{780.9}x100\% = \boxed{30.9\%}$

b) $\boldsymbol{V_{s\phi}} = 19.92 \times 10^3\underline{/0} + (0.78 + j3.9) \times 540\underline{/-41.4} = 21.67\underline{/3.4}$, $|V_{SL}| = 37.53$ kV

$$VR = \frac{37.53 - 34.5}{34.5}x100\% = \boxed{8.78\%}$$

c) $\boldsymbol{S_L} = 0.691 \times 15.56\underline{/41.4} = 10.75\underline{/41.4} = 8.063 + j7.11$ MVA/ph

$$S_R = \frac{8.063\angle\cos^{-1}0.98}{0.98} = 8.227\angle 11.5 \text{ MVA/ph} = 8.063 + j1.637$$

$Q_C = 7.11 - 1.637 = 5.47$ MVAR/ph $\quad X_{CY} = \frac{(19.92 \times 10^3)^2}{5.47x10^6} = 72.52\ \Omega/\text{ph}$

$$C_Y = \frac{1}{377 \times 72.52} = \boxed{36.6\ \mu F/ph}$$

d) $\boldsymbol{I} = \frac{8.227x10^6\angle -11.5}{19.92x10^3\angle 0} = 413\angle -11.5\ A$

$\boldsymbol{V_{s\phi}} = 19.92 \times 10^3\underline{/0} + (0.78 + j3.9) \times 413\underline{/-11.5} = 20.61\underline{/4.2}$, $|V_{SL}| = 35.7$ kV

$$VR = \frac{35.7 - 34.5}{34.5}x100\% = \boxed{3.48\%}$$

4.

a) Determine the necessary shaft speed (rpm) and torque (Nm) of a hydro-electric turbine that drives a 50 Hz, 32 pole synchronous generator, rated 22 MW if the generator's rated efficiency is 90%. (187.5 rpm, 1.25 MNm)

b) The governor of a 60 Hz, 4 pole turbo-alternator is set at 1746 rpm. Determine the percent error in the developed frequency. (3%)

a) The necessary shaft speed is the synchronous speed $\boxed{n_s = \frac{60 \text{ x } 50}{\left(32/2\right)} = 187.5 \quad \text{rpm}}$

Torque (T) is shaft power (P_s) divided by shaft speed (ω_s) in rad/s. The shaft power is the output power divided by efficiency, i.e.

$$P_S = \frac{22}{0.90} = 24.44 \quad \text{MW} \quad \text{and} \quad \omega_S = {2\pi \text{x} 187.5}/{60} = 19.64 \quad \text{rad/s}$$

Then: $$\boxed{T = \frac{24.44 \text{x} 10^6}{19.64} = 1.25 \quad \text{MNm}}$$

b) Since the machine is 4 pole, 60 Hz, the synchronous speed is:

$n_s = \frac{60 \text{ x } 60}{\left(4/2\right)} = 1800 \quad \text{rpm}$, this is the speed that will produce 60 Hz.

Since the actual shaft speed is 1746 rpm, the frequency is **low** and the error is given by:

$$\boxed{\text{Error} = \frac{1746 - 1800}{1800} \text{x} 100\% = -3\%}$$

5. One of the powerhouses of a hydroelectric development has three penstocks, each passes 400 m^3/s of water with velocity 30 m/s when the average head behind the dam is 54 m. The generators operate at 0.96 lag pf and the electricity is transmitted at 230 kV on two parallel transmission lines.

a) Calculate the penstock efficiency.

b) Assuming the coefficient of performance of the turbine is 0.53 and the generator efficiency is 94⅓%, what is the generated electrical power for the powerhouse?

c) Calculate the magnitude of the current in the individual transmission lines.

a) Since: $\eta_{pen} = \dfrac{v^2}{2gH} = \dfrac{30^2}{2 \text{ x } 9.81 \text{ x } 54} = \boxed{85\%}$

b) $P_{elec} = \eta_{gen} C_P P_{mech}$

and $P_{mech} = \dfrac{m_{rate} v^2}{2} = \dfrac{400 \text{ x } 10^3 \text{ x } 30^2}{2} = 180 \text{ MW / Penstock}$

Then: P_{elec} = 0.9433 x 0.53 x 180 = 90 MW for each penstock, resulting in:

$P_{total} = 3 \text{ x } 90 = \boxed{270 \text{ MW}}$ for the powerhouse.

c) The Apparent Power is given by: $S = \dfrac{270}{0.96} = 281.2 \text{ MVA}$

The total current is: $I = \dfrac{281.2 \text{ x } 10^6}{\sqrt{3} \text{ x } 230 \text{ x } 10^3} = 706 \text{ A}$

Half of this flows in each circuit giving: $\boxed{353 \text{ A}}$

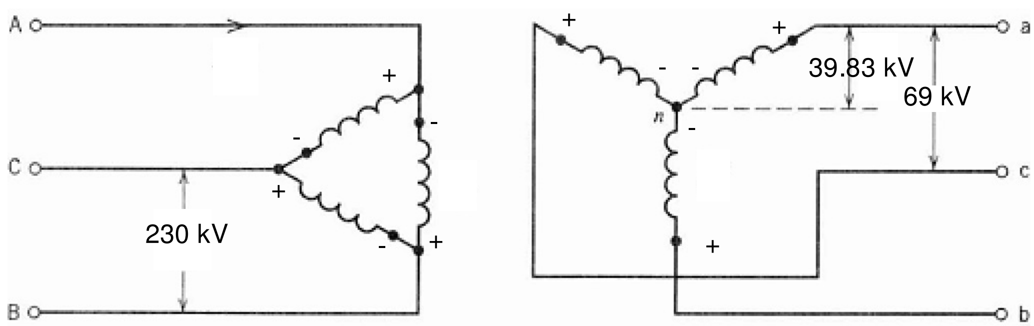
6. You have a requirement for a three-phase transformer to handle 100 MVA with a voltage ratio of 69 kV : 230 kV. You are provided with three single-phase transformers each rated 40 MVA with voltage ratios of 47.8 kV : 276 kV.

a) Draw a diagram showing how each transformer should be connected. Be sure to indicate the voltage applied to each winding.

b) By what percentage is the bank underloaded or overloaded?

a) Given: $a = \frac{276}{47.8} = 5.774$ and required $a' = \frac{230}{69} = 3.333$

Then $\frac{a}{a'} = \sqrt{3}$ $\therefore$ Δ–Y required. The configuration is shown below.



b) For 100 MVA @ 69 kV $|I_L| = \frac{100 \times 10^6}{\sqrt{3} \times 69 \times 10^3} = 836.7 \text{ A}$

Transformer secondary rating $|I_S| = \frac{40 \times 10^6}{47.8 \times 10^3} = 836.8 \text{ A}$

Load Ratio = $\frac{837}{837} = 1.0$

Transformers are at rated load

7. a) Calculate the volume of U^{235} required to produce an average of 750 MW of electricity annually, if the plant is 40% efficient.

b) Compare this with a coal power plant that employs a condenser that extracts 6 kWh for each kg of burned fuel.

Given: $\rho_{Ur} = 20 \times 10^3$ kg/m³, $\rho_{coal} = 4 \times 10^3$ kg/m³, $TEC_{coal} = 8$ kWh/kg

a) Since one joule requires 31×10^9 fission events, one watt will require 31×10^9 fission events per second.

750 MW requires $750 \times 10^6 \times (31 \times 10^9) = 23.25 \times 10^{18}$ fission events per second.

750 MW for 1 h requires $3600 \times (23.25 \times 10^{18}) = 83.7 \times 10^{21}$ fission events.

750 MW for 1 year requires $8760 \times (83.7 \times 10^{21}) = 733.2 \times 10^{24}$ fission events if the plant is 100% efficient. For 40% efficiency this becomes 1.833×10^{27}.

Since 1 kg of U^{235} can have 2.54×10^{24} fission events, the mass of fuel needed for the reactor annually is:

$$\text{Mass of } U^{235} \text{ annually} = \frac{1.833 \times 10^{27}}{2.54 \times 10^{24}} = 721.7 \text{ kg}$$

$$\text{Volume of } U^{235} \text{ annually} = \frac{721.7}{20 \times 10^3} = \boxed{0.0361 \text{ m}^3}$$

b) For the coal plant: $W_{Coal} = 8 - 6 = 2$ kWh/kg $\quad \eta_{ideal} = \frac{2}{8} = 25\%$

1 kg of coal contains 8 kWh of thermal energy and since the plant is 25% efficient, we only get 2 kWh/kg as output. To produce 750 MW of thermal power for one hour, we need to burn 375×10^3 kg of coal.

To produce 750 MW of thermal power annually, we need to burn:

$8760 \times 375 \times 10^3 = 3.285 \times 10^9$ kg of coal.

$$\text{Volume of Coal annually} = \frac{3.285 \times 10^9}{4000} = \boxed{821.25 \times 10^3 \text{ m}^3}$$