ECE370 POWER & ENERGY SYSTEMS
Homework Set 3 – Solutions

3.1

a) A coal-fired power plant employs a condenser that extracts 16 MJ for each kg of burned fuel. A neighboring oil-fired power plant employs a condenser that extracts 17 MJ for each kg of burned fuel. Calculate the mechanical energy of their turbines and their ideal efficiencies for:

i) Petroleum with TEC = 40 MJ/kg   ii) Coal with TEC = 24 MJ/kg

b) Which plant will produce cheaper electricity if petroleum costs 75¢/kg and coal costs \$250 per tonne if both generating units have the same efficiency (equal η_{gen}).

a) i) For Petroleum: $W_{Pet} = 40 - 17 = \boxed{23 \text{ MJ/kg}}$ $\eta_{ideal} = \frac{W_{Pet}}{Q_1} = \frac{23}{40} = \boxed{57.5\ \%}$

ii) For Coal: $W_{Coal} = 24 - 16 = \boxed{8 \text{ MJ/kg}}$ $\eta_{ideal} = \frac{W_{Coal}}{Q_1} = \frac{8}{24} = \boxed{33\tfrac{1}{3}\ \%}$

b) 1 kg of petroleum produces 23 MJ and costs 75 ¢, so the energy cost is 3.261 ¢/MJ.

Coal costs \$250/tonne, which is 25 ¢/kg.

1 kg of coal produces 8 MJ and costs 25 ¢, so the energy cost is 3.125 ¢/MJ.

Coal is 95.8% the price of Oil. (Coal is $4\tfrac{1}{6}$% cheaper.)

3.2

a) Calculate the mass of U^{235} required to produce an average of 750 MW of electricity annually, if the plant is 33⅓% efficient.

b) Compare this with the equivalent mass of coal for a 25% efficient plant.

c) Determine the ratio of volume of coal : volume of uranium.

Assume: $\rho_{Ur} = 21 \times 10^3$ kg/m^3, $\rho_{coal} = 4\frac{1}{4} \times 10^3$ kg/m^3, $TEC_{coal} = 6.0$ kWh/kg

Since one joule requires 31×10^9 fission events, one watt will require 31×10^9 fission events per second.

Then 750 MW requires $750 \times 10^6 \times (31 \times 10^9) = 23.25 \times 10^{18}$ fission events per second.

750 MW for 1 h requires $3600 \times (23.25 \times 10^{18}) = 83.7 \times 10^{21}$ fission events.

750 MW for 1 year requires $8760 \times (83.7 \times 10^{21}) = 733.2 \times 10^{24}$ fission events.

The number of annual fission events is 33.2×10^{24} if the plant is 100% efficient. For 33⅓% efficiency this becomes 2.2×10^{27}.

Since 1 kg of U^{235} can have 2.54×10^{24} fission events, the fuel needed for the reactor annually is:

a) Mass of U^{235} annually = $\dfrac{2.2 \times 10^{27}}{2.54 \times 10^{24}} = 866\ kg$

b) If 1 kg of coal contains 6.0 kWh of thermal energy and the plant is 25% efficient, we only get 1.5 kWh/kg as output. To produce 750 MW of thermal power for one hour, we need to burn 500×10^3 kg of coal.

To produce 750 MW of thermal power annually, we need to burn:

$8760 \times 500 \times 10^3 = 4.38 \times 10^9$ kg of coal.

c) Mass of coal/mass of uranium = 5.06×10^6

$$\frac{Volume\ Coal}{Volume\ Uranium} = 5.06 \times 10^6 \times \frac{21}{4.25} = \boxed{25 \times 10^6 : 1}$$

3.3 A 250 kVA, 3600/240 V, single-phase transformer has the following test data:

	Voltage (V)	Current (A)	Power (W)
O/C Test	240	57.85	4985
S/C Test	187	69.45	4823

Use the approximate equivalent circuit to calculate:

a) The voltage regulation and efficiency when the load takes 1100 A at 220 V and 0.6 lag pf. (NOTE: this is ***not*** rated load).

b) The voltage regulation and efficiency at rated load conditions and 0.8 lag pf.

From the OC Test:

$$pf_{oc} = \frac{4985}{240\ x\ 57.85} = 0.359 \text{ lag} \qquad \text{then:} \quad \theta_{oc} = \cos^{-1} 0.359 = 69^0$$

$\boldsymbol{I_{oc}} = 57.85\angle\text{-}69 = 20.77 - j54$ A

$$R_c = \frac{240}{20.77} = 11.55\ \Omega \qquad \text{and:} \quad X_m = \frac{240}{54} = 4.445\ \Omega$$

These values are referred to the *lv* side. Referring to the *hv* side gives:

$R_c = 2600\ \Omega$ and $X_m = 1000\ \Omega$

From the SC Test:

$$pf_{sc} = \frac{4823}{187\ x\ 69.45} = 0.3714 \text{ lag} \qquad \text{then:} \quad \theta_{sc} = \cos^{-1} 0.3713 = 68.2^0$$

$\boldsymbol{I'_2} = 69.45\angle\text{-}68.2$ A

$$Z = \frac{187\angle 0}{69.45\angle -68.2} = 2.693\angle 68.2\ \Omega \qquad \text{then convert to rectangular to get:}$$

R = 1.0 Ω and X = 2.5 Ω, which are referred to the *hv* side

a) $\boldsymbol{I_2} = 1100\angle\text{-}53.1$ A $\therefore$ $\boldsymbol{I'_2} = 73⅓\angle\text{-}53.1$ A $\quad \boldsymbol{V_2} = 220\angle 0$ V $\quad \boldsymbol{V'_2} = 3300\angle 0^0$

$\boldsymbol{V_p} = 3300\angle 0^0 + (1.0 + j2.5) \text{ x } 73⅓\angle\text{-}53.1^0 = 3491\angle 0.8^0$ V

$$VR = \frac{3491 - 3300}{3300} x100\% = 5.79\%$$

$$I_p = I'_2 + \frac{V_P}{R_c} + \frac{V_p}{jX_m} = 73.33\angle -53.1 + \frac{3491\angle 0.8}{2600} + \frac{3491\angle 0.8}{j1000} = 76.95\angle\text{-}53.9^0 \text{ A}$$

$P_{in} = \text{Re}\{3491\angle 0.8 \text{ x } 76.95\angle{+53.9}\} = 155.3$ kW

and $P_{out} = 220 \text{ x } 1100 \text{ x } 0.6 = 145.2$ kW

$$\eta = \frac{145.2}{155.3} x100\% = 93.5\%$$

b) $\boldsymbol{I_2} = \frac{250 \, x \, 10^3}{240} \angle - \cos^{-1} 0.8 = 1042\angle -36.9^0$ A $\quad \therefore \quad \boldsymbol{I'_2} = \frac{I_2}{a} = 69.44\angle -36.9^0$ A

$\boldsymbol{V_p} = 3600\angle 0^0 + (1.0 + j2.5) \text{ x } 69.44\angle -36.9^0 = 3761\angle 1.5^0$ V

$$VR = \frac{3761 - 3600}{3600} x 100\% = 4.47\%$$

$$I_p = I_2^{'} + \frac{V_P}{R_c} + \frac{V_p}{jX_m} = 69.44\angle -36.9 + \frac{3761\angle 1.5}{2600} + \frac{3761\angle 1.5}{j1000} = 72.94\angle -38.5^0 \text{ A}$$

$P_{in} = \text{Re}\{3761\angle 1.5 \text{ x } 72.94\angle +38.5^0\} = 210.3$ kW

and $P_{out} = 250 \text{ x } 10^3 \text{ x } 0.8 = 200$ kW

$$\eta = \frac{200}{210.3} x 100\% = 95.1\%$$

3.4 A hydroelectric development has two identical powerhouses each one has three penstocks and each penstock passes 200 m^3/s of water with velocity 35 m/s when the average head behind the dam is 83.25 m. The generators operate at 0.9185 lag pf and the electricity is transmitted at 345 kV on three parallel transmission lines.

a) Calculate the penstock efficiency.

b) Assuming the coefficient of performance of the turbine is 0.6 and the generator efficiency is 95¼%, what is the total generated real electrical power for the development?

c) Calculate the magnitude of the current in the individual transmission lines.

a) Since: $$\eta_{pen} = \frac{v^2}{2gH} = \frac{35^2}{2 \times 9.81 \times 83.25} = \boxed{75\%}$$

b) $$P_{elec} = \eta_{gen} C_P P_{mech}$$

and $$P_{mech} = \frac{m_{rate} v^2}{2} = \frac{200 \times 10^3 \times 35^2}{2} = 122.5\ MW / Penstock$$

Then: P_{elec} = 0.9525 x 0.6 x 122.5 = 70 MW for each penstock, resulting in:

P_{total} = 6 x 70 = $\boxed{420\ \text{MW}}$ for the development.

c) The Apparent Power is given by: $$S = \frac{420}{0.9185} = 457.3\ MVA$$

The total current is: $$I = \frac{457.3 \times 10^6}{\sqrt{3} \times 345 \times 10^3} = 765.2\ A$$

A third of this flows in each circuit giving: $\boxed{255\ \text{A/circuit}}$

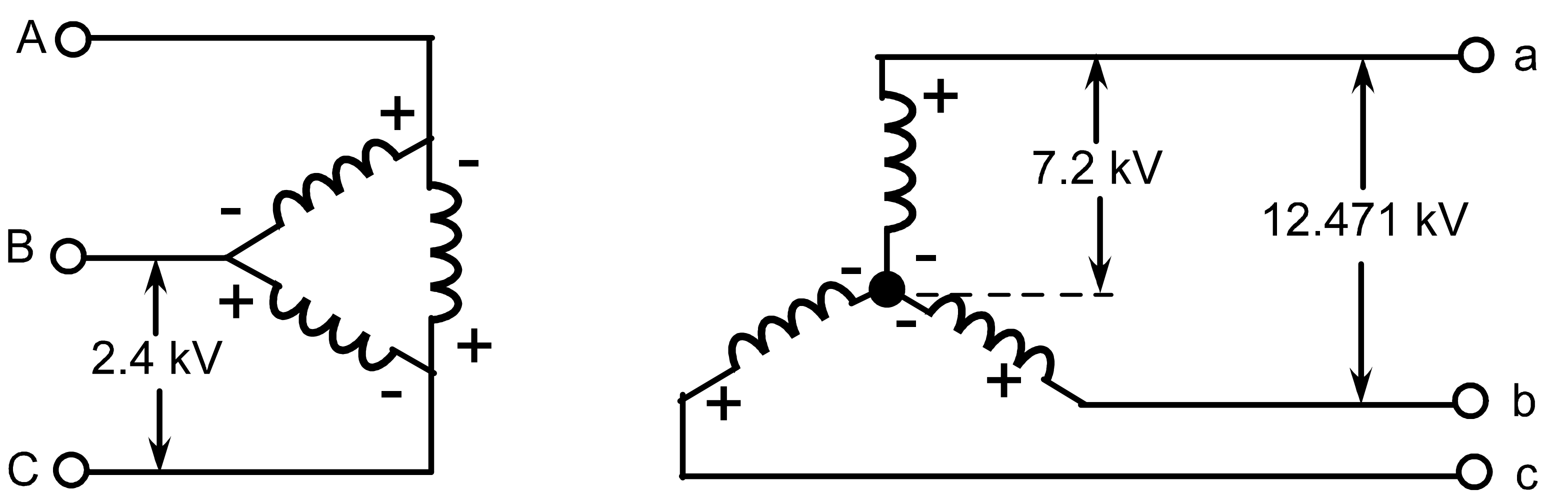
3.5 You have a requirement for a three-phase transformer to handle 10 MVA with a voltage ratio of 2.4 kV: 12.471 kV kV. You are provided with three single-phase transformers each rated 15.75 MVA with voltage ratios of 12.471 kV : 37.4 kV.

a) Draw a diagram showing how each transformer should be connected. Be sure to indicate the voltage applied to each winding. The tolerance on voltage is $\pm$0.25 kV.

b) By what percentage is the bank underloaded or overloaded?

a) Given: $a = \dfrac{12.471}{37.4} = \dfrac{1}{3}$ and required $a' = \dfrac{2.4}{12.471} = 0.1924$

Then $\dfrac{a}{a'} = \sqrt{3}$ $\therefore$ Δ–Y required. The configuration is shown below.

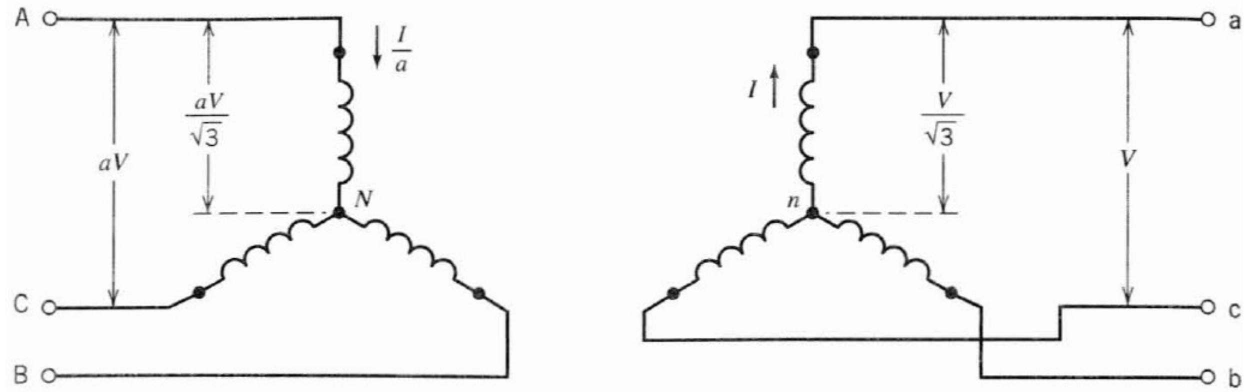


b) For 10 MVA @ 12.47 kV $|I_L| = \dfrac{10\ x\,10^6}{\sqrt{3}\ x\,12.47\ x\,10^3} = 463\ A$

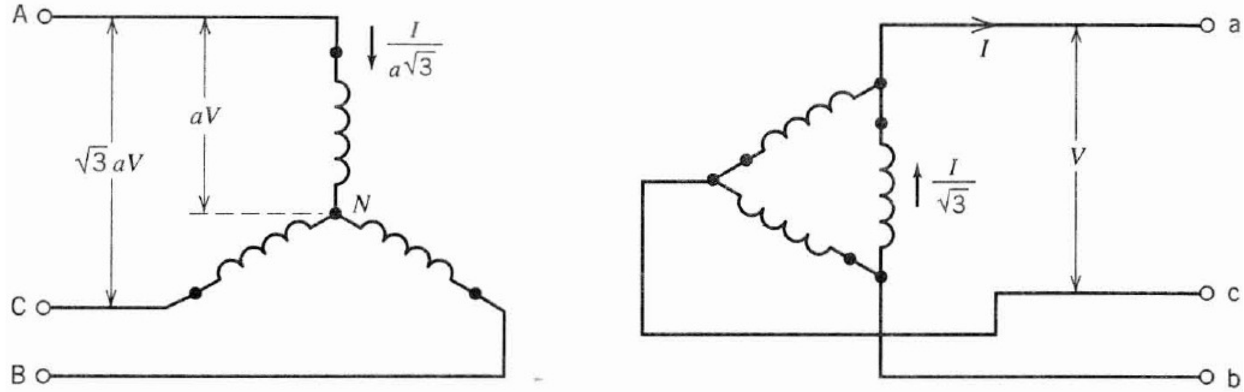
Transformer primary rating $|I_S| = \dfrac{15.75\ x\,10^6}{37.4\ x\,10^3} = 421.1\ A$

Load Ratio = $\dfrac{463}{421.1} = 1.099$

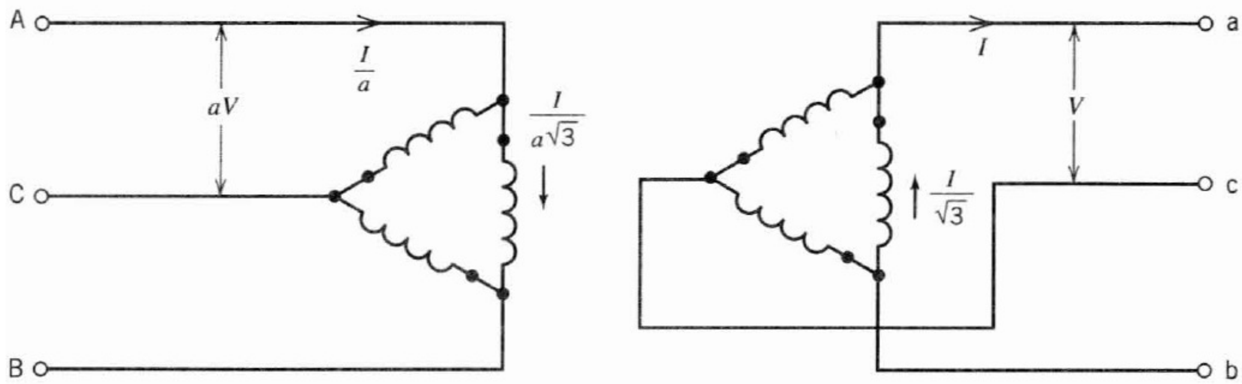
Transformers are 9.9% overloaded



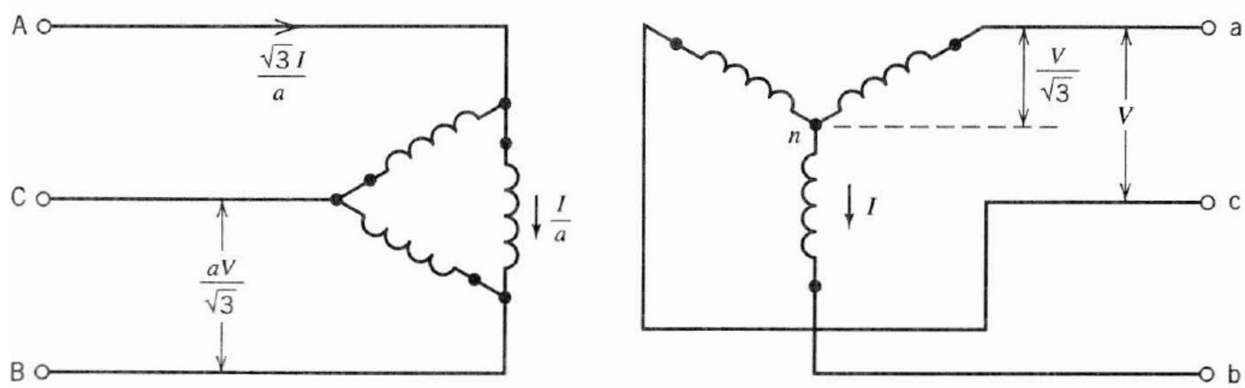
(a) Y – Y



(b) Y – Δ



(c) Δ – Δ



(d) Δ – Y